Optimization of Quantum Circuits

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Introduction

Quantum algorithms are implemented with quantum circuits by cascading quantum gates. The initial circuits obtained from algorithms are often large and it requires further optimization to minimize the costs of compution in quantum devices. This work analyzes an optimization heuristic [1] to extend, and validates the results by simulating circuits with **IBM quantum computer** [2]. The algorithm shows outperforming results.

Literature Review

The fundamental information in quantum computation is the state of a qubit. A **state** is represented by a vector in a two-dimensional complex space [3]. A generic qubit can be in supperposition of states $|0\rangle$ and $|1\rangle$ written as

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \quad (1)$$

where α and β are probability amplitudes and $|\alpha|^2 + |\beta|^2 = 1$. The states $|0\rangle$ and $|1\rangle$ are computational basis. They can be visualized as the north and south poles of Bloch Sphere of radius 1.

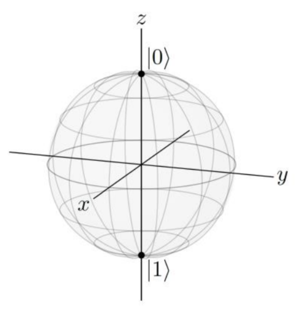


Figure 1. Representation of quantum states in Bloch Sphere.

Two qubits in **superposition** of states is expressed as

$$|\psi\rangle = \lambda_1|00\rangle + \lambda_2|01\rangle + \lambda_3|10\rangle + \lambda_4|11\rangle = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{pmatrix}. \quad (2)$$

The state $|\psi\rangle$ is separable if $\lambda_1\lambda_4 = \lambda_2\lambda_3$. Otherwise, the state is **entangled**. **Gates** are state manipulation devices represented by uniterary matrices such as in Table 1.

Table 1. Quantum gates.

Gate	Symbols	Uniterary Matrices
$Hadamard$	H	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
NOT	X	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$CNOT$		$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

The **quantum circuit** shown in Figure 2 realizes an entanled state $|\gamma\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

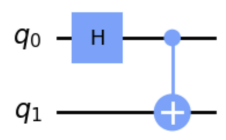


Figure 2. A circuit realizes entangled states.

Research Objectives

- To analyze and validate template matching algorithms
- To simulate practical problems with quantum computer
- To identify area to improve the algorithms

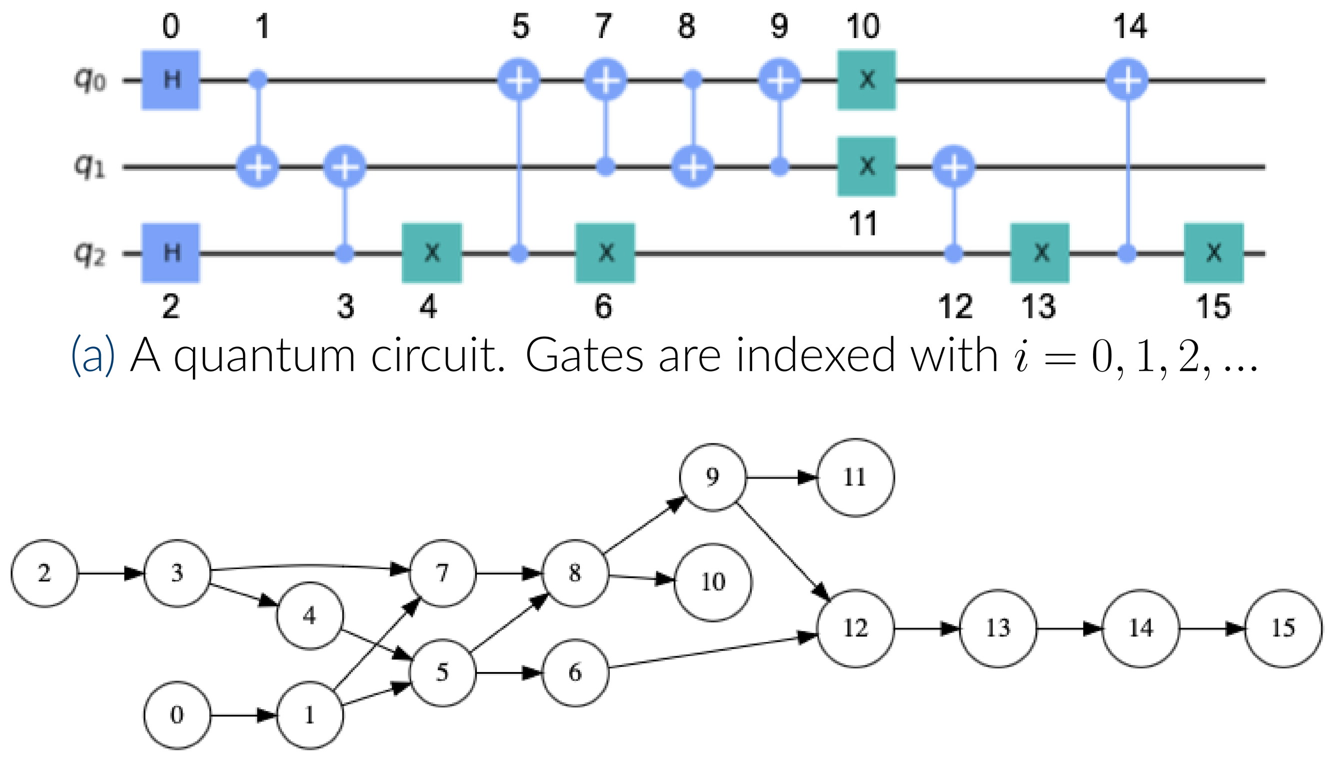
Methodologies

Algorithms

For a given gate library, a **template** T is a circuit (realizes identity function) such that a sequence of gates of size $\lfloor |T|/2 \rfloor + 1$ in T cannot be reduced by any other template. If a sequence of gates in a circuit matches with a sequence of gates of size $S_1 > \lfloor |T|/2 \rfloor$ in a template T , the matched sequence of gates in the circuit can be replaced with the inverse of the remaining sequence of T of size $S_2 < \lfloor |T|/2 \rfloor$ – called **template matching** [1]. The algorithm ensures that if a match is success (a template is applied), then the no. of gates in the circuit decreases.

Implementation

Gates in a circuit can often be reordered without affecting the function of the circuit. Mobility of gates in a circuit are captured in Directed Acyclic Graph (DAG). A graph-base algorithms are implemented in C/C++ for template matching.

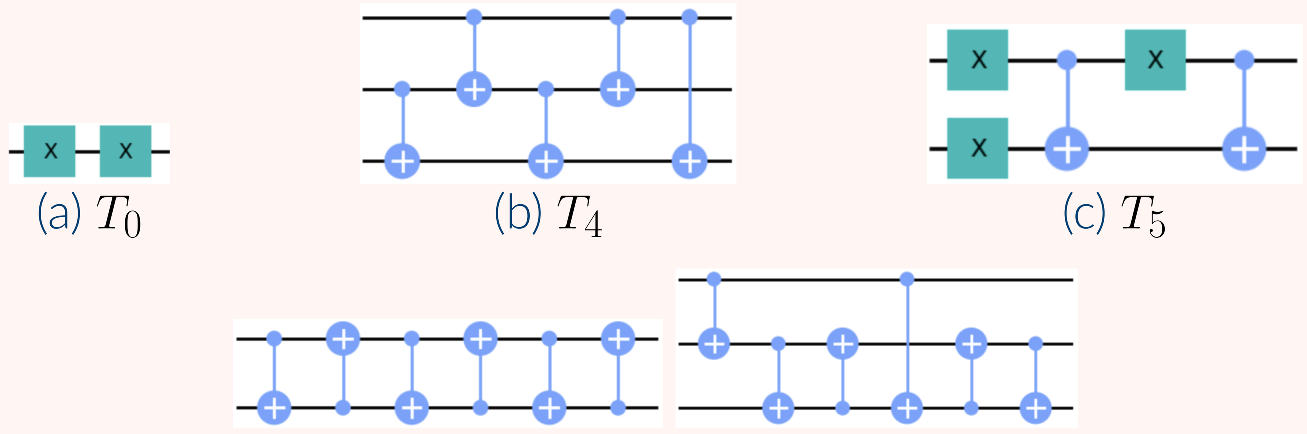


(a) A quantum circuit. Gates are indexed with $i = 0, 1, 2, \ldots$

(b) DAG of the circuit in Figure 3(a)

Figure 3. A quantum circuit and it's DAG.

Illustration of Template Matching



(a) T_0 (b) T_4 (c) T_5 (d) T_6 (e) T_9

Figure 4. Templates that are applied to optimize the circuit in Figure 5.

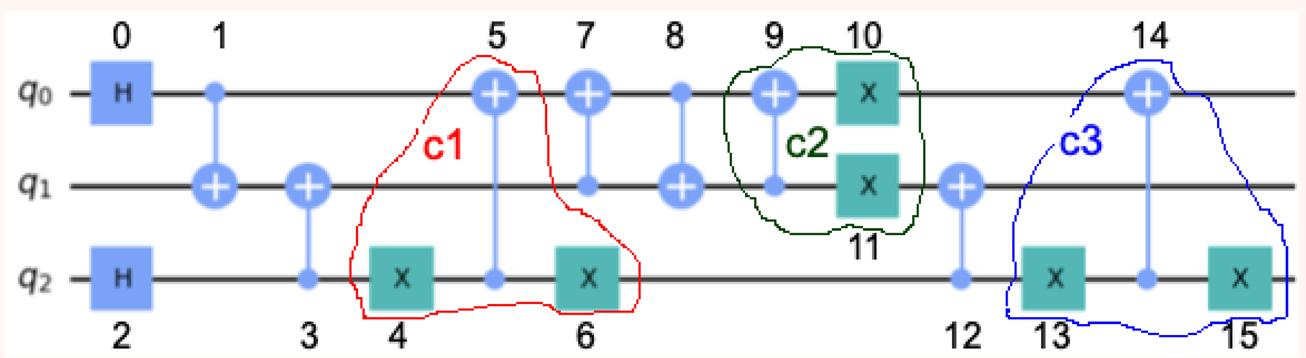


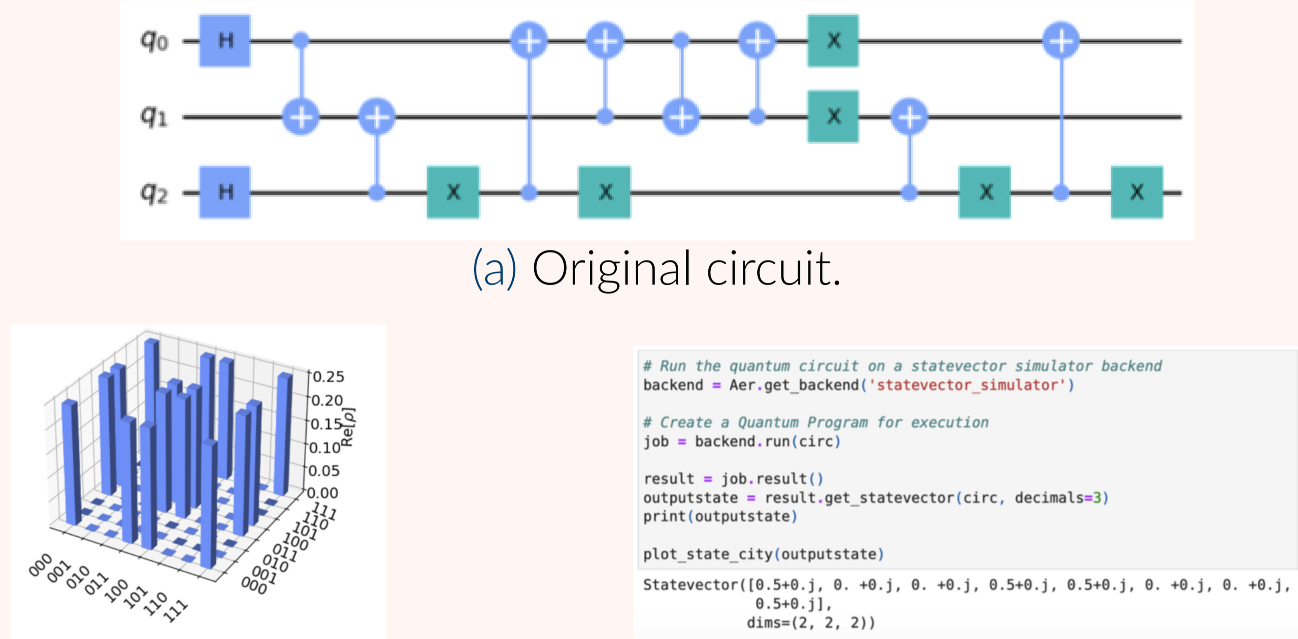
Figure 5. The sub-circuits c_1, c_2, c_3 can be optimized by template T_5. The order of templates applications in optimization is shown in Table 2. The final optimized circuit is shown in Figure 7(a).

Table 2. Statistics of template applications.

$TemplateIndex$	0 4 5 6 9
$NumberofTimeApplied$	1 2 4 1 1
$ApplicationsOrder$	5 4 5 5 5 0 4 9 6

Simulation with IBM Quantum Computer

The circuit shown in Figure 6(a) is taken from an Enigma problem demonstrated for QHACK competition [4]. The circuit 6(a) and the optimized circuit 7(a) are implemente with IBM qiskit. Simulation results from IBM quantum device for both circuits are shown in Figure 6(b) and 7(b) respectively that are equal.

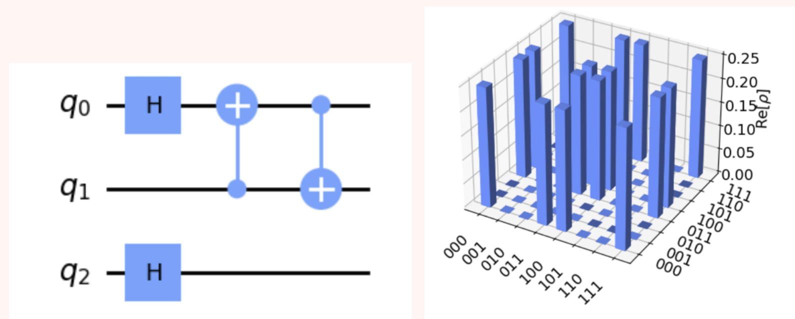


(a) Original circuit.

(b) Output states.

(c) Device API calls

Figure 6. Simulation of original circuit.



(a) Optimized (b) Output states.

Figure 7. Simulation of optimized circuit.

Results and Discussion

The presented algorithm shows a significant reduction of gates in the circuit. In fact, the first CNOT gate in Figure 7(a) can be removed without affecting the results. Moreover, the qubit q_2 is no longer entangled with other qubits q_0 and q_1 whereas all 3 qubits are in entangled in the original circuit. This reveals that the quantum algorithm that is implemented in the circuit shown Figure 6(a) can be redefined with a new formulation. For example "DO NOT LIE" in Enigma problem [4]. The future plan of this work is to extend the optimization algorithms for a wide gate library, and create an online tools for supporitng communities.

Acknowledgement

I want to express my gratitude to **Professor Andrew Gerber**, Department of Mechanical Engineering for his invaluable support and briging me in HPC research at UNB. I am thankful to **Professor De Baerdemacker** and his PhD student **Seyed Ehsan Ghasempouri** for participating in disscussion of validating this presented algorithms.

References

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