PROBLEMS

1. Indicate which of the following is an all-integer linear program and which is a mixed-integer linear program. Write the LP Relaxation for the problem but do not attempt to solve.

   a. Max $30x_1 + 25x_2$

   s.t.

   $$3x_1 + 1.5x_2 \le 400$$
   $$1.5x_1 + 2x_2 \le 250$$
   $$1x_1 + 1x_2 \le 150$$
   $$x_1, x_2 \ge 0 \text{ and } x_2 \text{ integer}$$

   b. Min $3x_1 + 4x_2$

   s.t.

   $$2x_1 + 4x_2 \ge 8$$
   $$2x_1 + 6x_2 \ge 12$$
   $$x_1, x_2 \ge 0 \text{ and integer}$$

SELF test

2. Consider the following all-integer linear program.

   Max $5x_1 + 8x_2$

   s.t.

   $$6x_1 + 5x_2 \le 30$$
   $$9x_1 + 4x_2 \le 36$$
   $$1x_1 + 2x_2 \le 10$$
   $$x_1, x_2 \ge 0 \text{ and integer}$$

   a. Graph the constraints for this problem. Use dots to indicate all feasible integer solutions.

   b. Find the optimal solution to the LP Relaxation. Round down to find a feasible integer solution.

   c. Find the optimal integer solution. Is it the same as the solution obtained in part (b) by rounding down?

3. Consider the following all-integer linear program.

   Max $1x_1 + 1x_2$

   s.t.

   $$4x_1 + 6x_2 \le 22$$
   $$1x_1 + 5x_2 \le 15$$
   $$2x_1 + 1x_2 \le 9$$
   $$x_1, x_2 \ge 0 \text{ and integer}$$

   a. Graph the constraints for this problem. Use dots to indicate all feasible integer solutions.

   b. Solve the LP Relaxation of this problem.

   c. Find the optimal integer solution.

4. Consider the following all-integer linear program.

   Max $10x_1 + 3x_2$

   s.t.

   $$6x_1 + 7x_2 \le 40$$
   $$3x_1 + 1x_2 \le 11$$
   $$x_1, x_2 \ge 0 \text{ and integer}$$

a. Formulate and solve the LP Relaxation of the problem. Solve it graphically, and round down to find a feasible solution. Specify an upper bound on the value of the optimal solution.
b. Solve the integer linear program graphically. Compare the value of this solution with the solution obtained in part (a).
c. Suppose the objective function changes to Max $3x_1 + 6x_2$. Repeat parts (a) and (b).

SELF test

5. Consider the following mixed-integer linear program.

$$
\begin{aligned}
\text{Max} \quad & 2x_1 + 3x_2 \\
\text{s.t.} \quad & \\
& 4x_1 + 9x_2 \le 36 \\
& 7x_1 + 5x_2 \le 35 \\
& x_1, x_2 \ge 0 \text{ and } x_1 \text{ integer}
\end{aligned}
$$

a. Graph the constraints for this problem. Indicate on your graph all feasible mixed-integer solutions.
b. Find the optimal solution to the LP Relaxation. Round the value of x_1 down to find a feasible mixed-integer solution. Is this solution optimal? Why or why not?
c. Find the optimal solution for the mixed-integer linear program.

6. Consider the following mixed-integer linear program.

$$
\begin{aligned}
\text{Max} \quad & 1x_1 + 1x_2 \\
\text{s.t.} \quad & \\
& 7x_1 + 9x_2 \le 63 \\
& 9x_1 + 5x_2 \le 45 \\
& 3x_1 + 1x_2 \le 12 \\
& x_1, x_2 \ge 0 \text{ and } x_2 \text{ integer}
\end{aligned}
$$

a. Graph the constraints for this problem. Indicate on your graph all feasible mixed-integer solutions.
b. Find the optimal solution to the LP Relaxation. Round the value of x_2 down to find a feasible mixed-integer solution. Specify upper and lower bounds on the value of the optimal solution to the mixed-integer linear program.
c. Find the optimal solution to the mixed-integer linear program.

SELF test

7. The following questions refer to a capital budgeting problem with six projects represented by 0-1 variables x_1, x_2, x_3, x_4, x_5, and x_6.
a. Write a constraint modeling a situation in which two of the projects 1, 3, 5, and 6 must be undertaken.
b. Write a constraint modeling a situation in which, if projects 3 and 5 must be undertaken, they must be undertaken simultaneously.
c. Write a constraint modeling a situation in which project 1 or 4 must be undertaken, but not both.
d. Write constraints modeling a situation where project 4 cannot be undertaken unless projects 1 and 3 also are undertaken.
e. Revise the requirement in part (d) to accommodate the case in which, when projects 1 and 3 are undertaken, project 4 also must be undertaken.

8. Spencer Enterprises must choose among a series of new investment alternatives. The potential investment alternatives, the net present value of the future stream of returns, the capital requirements, and the available capital funds over the next three years are summarized as follows:

		Capital Requirements ($)		
Alternative	**Net Present Value ($)**	**Year 1**	**Year 2**	**Year 3**
Limited warehouse expansion	4,000	3,000	1,000	4,000
Extensive warehouse expansion	6,000	2,500	3,500	3,500
Test market new product	10,500	6,000	4,000	5,000
Advertising campaign	4,000	2,000	1,500	1,800
Basic research	8,000	5,000	1,000	4,000
Purchase new equipment	3,000	1,000	500	900
Capital funds available		10,500	7,000	8,750

a. Develop and solve an integer programming model for maximizing the net present value.
b. Assume that only one of the warehouse expansion projects can be implemented. Modify your model of part (a).
c. Suppose that, if test marketing of the new product is carried out, the advertising campaign also must be conducted. Modify your formulation of part (b) to reflect this new situation.

9. Hawkins Manufacturing Company produces connecting rods for 4- and 6-cylinder automobile engines using the same production line. The cost required to set up the production line to produce the 4-cylinder connecting rods is $2000, and the cost required to set up the production line for the 6-cylinder connecting rods is $3500. Manufacturing costs are $15 for each 4-cylinder connecting rod and $18 for each 6-cylinder connecting rod. Hawkins makes a decision at the end of each week as to which product will be manufactured the following week. If a production changeover is necessary from one week to the next, the weekend is used to reconfigure the production line. Once the line has been set up, the weekly production capacities are 6000 6-cylinder connecting rods and 8000 4-cylinder connecting rods. Let

x_4 = the number of 4-cylinder connecting rods produced next week
x_6 = the number of 6-cylinder connecting rods produced next week
s_4 = 1 if the production line is set up to produce the 4-cylinder connecting rods; 0 if otherwise
s_6 = 1 if the production line is set up to produce the 6-cylinder connecting rods; 0 if otherwise

a. Using the decision variables x_4 and s_4, write a constraint that limits next week's production of the 4-cylinder connecting rods to either 0 or 8000 units.
b. Using the decision variables x_6 and s_6, write a constraint that limits next week's production of the 6-cylinder connecting rods to either 0 or 6000 units.
c. Write three constraints that, taken together, limit the production of connecting rods for next week.
d. Write an objective function for minimizing the cost of production for next week.