Linear Programs Size Reduction via Feature Sharing in Sparktope Compiler

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Motivation

- Building polynomial Linear Program (LP) models for problems that only have Exponential Extension Complexity (Rothvoss [1])

$$\mathrm{EP}_n = \mathrm{CH}\{x \in \{0,1\}^{\binom{n}{2}} : x \text{ is the edge-vector of a perfect matching in } K_n\}$$

$$\boxed{\sum_{ij \in \delta(S)} x_{ij} \geqslant 1, \quad S \subseteq V,\ |S| \geqslant 3,\ |S| \text{ is odd}}$$

$$\sum_{ij \in \delta(i)} x_{ij} = 1 \quad i = 1, 2, \ldots, n$$

$$0 \leqslant x_{ij} \leqslant 1, \quad 1 \leqslant i < j \leqslant n$$

Exponential

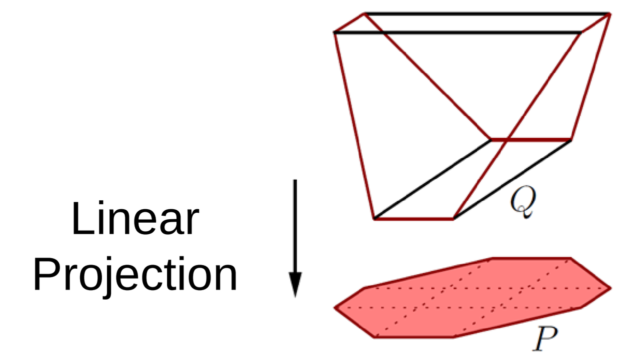
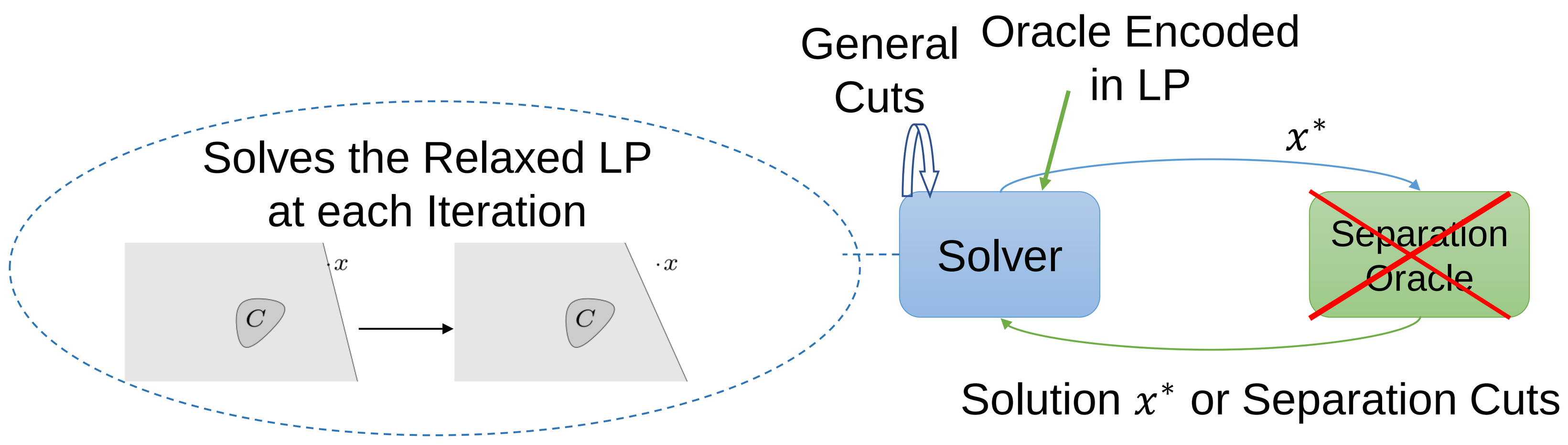


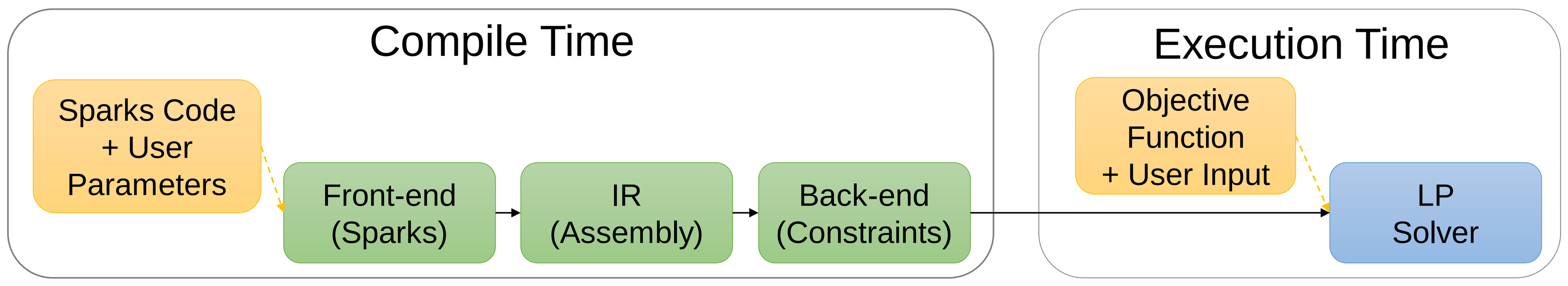
Fig1: Extended Formulation [2]

- Possibility of modeling Integer Programs (IP) as a single compact IP that has a polynomial time oracle encoded in the LP



- Modeling LPs through more intuitive higher level programming languages in comparison to Algebraic Modeling Systems (AMS)

Sparktope Compiler



- Q is Weak Extended Formulation (WEF) if
  - x−0/1 property
  - If "yes" $z^* = m + d$

$$Q = \{(x, w, s) : x \in [0,1]^q, w \in [0,1], s \in [0,1]^r, Ax + bw + Cs \leqslant h\}$$

$$z^* = \max\ \{c^T x + dw : (x, w, s) \in Q\}$$

Where $c_j = \begin{cases} 1 & \text{if } \bar{x}_j = 1 \\ -1 & \text{if } \bar{x}_j = 0 \end{cases} \qquad 0 < d \leqslant 1/2$

Problem

- Sparktope produces extremely large LPs for reasonably small codes which passes solver's limit on the number of constraints.

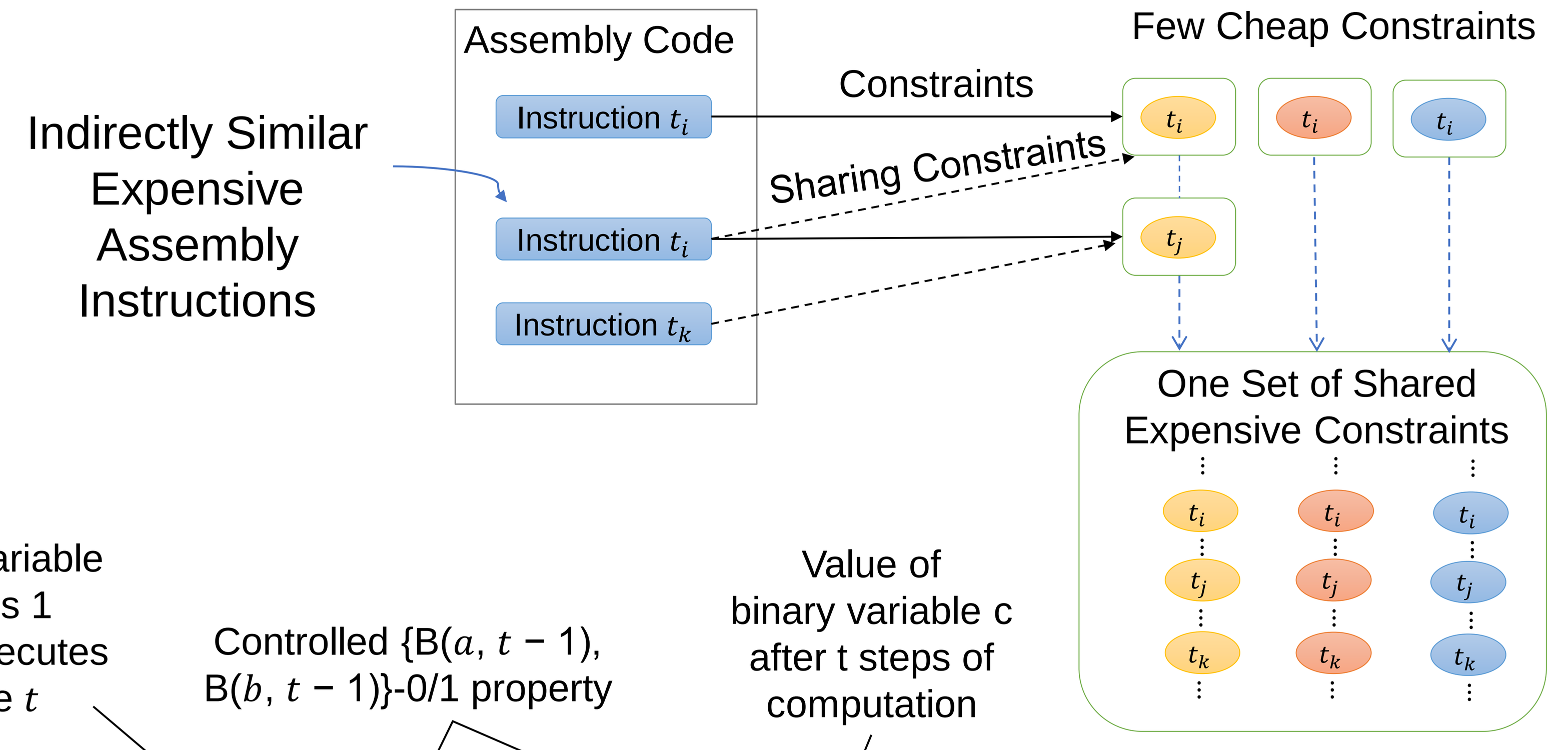
name	n	max steps	main.LB	init.UB	rows	columns	non-zeros	GB
mm8.lp	8	4000 (9747)	307	393	21,490,809	2,567,920	80,568,489	1.4 (3.4)
mm10.lp	10	7000 (19629)	472	611	54,809,388	5,354,967	210,572,706	3.6 (11)
mm12.lp	12	10000 (34771)	673	877	94,860,776	8,200,011	371,213,800	6.3 (23)
mm16.lp	16	16000 (83003)	1183	1553	212,451,096	14,288,092	854,715,828	15 (80)

Table1: LPs produced for the Maximum Matching problem with n nodes [3]

Proposed Methods

Reduce the size of LP by sharing different LP and Sparks features.

- **Constraint sharing**
  - Controlled x-0/1 property and Unique Execution Step constraints



Control variable $S(i,t)$ is 1 if line i executes at time t

Controlled {B(a, t − 1), B(b, t − 1)}-0/1 property

Value of binary variable c after t steps of computation

Assembly: set c and a b

LP:

$$S(i,t) + B(a,t-1) + B(b,t-1) - B(c,t) \le 2$$
$$S(i,t) - B(a,t-1) + B(c,t) \le 1$$
$$S(i,t) - B(b,t-1) + B(c,t) \le 1$$

- **Time sharing**
  - Multiple Clocks in the LP model for Semi-independent Blocks of Code (SIB)

$$S(1,1) = 1$$

$$\sum_{i=1}^{l} S(i,t) = 1$$

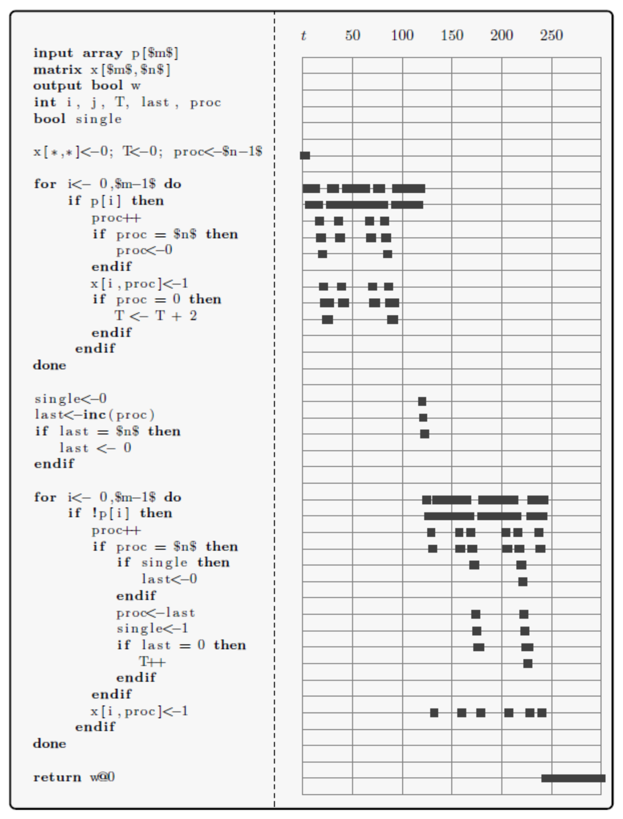
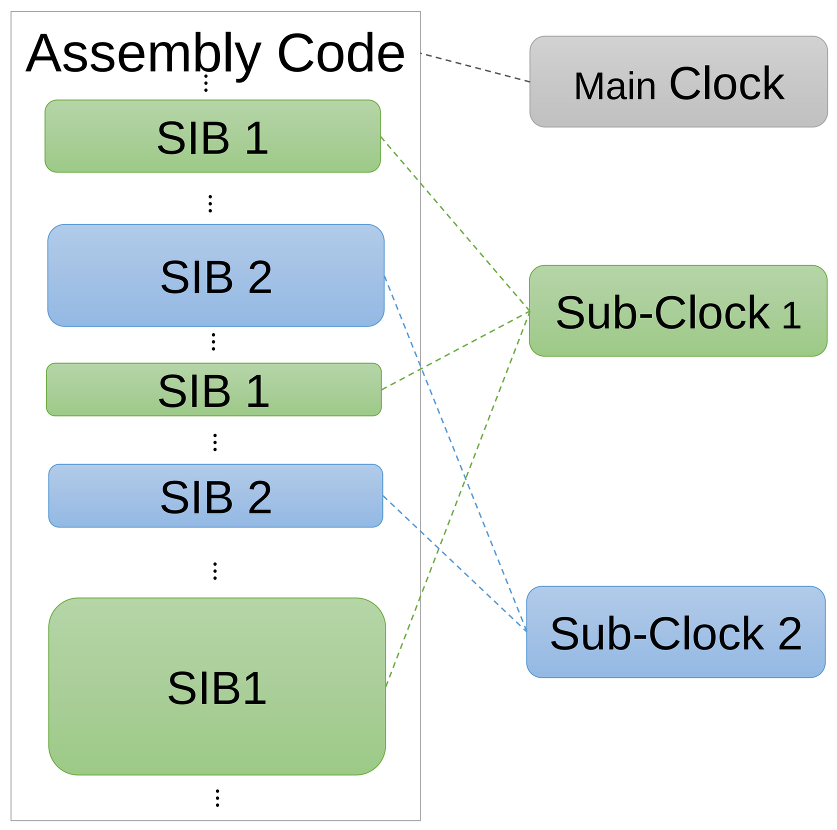
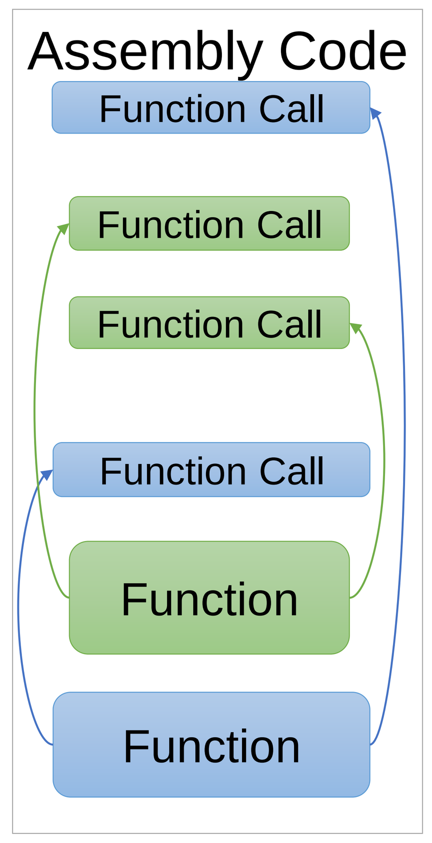


Fig2: Trace of run $S(i,t)$ [3]



- **Code sharing**
  - Allow functions by introducing new constraints for Goto statements based on register values
  - Eliminate expensive stack structure for non-recursive functions

$$S(i,t) - S(k,t+1) \leqslant 0$$



References

[1] Rothvoss, T. (2017) 'The Matching Polytope has Exponential Extension Complexity', Journal of the ACM, 64(6), p. 41:1-41:19. Available at: https://doi.org/10.1145/3127497.

[2] Fiorini, S. et al. (2012) 'Combinatorial Bounds on Nonnegative Rank and Extended Formulations'. arXiv. Available at: https://doi.org/10.48550/arXiv.1111.0444.

[3] Avis, D. and Bremner, D. (2020) 'Sparktope: linear programs from algorithms'. arXiv. Available at: https://doi.org/10.48550/arXiv.2005.02853.

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