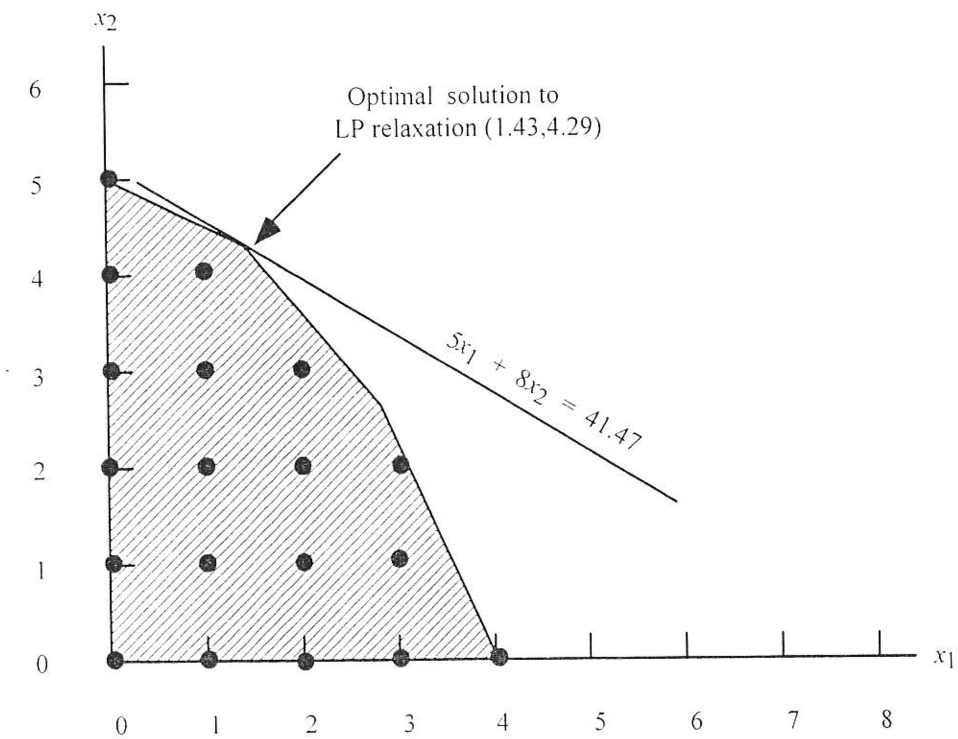
Solutions:

1. a. This is a mixed integer linear program. Its LP Relaxation is

$$\begin{aligned} \text{Max} \quad & 30x_1 + 25x_2 \\ \text{s.t.} \quad & \\ & 3x_1 + 1.5x_2 \le 400 \\ & 1.5x_1 + 2x_2 \le 250 \\ & x_1 + x_2 \le 150 \\ & x_1, x_2 \ge 0 \end{aligned}$$

   b. This is an all-integer linear program. Its LP Relaxation just requires dropping the words "and integer" from the last line.

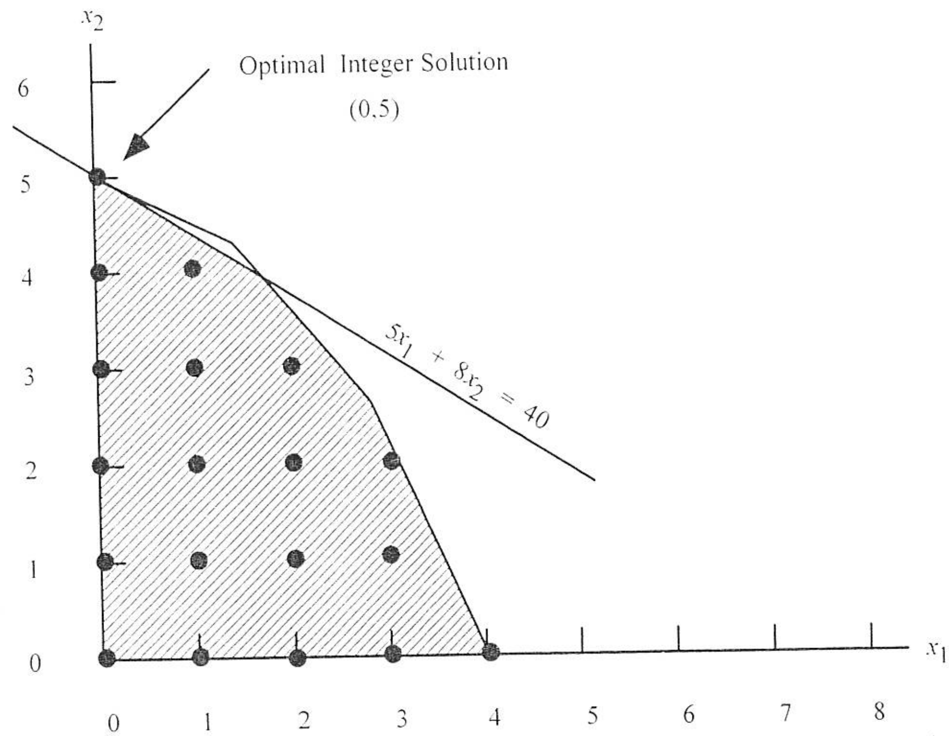
2. a.



   b. The optimal solution to the LP Relaxation is given by $x_1 = 1.43$, $x_2 = 4.29$ with an objective function value of 41.47.

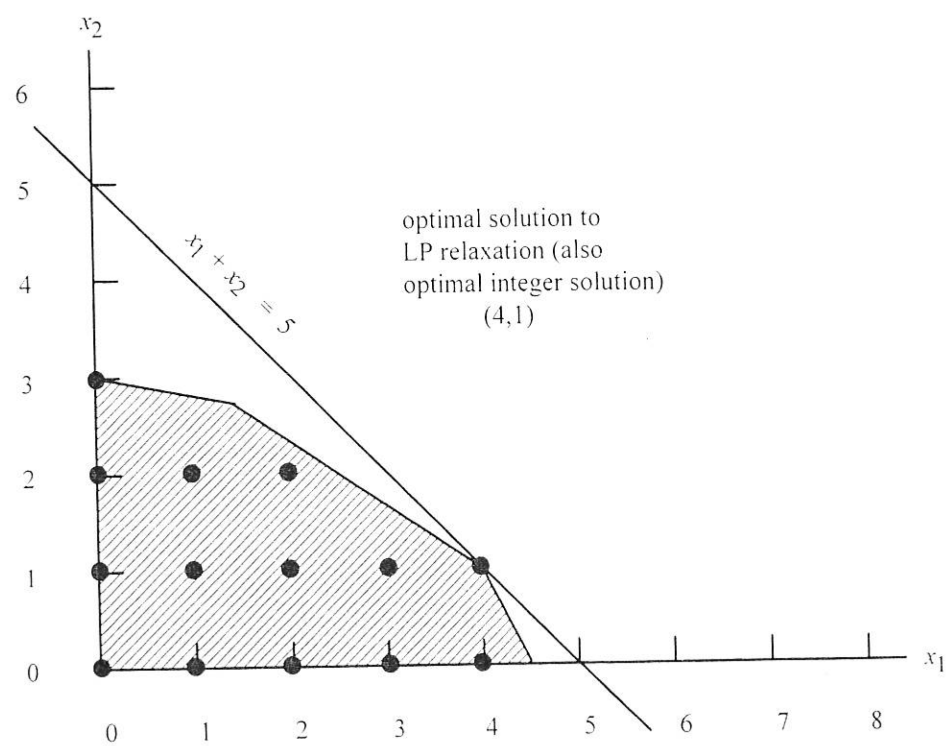
   Rounding down gives the feasible integer solution $x_1 = 1$, $x_2 = 4$. Its value is 37.

c.



The optimal solution is given by $x_1 = 0, x_2 = 5$. Its value is 40. This is not the same solution as that found by rounding down. It provides a 3 unit increase in the value of the objective function.

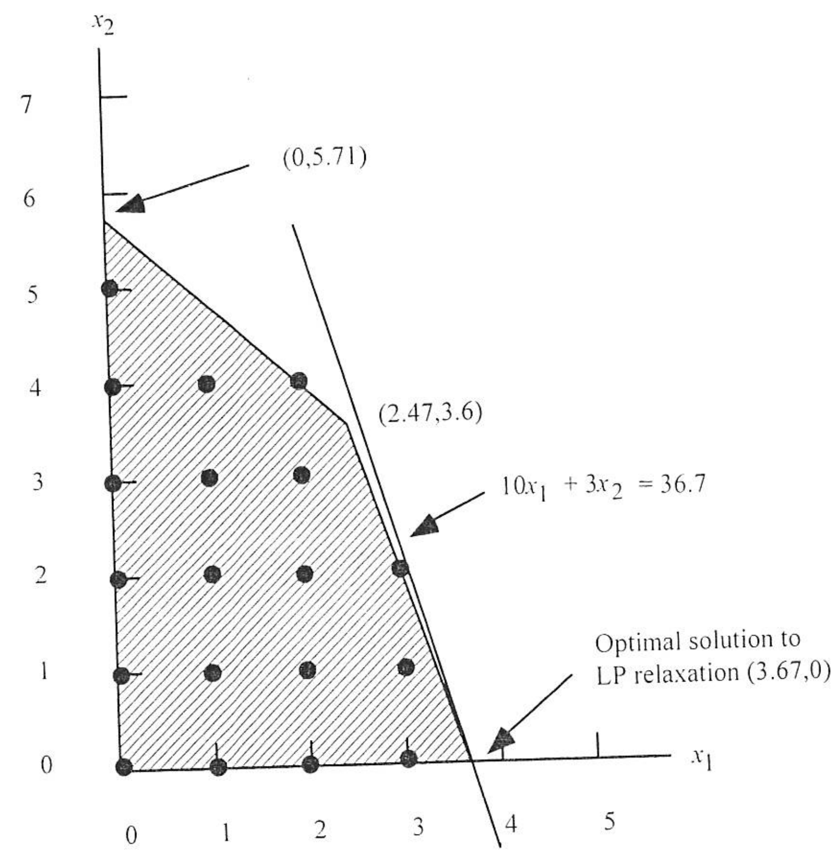
3. a.



b. The optimal solution to the LP Relaxation is shown on the above graph to be $x_1 = 4$, $x_2 = 1$. Its value is 5.

c. The optimal integer solution is the same as the optimal solution to the LP Relaxation. This is always the case whenever all the variables take on integer values in the optimal solution to the LP Relaxation.

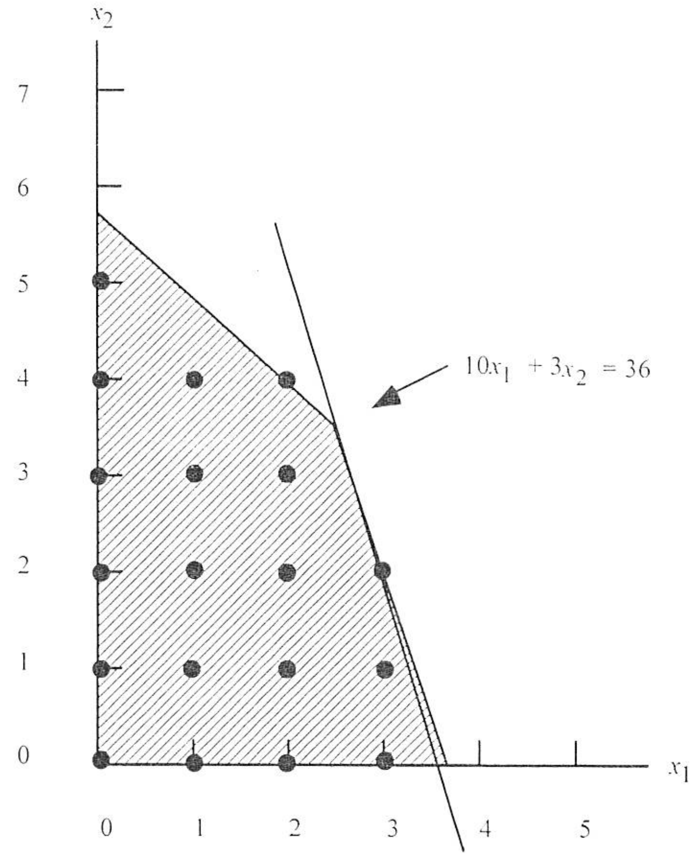
4. a.



The value of the optimal solution to the LP Relaxation is 36.7 and it is given by $x_1 = 3.67$, $x_2 = 0.0$. Since we have all less-than-or-equal-to constraints with positive coefficients, the solution obtained by "rounding down" the values of the variables in the optimal solution to the LP Relaxation is feasible. The solution obtained by rounding down is $x_1 = 3$, $x_2 = 0$ with value 30.

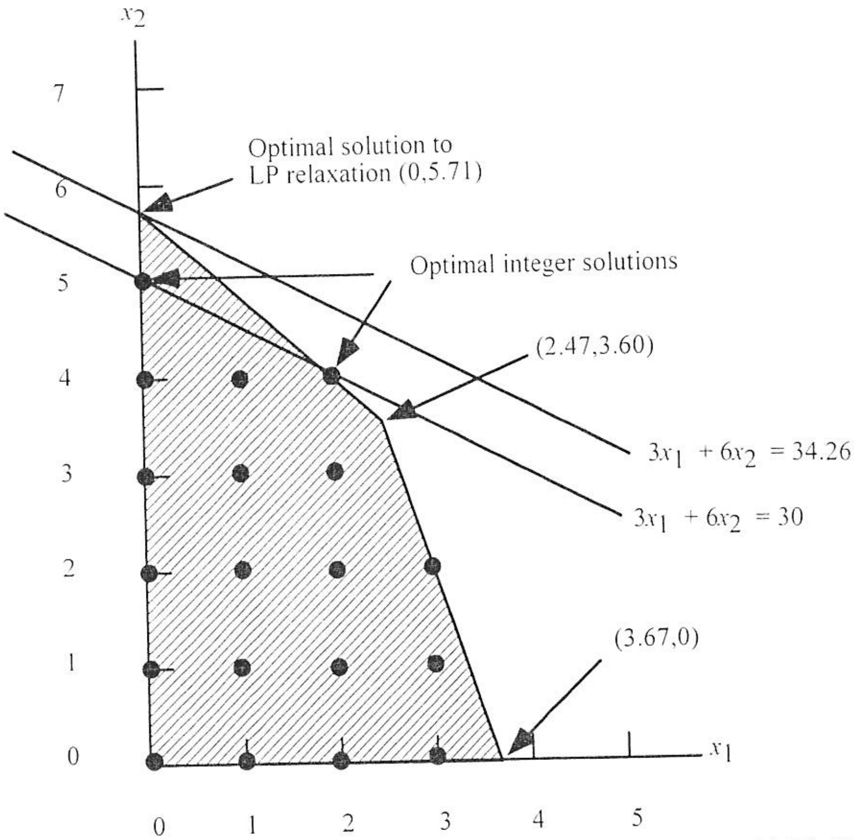
Thus a lower bound on the value of the optimal solution is given by this feasible integer solution with value 30. An upper bound is given by the value of the LP Relaxation, 36.7. (Actually an upper bound of 36 could be established since no integer solution could have a value between 36 and 37.)

b.



The optimal solution to the ILP is given by $x_1 = 3$, $x_2 = 2$. Its value is 36. The solution found by "rounding down" the solution to the LP relaxation had a value of 30. A 20% increase in this value was obtained by finding the optimal integer solution - a substantial difference if the objective function is being measured in thousands of dollars.

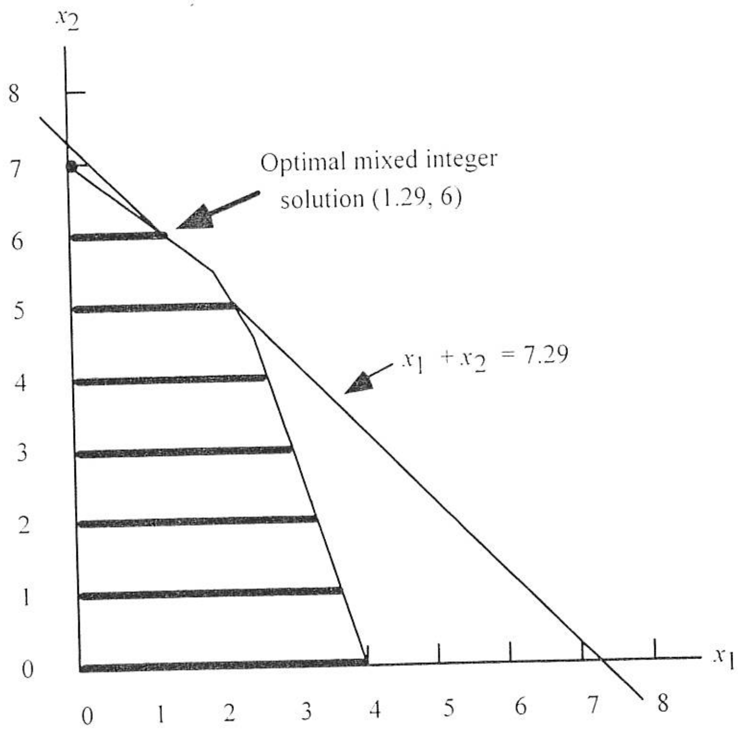
c.



The optimal solution to the LP Relaxation is $x_1 = 0$, $x_2 = 5.71$ with value = 34.26. The solution obtained by "rounding down" is $x_1 = 0$, $x_2 = 5$ with value 30. These two values provide an upper bound of 34.26 and a lower bound of 30 on the value of the optimal integer solution.

There are alternative optimal integer solutions given by $x_1 = 0$, $x_2 = 5$ and $x_1 = 2$, $x_2 = 4$; value is 30. In this case rounding the LP solution down does provide the optimal integer solution.

c.



The optimal solution to the MILP is $x_1 = 1.29$, $x_2 = 6$. Its value is 7.29.

The solution $x_1 = 2.22$, $x_2 = 5$ is almost as good. Its value is 7.22.

7. a. $x_1 + x_3 + x_5 + x_6 = 2$

b. $x_3 - x_5 = 0$

c. $x_1 + x_4 = 1$

d. $x_4 \leq x_1$

$x_4 \leq x_3$

e. $x_4 \leq x_1$

$x_4 \leq x_3$

$x_4 \geq x_1 + x_3 - 1$

8. a. Let $x_i = \begin{cases} 1 \text{ if investment alternative } i \text{ is selected} \\ 0 \text{ otherwise} \end{cases}$

$$\begin{array}{lrcrcrcrcrcrcl}
\max & 4000x_1 & + & 6000x_2 & + & 10500x_3 & + & 4000x_4 & + & 8000x_5 & + & 3000x_6 & & \\
\text{s.t.} & & & & & & & & & & & & & \\
& 3000x_1 & + & 2500x_2 & + & 6000x_3 & + & 2000x_4 & + & 5000x_5 & + & 1000x_6 & \leq & 10{,}500 \\
& 1000x_1 & + & 3500x_2 & + & 4000x_3 & + & 1500x_4 & + & 1000x_5 & + & 500x_6 & \leq & 7{,}000 \\
& 4000x_1 & + & 3500x_2 & + & 5000x_3 & + & 1800x_4 & + & 4000x_5 & + & 900x_6 & \leq & 8{,}750
\end{array}$$

$$x_1,\ x_2,\ x_3,\ x_4,\ x_5,\ x_6 = 0,\ 1$$

Optimal Solution found using *The Management Scientist* or LINDO

$x_3 = 1$
$x_4 = 1$
$x_6 = 1$

Value = 17,500

b. The following mutually exclusive constraint must be added to the model.

$x_1 + x_2 \leq 1$ No change in optimal solution.

c. The following co-requisite constraint must be added to the model in b.

$x_3 - x_4 = 0$. No change in optimal solution.

9. a. $x_4 \leq 8000\ s_4$

b. $x_6 \leq 6000\ s_6$

c. $x_4 \leq 8000\ s_4$
$x_6 \leq 6000\ s_6$
$s_4 + s_6 = 1$

d. Min $15\ x_4 + 18\ x_6 + 2000\ s_4 + 3500\ s_6$