

# A New Communication-Efficient Privacy-Preserving Range Query Scheme in Fog-Enhanced IoT Rongxing Lu

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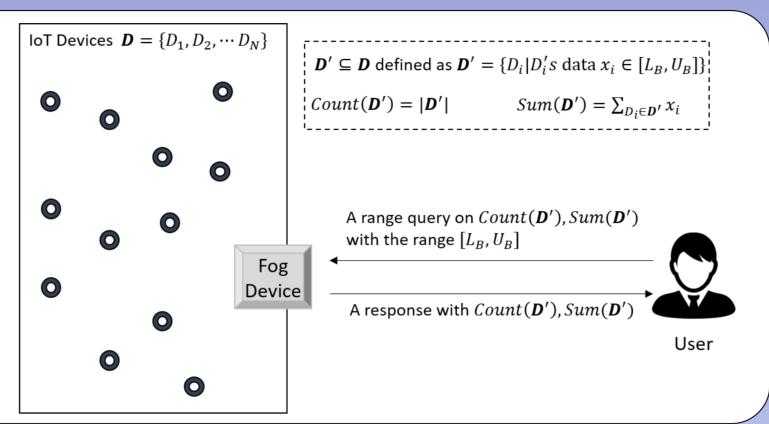
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# ABSTRACT

Fog-enhanced Internet of Things (IoT) has received considerable attention in recent years, as fog devices deployed at the network edge can not only improve the performance of IoT applications, but also enhance the security and privacy of IoT. In this work, we present a new communication-efficient privacy-preserving range query scheme in Fog-enhanced IoT. With the proposed scheme, both the query range and individual IoT device's data can be privacy-preserved by using BGN homomorphic encryption technique. In addition, the proposed scheme employs a range query expression, decomposition, and composition technique to reorganize the range query, which can achieve  $O(\sqrt{n})$  communication efficiency. Extensive experiments are conducted, and the results indicate that the proposed scheme is efficient in terms of communication overhead.

## System Model

- **IoT devices**  $D = \{D_1, \dots, D_N\}$ : Each  $D_i \in D$  is equipped with sensing and periodically sends the sensed data  $w_i \in [1, n]$  to the fog device.
- Fog device: The fog device can process the data collected from IoT devices and handle the range query request from the query user.
- Query user: A query user will launch range queries to the fog device and gain the desirable result from the fog device.



## **Design Goals**

- The proposed scheme should be privacy-preserving,
  i.e., the query range [L<sub>B</sub>, U<sub>B</sub>] and the elements of subset D' should be privacy-preserving.
- The proposed scheme should be communication efficient, i.e., achieving  $\sqrt{n}$  query communication efficiency.

A. Range Query Expression, Decomposition, and Composition

(1) Range Query Expression

A range query  $[L_B, U_B]$   $(1 \le L_B \le U_B \le n)$  is first represented to an array A[1..n] as shown in Fig. 1 and then reorganized into an  $m \times m$   $(n = m^2)$  matrix R as

 $R(i,j) = \begin{cases} 1 & L_B \le k = (i-1) \times m + j \le U_B \\ 0 & otherwise \end{cases}$ 

### (2) Range Query Decomposition

Rol is a row in which not all elements are 0s or 1s;

Bol is a set of continuous rows in which all elements are 1s; Then, matrix *R* can be decomposed as the following steps. **Step-1**: Break down *R* into three matrices  $R_1$ ,  $R_2$  and  $R_3$  such that  $R = R_1 \lor R_2 \lor R_3$ , where  $R_1$  and  $R_3$  include at most one Rol respectively, and  $R_2$  includes at most one Bol.

**Step-2**: Decompose  $R_w$  into two matrices  $R_{1-X_wY_w}$  and  $R_{X'_wY'_w}$  such that  $R_w = R_{1-X_wY_w} \wedge R_{X'_wY'_w}$  for w = 1,2,3.

• Generate  $R_{1-X_WY_W}$  matrix: Set  $X_w = (x_{w1}, x_{w2}, \dots, x_{wm})$ with the row rule: if the *i*-th row in  $R_w$  are all 1s, set  $x_{wi} =$ 0 and set  $x_{wi} = 1$  otherwise. Set  $Y_w = (y_{w1}, y_{w2}, \dots, y_{wm})$ with the column rule: if Rol (Bol) in  $R_w$  has an element 1 in column *j*, set  $y_{wj} = 0$ ; and set  $y_{wj} = 1$  otherwise. Then,

 $R_{1-X_{w}Y_{w}}(i,j) = 1 - x_{wi}y_{wj}$ 

• Generate  $R_{X'_wY'_w}$  matrix: Set  $X'_w = (x'_{w1}, x'_{w2}, \dots, x'_{wm})$  with the row rule: if Rol (Bol) in  $R_w$  has an element 1 in row *i*, set  $x'_{wi} = 1$ ; and set  $x'_{wi} = 0$  otherwise. Set  $Y'_w = (y'_{w1}, y'_{w2}, \dots, y'_{wm}) = (1, 1, \dots, 1)$ . Then,

 $R_{X'_{w}Y'_{w}}(i,j) = x'_{wi}y'_{wj}$ 

## (3) Range Query Composition

The matrix *R* can be recovered by twelve vectors  $(X_1, Y_1, X'_1, Y'_1, X_2, Y_2, X'_2, Y'_2, X_3, Y_3, X'_3, Y'_3)$  as  $R(i, j) = R_1(i, j) \lor R_2(i, j) \lor R_3(i, j)$ 

- $= \vee_{w=1}^{3} \left( R_{1-X_{w}Y_{w}}(i,j) \wedge R_{X'_{w}Y'_{w}}(i,j) \right)$
- $= \vee_{w=1}^{3} \left( (1 x_{wi} y_{wj}) \wedge x'_{wi} y'_{wj} \right)$

 $= \sum_{w=1}^{3} ((1 - x_{wi} y_{wj}) \cdot x'_{wi} y'_{wj})$ 

Since all elements in vectors  $(X_1, X_3, Y'_1, Y'_2, Y'_3)$  are 1s and all elements in the  $Y_2$  are all 0s when  $R_2$  includes one BoI. Then,  $R(i,j) = \sum_{w=1}^{3} ((1 - x_{wi}y_{wj}) \cdot x'_{wi}y'_{wj})$ 

 $= (1 - y_{1j}) \cdot x'_{1i} + x'_{2i} + (1 - y_{3j}) \cdot x'_{3i}$  $= \begin{cases} 1 & \text{within the query range} \\ 0 & \text{otherwise} \end{cases}$ 

Thus, the matrix *R* related to query can be recovered by  $(Y_1, X'_1, X'_2, Y_3, X'_3)$  with size  $O(5m) = O(\sqrt{n})$ .

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
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$n = m^2$ for example, $n = 36, m = 6$ <b>1-XY Matrix:</b> Row Rule If all 1s in a row, set 0; else set 1. Column Rule If Rol (Bol) has 1 in a column, set 0; else set 1.																																			
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$X_1 = (1,1,1,1,1,1)$ $X'_1 = (0,1,0,0,0,0)$ $Y_1 = (1,1,0,0,0,0)$ $Y'_1 = (1,1,1,1,1,1)$											$X_2 =$ $Y_2 =$	-			-		$X'_{2} = (0,0,1,1,0,0)$ $Y'_{2} = (1,1,1,1,1,1)$						$X_3 = (1,1,1,1,1,1) Y_3 = (0,0,0,1,1,1)$						$X'_{3} = (0,0,0,0,1,0)$ $Y'_{3} = (1,1,1,1,1,1)$						
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**Fig. 1**. An example for range query expression, decomposition, and composition

## B. The proposed privacy-preserving range query scheme in fog-enhanced IoT

(1) Query User Key Generation: the query user generates the BGN public key pk and private key sk.

**Step-1**:  $D_k$  converts the sensed data  $w_k$  into (i, j) such that

 $w_k = (i-1) \times m + j$ 

 $C \xrightarrow{aec} Count(D') = |D'| = \sum R_k(i,j)$ 

#### (2) Range Query Generation at Query User

Represent the range query to a matrix *R* and apply the decomposition rules to prepare vectors  $(Y_1, X'_1, X'_2, Y_3, X'_3)$ . Then, the query user computes

 $\overline{Y_1} = (\overline{y_{11}} = 1 - y_{11}, \overline{y_{12}} = 1 - y_{12}, \cdots, \overline{y_{1m}} = 1 - y_{1m},)$   $\overline{Y_3} = (\overline{y_{31}} = 1 - y_{31}, \overline{y_{32}} = 1 - y_{32}, \cdots, \overline{y_{3m}} = 1 - y_{3m},)$ Since R(i,j) =  $(1 - y_{1j}) \cdot x'_{1i} + x'_{2i} + (1 - y_{3j}) \cdot x'_{3i}$  $= \overline{y_{1j}} \cdot x'_{1i} + x'_{2i} + \overline{y_{3j}} \cdot x'_{3i}$ 

Then, *R* can be represented by  $(\overline{Y}_1, X'_1, X'_2, \overline{Y}_3, X'_3)$ .

- Use BGN to encrypt these vectors as  $(E(\overline{Y_1}), E(X'_1), E(X'_2), E(\overline{Y_3}), E(X'_3)).$
- Send  $(E(\overline{Y_1}), E(X'_1), E(X'_2), E(\overline{Y_3}), E(X'_3))$  as a query to all IoT devices via the fog device.

## (3) Query Response at IoT Device

Each  $D_k$  with sensed data  $w_k$  performs the following steps.

**Step-2**:  $D_k$  picks up  $E(\overline{y_{1j}}), E(x'_{1i}), E(x'_{2i}), E(\overline{y_{3j}}), E(x'_{3i})$ , and chooses two random numbers  $r_{k_1}$  and  $r_{k_2}$ . Then, it computes  $c_k = e(E(\overline{y_{1j}}), E(x'_{1i})) \cdot e(E(x'_{2i}), g) \cdot e(E(\overline{y_{3j}}), E(x'_{3i})) \cdot e(g, h)^{r_{k_1}}$  $= E_T(\overline{y_{1j}}x'_{1i} + x'_{2i} + \overline{y_{3j}}x'_{3i}) = E_T(R_k(i, j))$  $s_k = c_k^{w_k} \cdot e(g, h)^{r_{k_2}} = E_T(R_k(i, j))^{w_k} \cdot e(g, h)^{r_{k_2}} = E_T(R_k(i, j) \cdot w_k)$ **Step-3**:  $D_k$  forwards  $(c_k, s_k)$  to the fog device.

## (4) Response Aggregation at Fog Device

After receiving all  $(c_k, s_k)$  from all  $D_k \in \mathbf{D}$ , the fog device computes

$$S \mid C = \prod_{D_k \in \mathbf{D}} c_k = E_T(\sum_{D_k \in \mathbf{D}} R_k(i,j))$$

 $S = \prod_{D_k \in D} s_k = E_T(\sum_{D_k \in D} R_k(i, j) \cdot w_k)$ and returns (*C*, *S*) as the response to the query user.

## (5) Response Recovery at Query User

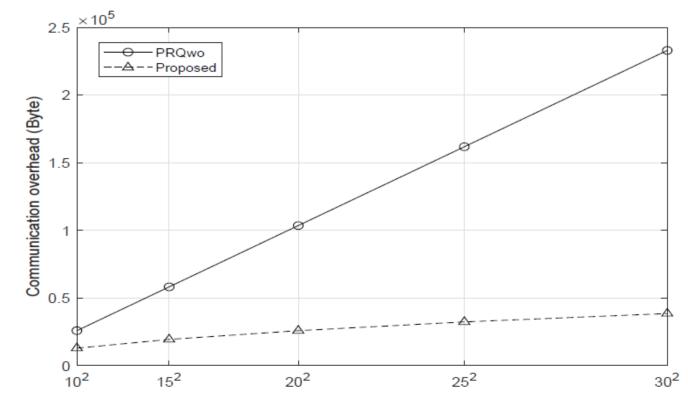
On receiving (C,S), the query user uses the private key sk to recover the query results as

 $D_k \in D$ 

$$S \xrightarrow{dec} Sum(D') = \sum_{D_i \in D'} w_i = \sum_{D_k \in D} R_k(i,j) \cdot w_k$$

#### **Communication Overhead Analysis**

The query user just sends 5 encrypted *m*-dimensional vectors to the fog device. Therefore, the communication overhead of the query user is  $O(5m) = O(\sqrt{n})$ .



**Fig. 2**. Communication overhead comparisons between the proposed scheme and traditional scheme

\* This work has been accepted by IEEE Internet of Things Journal in 2018.