Technical Report of A Transformation Approach for Classifying $\mathcal{ALCHI}(\mathcal{D})$ Ontologies with a Consequence-based $\mathcal{ALCH}$ Reasoner

by

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Abstract

Consequence-based techniques have been developed to provide efficient classification for less expressive languages. Ontology transformation techniques are often used to approximate axioms in a more expressive language by axioms in a less expressive language. In this paper, we present an approach to use a fast consequence-based $\mathcal{ALCH}$ reasoner to classify an $\mathcal{ALCHI}(\mathcal{D})$ ontology with a subset of OWL 2 datatypes and facets. We transform datatype and inverse role axioms into $\mathcal{ALCH}$ axioms. The transformed ontology preserves sound and complete classification w.r.t the original ontology. The proposed approach has been implemented in the prototype WSClassifier which exhibits the high performance of consequence reasoning. The experiments show that for classifying large and highly cyclic $\mathcal{ALCHI}(\mathcal{D})$ ontologies, WSClassifier's performance is significantly faster than tableau-based reasoners.

1 Introduction

Ontology classification is the foundation of many ontology reasoning tasks. Recently, consequence-based techniques have been developed to provide efficient classification for sublanguages of OWL 2 DL profile, e.g. $\mathcal{EL}^{++}$ [2, 3, 8], Horn-$\mathcal{SHIQ}$ [7], $\mathcal{EL}^{\perp}(\mathcal{D})$ [9], $\mathcal{ALCH}$ [13]. There have been some approaches to use existing consequence-based reasoners to classify more expressive ontologies, like MORe [1]. In this paper, we propose an approach to use a consequence $\mathcal{ALCH}$ reasoner to classify an $\mathcal{ALCHI}(\mathcal{D})$ ontology by transforming it into an $\mathcal{ALCH}$ ontology with soundness and completeness preserved. The purpose of the approach is to extend the expressiveness of the existing consequence-based reasoner without changing its complex inference rules and implementation.

Ontology transformation is often accomplished by approximating non-Horn ontologies/theories by Horn replacements [12, 11, 15]. These approximations can be used to optimize reasoning by exploiting more efficient inference for Horn ontologies/theories. The approximation $\mathcal{O}'$ in Ren *et al.* [11] is a lower bound of the original ontology $\mathcal{O}$, i.e. $\mathcal{O}'$ entails no more subsumptions than $\mathcal{O}$ does. In contrast, approximation in Zhou *et al.* [15] provides an upper bound. Kautz *et al.* [12] computes both upper and lower bounds of propositional logic theories. Another approach preserves both soundness and completeness of classification results such as the elimination of transitive roles in Kazakov [7]. Our work of this paper is of the second kind. We classify an $\mathcal{ALCHI}(\mathcal{D})$ ontology $\mathcal{O}$ in two stages: (1) transform $\mathcal{O}$ into an $\mathcal{ALCHI}$ ontology $\mathcal{O}^{-}_{\mathcal{D}}$ s.t. $\mathcal{O} \models A \sqsubseteq B$ iff $\mathcal{O}^{-}_{\mathcal{D}} \models A \sqsubseteq B$; (2) transform $\mathcal{O}^{-}_{\mathcal{D}}$ into an $\mathcal{ALCH}$ ontology $\mathcal{O}^{-}_{\mathcal{ID}}$ s.t. $\mathcal{O}^{-}_{\mathcal{ID}} \models A \sqsubseteq B$ iff $\mathcal{O}^{-}_{\mathcal{D}} \models A \sqsubseteq B$. We use these approaches to implement a reasoner called WSClassifier which transforms an $\mathcal{ALCHI}(\mathcal{D})$ ontology into an $\mathcal{ALCH}$ ontology and classifies it with a fast consequence $\mathcal{ALCH}$ reasoner ConDOR [13]. WSClassifier is significantly faster than tableau-based reasoners on large and highly cyclic ontologies.

In our previous work [14] we approximated an $\mathcal{ALCHOI}$ ontology by an $\mathcal{ALCH}$ ontology which was then classified by a hybrid of consequence- and tableau-based reasoners. Unlike [14], in this paper we claim completeness for $\mathcal{I}$'s transformation. Calvanese *et al.* [4] introduces a general approach to eliminate inverse roles and functional restrictions from $\mathcal{ALCFI}$ to $\mathcal{ALC}$. For eliminating $\mathcal{I}$, the approach needs to add one axiom for each inverse role and each concept. So the number of axioms added can be very large. Ding *et al.* [6] introduces a new mapping from $\mathcal{ALCI}$ to $\mathcal{ALC}$ and further extends it to a mapping from $\mathcal{SHI}$ to $\mathcal{SH}$ in [5]. The approach allows tableau-based decision procedures to use some caching techniques and improve the reasoning performance in practice. Both approaches in [4, 5] preserve the soundness and completeness of inference after elimination of $\mathcal{I}$. Our approach is similar to the one in [6, 5]. However, the NNF normalized form in [6, 5] in which $\top$ appears in the left side of all axioms will dramatically degrade the performance of our consequence-based $\mathcal{ALCH}$ reasoner. Thus we eliminate the inverse role based on our own normalized form and our approach is more suitable for consequence-based reasoners.

2 Preliminary

The syntax of $\mathcal{ALCHI}(\mathcal{D})$ uses atomic concepts N_C, atomic roles N_R and features N_F. We use A, B for atomic concepts, C, D for concepts, r, s for atomic roles, R, S for roles, F, G for features. The parameter $\mathcal{D}$ defines a *datatype map* $\mathcal{D} = (N_{DT}, N_{LS}, N_{FS}, \cdot^{\mathcal{D}})$, where: (1) N_{DT} is a set of datatype names; (2) N_{LS} is a function assigning to each $d \in N_{DT}$ a set of constants $N_{LS}(d)$; (3) N_{FS} is a function assigning to each $d \in N_{DT}$ a set of facets $N_{FS}(d)$, each $f \in N_{FS}(d)$ has the form (p_f, v); (4) $\cdot^{\mathcal{D}}$ is a function assigning a datatype interpretation $d^{\mathcal{D}}$ to each $d \in N_{DT}$ called the *value space* of d, a data value $v^{\mathcal{D}} \in d^{\mathcal{D}}$ for each $v \in N_{LS}(d)$[1], and a facet interpretation $f^{\mathcal{D}}$ for each $f \in \bigcup_{d \in N_{DT}} N_{FS}(d)$.[2]

The basic forms of data ranges in $\mathcal{D}$ are $\top_{\mathcal{D}}$, d, $d[f]$ or $\{v\}$, which we call *atomic data range*s and denote by ADR in this paper. A data range dr is defined recursively using the corresponding constructors $\sqcap$, $\sqcup$, and $\neg$, and their interpretations are shown in Table 1. A role R is either atomic role r or an *inverse role* r^-. A complex concept is defined from N_C using the constructors shown in the first two columns of the lower part of Table 1. Axioms are constructed using the concepts, roles and features, and an ontology is a set of axioms.

Table 1: Model Theoretic Semantics of $\mathcal{ALCHI}(\mathcal{D})$

	Semantics of Data Ranges	
$(\top_{\mathcal{D}})^{\mathcal{D}} = \Delta^{\mathcal{D}}$	$\{v\}^{\mathcal{D}} = \{v^{\mathcal{D}}\}$	$(dr_1 \sqcap dr_2)^{\mathcal{D}} = dr_1^{\mathcal{D}} \cap dr_2^{\mathcal{D}}$
$(d[f])^{\mathcal{D}} = d^{\mathcal{D}} \cap f^{\mathcal{D}}$	$(\neg dr)^{\mathcal{D}} = \Delta^{\mathcal{D}} \setminus dr^{\mathcal{D}}$	$(dr_1 \sqcup dr_2)^{\mathcal{D}} = dr_1^{\mathcal{D}} \cup dr_2^{\mathcal{D}}$
	Semantics of Concepts, Roles and Axioms	
$\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$	$(\exists R.C)^{\mathcal{I}} = \{x \mid R^{\mathcal{I}}(x) \cap C^{\mathcal{I}} \neq \emptyset\}$	$(R^-)^{\mathcal{I}} = \{(x, y) \mid (y, x) \in R^{\mathcal{I}}\}$
$(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$	$(\forall R.C)^{\mathcal{I}} = \{x \mid R^{\mathcal{I}}(x) \subseteq C^{\mathcal{I}}\}$	$C \sqsubseteq D \Rightarrow C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
$(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$	$(\exists F.dr)^{\mathcal{I}} = \{x \mid F^{\mathcal{I}}(x) \cap dr^{\mathcal{D}} \neq \emptyset\}$	$R \sqsubseteq S \Rightarrow R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$
$(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$	$(\forall F.dr)^{\mathcal{I}} = \{x \mid F^{\mathcal{I}}(x) \subseteq dr^{\mathcal{D}}\}$	$F \sqsubseteq G \Rightarrow F^{\mathcal{I}} \subseteq G^{\mathcal{I}}$

Semantics of $\mathcal{ALCHI}(\mathcal{D})$ is defined via an *interpretation* $\mathcal{I} = (\Delta^{\mathcal{I}}, \Delta^{\mathcal{D}}, \cdot^{\mathcal{I}}, \cdot^{\mathcal{D}})$ where $\Delta^{\mathcal{I}}$ and $\Delta^{\mathcal{D}}$ are disjoint non-empty sets called *object domain* and *data domain*. $d^{\mathcal{D}} \subseteq \Delta^{\mathcal{D}}$ for each $d \in N_{DT}$. $\cdot^{\mathcal{D}}$ has been described above. The object interpretation function $\cdot^{\mathcal{I}}$ assigns a set $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ to each $A \in N_C$, a relation $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ to each $R \in N_R$ and a relation $F^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{D}}$ to each $F \in N_F$. $\cdot^{\mathcal{I}}$ can be further extended to all the concepts shown in Table 1, in which $R^{\mathcal{I}}(x) = \{y \mid (x, y) \in R^{\mathcal{I}}\}$ and $F^{\mathcal{I}}(x) = \{v \mid (x, v) \in F^{\mathcal{I}}\}$. If the language of an ontology does not support the datatype map, then $\Delta^{\mathcal{D}}$ and $\cdot^{\mathcal{D}}$ does not exist and the interpretation is represented by $(\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$. An interpretation is

[1]We assume $N_{LS}(d)$ for different datatypes are mutually disjoint

[2]Since one facet may be shared by multiple datatypes, we define its interpretation as containing subsets of all the relevant datatypes.

a model of an ontology $\mathcal{O}$ if it satisfies all the axioms in $\mathcal{O}$ according to the semantics shown in the third column of the lower part of Table 1.

All atomic concepts, atomic roles, and features of an ontology $\mathcal{O}$ is called the signature of $\mathcal{O}$, denoted by $\mathsf{Sig}(\mathcal{O})$. Also in the rest of the paper, we write $N_{DT}(\mathcal{O})$ and $ADR(\mathcal{O})$ for all datatypes and atomic data ranges in $\mathcal{O}$. And $ADR_d(\mathcal{O})$ denotes the subset of $ADR(\mathcal{O})$ in datatype d, i.e. of the form d, $d[f]$ or $\{v\}$ where $v \in N_{LS}(d)$.

3 Transformation for Datatypes

In this section we introduce how we transform an $\mathcal{ALCHI}(\mathcal{D})$ ontology $\mathcal{O}$ into an $\mathcal{ALCHI}$ ontology $\mathcal{O}_{\mathcal{D}}^{-}$ such that $\mathcal{O} \models A \sqsubseteq B$ iff $\mathcal{O}_{\mathcal{D}}^{-} \models A \sqsubseteq B$. We assume value spaces of all the datatypes in $\mathcal{D}$ are disjoint, as do Motik et al [10]. We apply our approach to some commonly used datatypes: (1) real with facets rational, decimal, integer, $>_a$, $\geq_a$, $<_a$ and $\leq_a$; (2) strings with equal value; (3) boolean values.

Our basic idea is to produce $\mathcal{O}_{\mathcal{D}}^{-}$ from $\mathcal{O}$ to encode features into roles and data ranges into concepts, and then add extra axioms to preserve the subsumptions between atomic concepts in $N_C \cap \mathsf{Sig}(\mathcal{O})$. Table 2 gives the definition of encoding function φ over atomic elements in $\mathcal{O}$, where A_d, A_f, A_v are fresh concepts and R_F is a fresh role. φ over complex data ranges, roles, concepts and axioms are defined recursively using corresponding constructors.

Table 2: Encoding φ for atomic concepts/roles/features/data ranges

$\varphi(\top_{\mathcal{D}}) = \top$	$\varphi(d[f]) = A_d \sqcap A_f$	$\varphi(\top) = \top$	$\varphi(A) = A$
$\varphi(d) = A_d$	$\varphi(\{v\}) = A_v$	$\varphi(R) = R$	$\varphi(F) = R_F$

The following theorem shows classification of $\varphi(\mathcal{O}) = \{\varphi(\alpha) \mid \alpha \in \mathcal{O}\}$ is sound w.r.t. $\mathcal{O}$.

Theorem 1 *Let α be an axiom s.t. $\mathsf{Sig}(\alpha) \subseteq \mathsf{Sig}(\mathcal{O})$. If $\varphi(\mathcal{O}) \models \varphi(\alpha)$, then $\mathcal{O} \models \alpha$.*

Proof. It is sufficient to prove every model $\mathcal{I}$ of $\mathcal{O}$ can be turned into a model $\mathcal{I}'$ of $\varphi(\mathcal{O})$ which preserves interpretations for symbols in $\mathsf{Sig}(\mathcal{O})$ after encoding. Let $\mathcal{I}'$ an interpretation of $\varphi(\mathcal{O})$ where:

- $\Delta^{\mathcal{I}'} = \Delta^{\mathcal{I}} \cup \Delta^{\mathcal{D}}$.
- For each $A \in N_C \cap \mathsf{Sig}(\varphi(\mathcal{O}))$, if $A \in \mathsf{Sig}(\mathcal{O})$, then $A^{\mathcal{I}'} = A^{\mathcal{I}}$, otherwise A is of the form A_d, A_f or A_v, which is interpreted as $d^{\mathcal{D}}$, $f^{\mathcal{D}}$ or $\{v^{\mathcal{D}}\}$, respectively.
- For each $r \in N_R \cap \mathsf{Sig}(\varphi(\mathcal{O}))$, $r^{\mathcal{I}'} = r^{\mathcal{I}}$ if $r \in \mathsf{Sig}(\mathcal{O})$, or $r^{\mathcal{I}'} = F^{\mathcal{I}}$ if $r = \varphi(F)$ and $F \in N_F \cap \mathsf{Sig}(\mathcal{O})$.

It is easy to see that under such definition, for every data range dr in $\mathcal{O}$, $\varphi(dr)^{\mathcal{I}'} = dr^{\mathcal{D}}$; for every role R in $\mathcal{O}$, $\varphi(R)^{\mathcal{I}'} = R^{\mathcal{I}}$. And for any concept C, by an induction over all forms C in Table 1, it is obvious that $\varphi(C)^{\mathcal{I}'} = C^{\mathcal{I}}$. Thus concept subsumption and role subsumption axioms in $\mathcal{O}$ are also satisfied after the encoding. Feature subsumption axioms $F \sqsubseteq G$ in $\mathcal{O}$ are encoded into $\varphi(F) \sqsubseteq \varphi(G)$ in $\varphi(\mathcal{O})$, which is trivially satisfied by $\mathcal{I}'$. Thus $\mathcal{I}'$ is a model of $\varphi(\mathcal{O})$, and the theorem is proved. □

Although $\varphi(\mathcal{O})$'s classification is sound w.r.t. $\mathcal{O}$, it may not be complete. In order to preserve completeness, extra axioms need to be added to $\varphi(\mathcal{O})$ to get $\mathcal{O}^-_{\mathcal{D}}$. Computation of $\mathcal{O}^-_{\mathcal{D}}$ is shown in Algorithm 1. In the procedure we use two functions $\mathsf{normalize}_d$ and $\mathsf{getAxioms}_d$ for each datatype $d \in N_{DT}(\mathcal{O})$. $\mathsf{normalize}_d$ rewrites data ranges $d[f]$ into normalized forms to reduce the kinds of facets used. $\mathsf{getAxioms}_d$ produces a set of $\mathcal{ALCHI}$ axioms $\mathcal{O}^+_d$ to be included into $\mathcal{O}^-_{\mathcal{D}}$. Details will be explained later for the datatypes and facets currently supported.

Algorithm 1: Datatype Transformation

Input: An $\mathcal{ALCHI}(\mathcal{D})$ ontology $\mathcal{O}$
Output: An $\mathcal{ALCHI}$ ontology $\mathcal{O}^-_{\mathcal{D}}$ with the same classification result as $\mathcal{O}$

foreach $d \in N_{DT}(\mathcal{O})$ **do**
  foreach $adr \in ADR_d(\mathcal{O})$ **do**
    Replace adr with $\mathsf{normalize}_d(adr)$ in $\mathcal{O}$;
Create an encoding φ for $\mathcal{O}$ and initialize $\mathcal{O}^-_{\mathcal{D}}$ with $\varphi(\mathcal{O})$;
foreach $d_1, d_2 \in N_{DT}(\mathcal{O}), d_1 \neq d_2$ **do** $\mathcal{O}^-_{\mathcal{D}} \leftarrow \mathcal{O}^-_{\mathcal{D}} \cup \{\varphi(d_1) \sqcap \varphi(d_2) \sqsubseteq \bot\}$;
foreach $d \in N_{DT}(\mathcal{O})$ **do** $\mathcal{O}^-_{\mathcal{D}} \leftarrow \mathcal{O}^-_{\mathcal{D}} \cup \mathsf{getAxioms}_d(ADR_d(\mathcal{O}), \varphi)$;
return $\mathcal{O}^-_{\mathcal{D}}$;

Next we give a sufficient condition for $\mathcal{O}^-_{\mathcal{D}}$ to be complete w.r.t. $\mathcal{O}$.

Definition 2 *Let ADR_d be a set of atomic data ranges of the forms d, $d[f]$, or $\{v\}$ where $v \in N_{LS}(d)$, and φ be an encoding function. We say* $\mathsf{getAxioms}_d$ **preserves data range relationships** *if for any ADR_d, φ, and different $ar_1, \ldots, ar_n, ar'_1, \ldots, ar'_m \in ADR_d, m, n \geq 0$, such that $(\bigsqcap_{i=1}^n ar_i) \sqcap (\bigsqcap_{j=1}^m \neg ar'_j)$ is unsatisfiable, $(\bigsqcap_{i=1}^n \varphi(ar_i)) \sqcap (\bigsqcap_{j=1}^m \neg\varphi(ar'_j))$ is unsatisfiable in the ontology $\mathcal{O}^+_d = \mathsf{getAxioms}_d(ADR_d, \varphi)$.*

Lemma 3 *Let $\mathcal{O}^-_{\mathcal{D}}$ be the ontology computed from $\mathcal{O}$ using Algorithm 1. If for every datatype $d \in N_{DT}(\mathcal{O})$, $\mathsf{getAxioms}_d$ preserves data range relationships, then for any model $\mathcal{I}'$ of $\mathcal{O}^-_{\mathcal{D}}$, there exists a $\Delta^{\mathcal{D}}$ such that for every $x \in \Delta^{\mathcal{I}'}$, there is a data value $t(x) \in \Delta^{\mathcal{D}}$ such that $x \in \varphi(dr)^{\mathcal{I}'} \leftrightarrow t(x) \in dr^{\mathcal{D}}$ for any data range dr appeared in $\mathcal{O}$.*

Proof. Let $ADR_d(\mathcal{O})$ be all the atomic data ranges in $\mathcal{O}$ which is of datatype

d. We define $DR_d^+(x), DR_d^-(x)$ and $DR_d(x)$ as

$$DR_d^+(x) = \{ar \mid ar \in ADR_d(\mathcal{O}) \wedge x \in \varphi(ar)^{\mathcal{I}'}\}$$
$$DR_d^-(x) = \{\neg ar \mid ar \in ADR_d(\mathcal{O}) \wedge x \notin \varphi(ar)^{\mathcal{I}'}\}$$
$$DR_d(x) = DR_d^+(x) \cup DR_d^-(x)$$

We first show that there is at most one $d^* \in N_{DT}(\mathcal{O})$ such that $DR_{d^*}^+(x) \neq \emptyset$. Note that if $ar \in DR_d^+(x)$, then $x \in \varphi(ar)^{\mathcal{I}'}$ and $ar^{\mathcal{D}} \subseteq d^{\mathcal{D}}$. Since $(ar \sqcap \neg d)^{\mathcal{D}} = \emptyset$ and $\mathsf{getAxioms}_d$ preserves data range relationships, $\varphi(ar) \sqcap \neg\varphi(d) \sqsubseteq \bot$. Hence if $DR_d^+(x) \neq \emptyset$, then $x \in \varphi(d)^{\mathcal{I}'}$. Line 5 of Algorithm 1 ensures $\mathcal{O}_{\mathcal{D}}^- \models \varphi(d_1) \sqcap \varphi(d_2) \sqsubseteq \bot$ for any different datatypes d_1 and d_2, thus x cannot belong to both $\varphi(d_1)^{\mathcal{I}'}$ and $\varphi(d_2)^{\mathcal{I}'}$.

If $DR_d^+(x) = \emptyset$ for all datatypes $d \in N_{DT}(\mathcal{O})$, then we can choose some $\Delta^{\mathcal{D}} \supsetneqq \cup_{d \in N_{DT}(\mathcal{O})} d^{\mathcal{D}}$, and $t(x)$ be a value which is not in any $d^{\mathcal{D}}$. Otherwise there can only be one $d^* \in N_{DT}(\mathcal{O})$ such that $DR_{d^*}^+(x) \neq \emptyset$. Let $h = \bigsqcap_{dr \in DR_{d^*}(x)} dr$. Since $x \in \varphi(h)^{\mathcal{I}'}$, $\varphi(h)$ is satisfiable. Since $\mathsf{getAxioms}_{d^*}$ preserves data range relationships, by Definition 2, there exists a $\Delta^{\mathcal{D}}$ and $t(x) \in \Delta^{\mathcal{D}}$ such that $t(x) \in h^{\mathcal{D}}$. Next we prove $x \in \varphi(dr)^{\mathcal{I}} \leftrightarrow t(x) \in dr^{\mathcal{D}}$ by induction over dr:

- $dr = ar$ If $DR_d^+(x) = \emptyset$ for all datatypes, then $x \notin \varphi(ar)^{\mathcal{I}'}$ and $t(x) \notin ar^{\mathcal{D}}$. Otherwise both d^* and h exist. If $x \in \varphi(ar)^{\mathcal{I}'}$, then ar is a conjunct of h. Hence $h^{\mathcal{D}} \subseteq ar^{\mathcal{D}}$. Since $t(x) \in h^{\mathcal{D}}$, $t(x) \in ar^{\mathcal{D}}$. Otherwise $x \notin \varphi(ar)^{\mathcal{I}'}$ then either $\neg ar$ is a conjunct of h or ar is not of type d^*. In both cases, $t(x) \notin ar^{\mathcal{D}}$. So in all cases, $x \in \varphi(ar)^{\mathcal{I}'} \leftrightarrow t(x) \in ar^{\mathcal{D}}$ holds.

- $dr = \neg dr'$

$$x \in \varphi(dr)^{\mathcal{I}'} \leftrightarrow x \notin \varphi(dr')^{\mathcal{I}'} \leftrightarrow t(x) \notin dr'^{\mathcal{D}} \leftrightarrow t(x) \in dr^{\mathcal{D}}$$

- $dr = dr_1 \sqcap dr_2$

$$\begin{aligned} x \in \varphi(dr)^{\mathcal{I}'} &\leftrightarrow x \in \varphi(dr_1)^{\mathcal{I}'} \wedge x \in \varphi(dr_2)^{\mathcal{I}'} \\ &\leftrightarrow t(x) \in dr_1^{\mathcal{D}} \wedge t(x) \in dr_2^{\mathcal{D}} \leftrightarrow t(x) \in dr^{\mathcal{D}} \end{aligned}$$

- $dr = dr_1 \sqcup dr_2$

$$\begin{aligned} x \in \varphi(dr)^{\mathcal{I}'} &\leftrightarrow x \in \varphi(dr_1)^{\mathcal{I}'} \vee x \in \varphi(dr_2)^{\mathcal{I}'} \\ &\leftrightarrow t(x) \in dr_1^{\mathcal{D}} \vee t(x) \in dr_2^{\mathcal{D}} \leftrightarrow t(x) \in dr^{\mathcal{D}} \end{aligned}$$

Therefore the lemma holds. □

Theorem 4 *Let $\mathcal{O}_{\mathcal{D}}^-$ be the ontology computed from $\mathcal{O}$ using Algorithm 1, and α be an axiom using symbols in $\mathsf{Sig}(\mathcal{O})$ or data ranges in $\mathcal{O}$. If for every datatype $d \in N_{DT}(\mathcal{O})$, $\mathsf{getAxioms}_d$ preserves data range relationships, then $\mathcal{O} \models \alpha$ implies $\mathcal{O}_{\mathcal{D}}^- \models \varphi(\alpha)$.*

Proof. We will show that if $\mathcal{O}_\mathcal{D}^- \not\models \varphi(\alpha)$, then $\mathcal{O} \not\models \alpha$. Let $\mathcal{I}'$ be a model of $\mathcal{O}_\mathcal{D}^-$ where $\mathcal{I}' \not\models \varphi(\alpha)$. Since all $\mathsf{getAxioms}_d$ preserve data range relationships, by Lemma 3 we can find a $\Delta^\mathcal{D}$ and for each $x \in \Delta^{\mathcal{I}'}$, there exists $t(x) \in \Delta^\mathcal{D}$ such that $x \in \varphi(dr)^\mathcal{I} \leftrightarrow t(x) \in dr^\mathcal{D}$. Using this $\Delta^\mathcal{D}$, we define an interpretation $\mathcal{I} = (\Delta^\mathcal{I}, \Delta^\mathcal{D}, \cdot^\mathcal{I})$ of $\mathcal{O}$, where the domain, atomic concepts and roles are interpreted the same as $\mathcal{I}'$, while any feature F is interpreted as

$$F^\mathcal{I} = \{(o, t(x)) \mid (o, x) \in \varphi(F)^{\mathcal{I}'}\}$$

Since $x \in \varphi(dr)^\mathcal{I} \leftrightarrow t(x) \in dr^\mathcal{D}$, it is easy to see $(\exists F.dr)^\mathcal{I} = (\varphi(\exists F.dr))^{\mathcal{I}'}$ and $(\forall F.dr)^\mathcal{I} = (\varphi(\forall F.dr))^{\mathcal{I}'}$. For any roles R, we also have $R^\mathcal{I} = \varphi(R)^{\mathcal{I}'}$. By induction over different types of concepts in Table 1, we know $C^\mathcal{I} = \varphi(C)^{\mathcal{I}'}$ for any concept C in $\mathcal{O}$. Hence $\mathcal{I}$ satisfies concept, role, and feature subsumption axioms and is a model of $\mathcal{O}$. Since α use only symbols in $\mathsf{Sig}(\mathcal{O})$ and data ranges in $\mathcal{O}$, for any concept or role X used in α, $X^\mathcal{I} = \varphi(X)^{\mathcal{I}'}$. Thus because $\mathcal{I}' \not\models \varphi(\alpha)$, if α is a concept or role subsumption axiom, $\mathcal{I} \not\models \alpha$; otherwise α is a feature subsumption axiom of the form $F \sqsubseteq G$, and $\mathcal{I} \not\models \alpha$ holds trivially from the definition of $F^\mathcal{I}$ and $G^\mathcal{I}$. Hence $\mathcal{O} \not\models \alpha$ and the theorem is proved. □

Next we discuss the implementation of $\mathsf{normalize}_d$ and $\mathsf{getAxioms}_d$ for some commonly used datatypes. For boolean type, we do not have any facets, so $\mathsf{normalize}_d$ does nothing. Since the only atomic data ranges are $xsd\!:\!boolean$, $\{true\}$ and $\{false\}$, $\mathsf{getAxioms}_d$ only needs to return two axioms $\varphi(xsd\!:\!boolean) \equiv \varphi(\{true\}) \sqcup \varphi(\{false\})$ and $\varphi(\{true\}) \sqcap \varphi(\{false\}) \sqsubseteq \bot$.

For string type, currently we do not support any facets, so $\mathsf{normalize}_d$ does nothing either. Atomic data ranges are either $xsd\!:\!string$ or of the form $\{c\}$, where c is a constant. We need to add $\varphi(\{c\}) \sqsubseteq \varphi(xsd : string)$ for each $\{c\} \in ADR(\mathcal{O})$, as well as pairwise disjoint axioms for all such $\varphi(\{c\})$.

Numeric datatypes are the most commonly used datatypes in ontologies. Here we discuss the implementation for $owl : real$, which we denote by $\mathbb{R}$. $owl : rational$, $xsd : decimal$ and $xsd : integer$ are treated as facets rat, dec and int of $\mathbb{R}$, respectively. Comparison facets of the forms $>_a$, $<_a$, $\geq_a$, $\leq_a$ are supported. For $\mathsf{normalize}_d$ with input ar, we need: (1) if $adr = \mathbb{R}[f]$, transform it to equivalent data ranges using only facets of the form $>_a$, e.g. $\mathbb{R}[<_a] = \mathbb{R} \sqcap \neg(\mathbb{R}[>_a] \sqcup \{a\})$; (2) replace any constant a used in ar with a normal form, so that any constants having the same interpretation becomes the same after normalization, e.g. integer constants +3 and 3 are both interpreted as real number 3, so they are normalized into the same form "3"^^$xsd\!:\!integer$. Algorithm 2 gives the details of $\mathsf{getAxioms}_\mathbb{R}$ for real numbers.

For boolean and string, it is obvious that the corresponding $\mathsf{getAxioms}_d$ preserves data range relationships. For $\mathsf{getAxioms}_\mathbb{R}$, we prove this property in the following lemma.

Lemma 5 *Algorithm 2's implementation of* $\mathsf{getAxioms}_\mathbb{R}$ *preserves data range relationship for* $owl\!:\!real$.

Proof. Let $h = \sqcap_{i=1}^m dr_i, m > 0$ where dr_i is ar or $\neg ar$. After normalization for real datatype, ar can only be of the forms $\mathbb{R}$, $\mathbb{R}[dec]$, $\mathbb{R}[rat]$, $\mathbb{R}[int]$, $\mathbb{R}[>_a]$ or

Algorithm 2: $\mathsf{getAxioms}_{\mathbb{R}}$ for $\mathbb{R}$

Input: A set of atomic data ranges $ADR_{\mathbb{R}}(\mathcal{O})$ of type $\mathbb{R}$, encoding function φ

Output: A set of axioms $\mathcal{O}^+_{\mathbb{R}}$

1 $\mathcal{O}^+_{\mathbb{R}} \leftarrow \emptyset$;

2 **foreach** $\{v\} \in ADR_{\mathbb{R}}(\mathcal{O})$ **do**

3   **if** $\mathbb{R}[int] \in ADR_{\mathbb{R}}(\mathcal{O})$ *and* $v^{\mathcal{D}} \in (\mathbb{R}[int])^{\mathcal{D}}$ **then** add $\varphi(\{v\}) \sqsubseteq \varphi(int)$ to $\mathcal{O}^+_{\mathbb{R}}$;

4   **if** $\mathbb{R}[dec] \in ADR_{\mathbb{R}}(\mathcal{O})$ *and* $v^{\mathcal{D}} \in (\mathbb{R}[dec])^{\mathcal{D}}$ **then** add $\varphi(\{v\}) \sqsubseteq \varphi(dec)$ to $\mathcal{O}^+_{\mathbb{R}}$;

5   **if** $\mathbb{R}[rat] \in ADR_{\mathbb{R}}(\mathcal{O})$ *and* $v^{\mathcal{D}} \in (\mathbb{R}[rat])^{\mathcal{D}}$ **then** add $\varphi(\{v\}) \sqsubseteq \varphi(rat)$ to $\mathcal{O}^+_{\mathbb{R}}$;

6 **if** $\mathbb{R}[int], \mathbb{R}[dec] \in ADR_{\mathbb{R}}(\mathcal{O})$ **then** add $\varphi(int) \sqsubseteq \varphi(dec)$ to $\mathcal{O}^+_{\mathbb{R}}$;

7 **if** $\mathbb{R}[int], \mathbb{R}[rat] \in ADR_{\mathbb{R}}(\mathcal{O})$ **then** add $\varphi(int) \sqsubseteq \varphi(rat)$ to $\mathcal{O}^+_{\mathbb{R}}$;

8 **if** $\mathbb{R}[dec], \mathbb{R}[rat] \in ADR_{\mathbb{R}}(\mathcal{O})$ **then** add $\varphi(dec) \sqsubseteq \varphi(rat)$ to $\mathcal{O}^+_{\mathbb{R}}$;

9 Put all $\mathbb{R}[>_a] \in ADR_{\mathbb{R}}(\mathcal{O})$ in *fArray* with ascending order of a;

10 **foreach** *pair of adjacent elements* $\mathbb{R}[>_a]$ *and* $\mathbb{R}[>_b]$ *($a < b$) in fArray* **do**

11   add $\varphi(>_b) \sqsubseteq \varphi(>_a)$ to $\mathcal{O}^+_{\mathbb{R}}$;

12   **if** $\mathbb{R}[int] \in ADR_{\mathbb{R}}(\mathcal{O})$ **then**

13         $M \leftarrow \{\{a_i\}\}^n_{i=1}$, where $a_1, \ldots, a_n$ are all integer constants in $(a, b]$;

14   **if** $M \subseteq ADR_{\mathbb{R}}(\mathcal{O})$ **then**

15   add $\varphi(int) \sqcap \varphi(>_a) \sqcap \neg\varphi(>_b) \sqcap (\bigsqcap^n_{i=1} \neg\varphi(\{a_i\})) \sqsubseteq \bot$ to $\mathcal{O}^+_{\mathbb{R}}$;

16   Let N be all v such that $\{v\} \in ADR_{\mathbb{R}}(\mathcal{O})$ and $v^{\mathcal{D}} \in (a, b]$;

17   **foreach** $v \in N$ **do** add $\varphi(\{v\}) \sqsubseteq \varphi(>_a)$, $\varphi(\{v\}) \sqcap \varphi(>_b) \sqsubseteq \bot$ to $\mathcal{O}^+_{\mathbb{R}}$;

18   **foreach** $v_1, v_2 \in N, v_1 \neq v_2$ **do** add $\varphi(\{v_1\}) \sqcap \varphi(\{v_2\}) \sqsubseteq \bot$ to $\mathcal{O}^+_{\mathbb{R}}$;

19 **return** $\mathcal{O}^+_{\mathbb{R}}$;

$\{v\}$. We write h^+ and h^- for the set of all positive and negative conjuncts of h, respectively. Next we perform a case-by-case analysis over all possible cases that $h^{\mathcal{D}} = \emptyset$. Without loss of generality, we assume h is minimal, i.e. if we remove any conjunct from h, then $h^{\mathcal{D}} \neq \emptyset$.

Case 1 $h^+ \subseteq \{\mathbb{R}, \mathbb{R}[dec], \mathbb{R}[rat], \mathbb{R}[int]\}$. There are two cases for $h^{\mathcal{D}} = \emptyset$:

Case 1.1 $\neg\mathbb{R} \in h^-$. $\varphi(\mathbb{R}[f]) \sqcap \neg\varphi(\mathbb{R}) \sqsubseteq \bot$ is implied by the definition of φ.

Case 1.2 There exists $f_1, f_2 \in \{rat, dec, itg\}$ such that $(\mathbb{R}[f_1] \sqcap \neg\mathbb{R}[f_2])^{\mathcal{D}} = \emptyset$. All such cases are captured by lines 6 to 8, and $\varphi(\mathbb{R}[f_1]) \sqcap \neg\varphi(\mathbb{R}[f_2]) \sqsubseteq \bot$.

Case 2 There exists $\mathbb{R}[>_a] \in h^+$. There are 3 cases for $h^{\mathcal{D}} = \emptyset$:

Case 2.1 $\neg\mathbb{R}[>_{a'}] \in h^-$ and $a > a'$. In this case axioms added by line 11, where $a = a'$ and $b = a$, ensures that $\mathcal{O}^+_{\mathbb{R}} \models \varphi(\mathbb{R}[>_a]) \sqcap \varphi(\neg\mathbb{R}[>_{a'}]) \sqsubseteq \bot$ for any a, a' such that $a > a'$.

Case 2.2 $\mathbb{R}[int] \in h^+, \neg\mathbb{R}[>_b], \neg\{a_1\}, \ldots, \neg\{a_n\} \in h^-$ such that $a_1, \ldots, a_n$ are all integers in the range $(a, b]$. This case is captured by lines 12 to 15.

Case 2.3 $\{v\} \in h^+$, and $v \leq a$. Line 17 ensures that $\varphi(\{v\}) \sqcap \varphi(\mathbb{R}[>_{a'}]) \sqsubseteq \bot$ for the minimum a' such that $v \leq a'$. Axioms added by line 11 ensures $\mathcal{O}^+_{\mathbb{R}} \models \varphi(\mathbb{R}[>_a]) \sqsubseteq \varphi(\mathbb{R}[>_{a'}])$. Thus $\mathcal{O}^+_{\mathbb{R}} \models \varphi(\{v\}) \sqcap \varphi(\mathbb{R}[>_a]) \sqsubseteq \bot$.

Case 3 There exists v such that $\{v\} \in h^+$, and no $\mathbb{R}[>_a] \in h^+$. There are 3 cases where $h^{\mathcal{D}} = \emptyset$:

Case 3.1 There exists $f \in \{rat, dec, itg\}$ such that $\neg\mathbb{R}[f] \in h^-$ and $v^{\mathcal{D}} \in \mathbb{R}[f]$. $\mathcal{O}^+_{\mathbb{R}} \models \varphi(\{v\}) \sqcap \neg\varphi(\mathbb{R}[f]) \sqsubseteq \bot$ is ensured by lines 2 to 5.

Case 3.2 $\neg\mathbb{R} \in h^-$. Since in OWL 2 no real constants beyond *owl*:*rational*, *xsd*:*decimal*, *xsd*:*integer* are supported. So there exists $f \in \{rat, dec, itg\}$ such that $v^{\mathcal{D}} \in \mathbb{R}[f]$. Lines 2 to 5 ensures $\varphi(\{v\}) \sqsubseteq \varphi(\mathbb{R}[f]) \sqsubseteq \varphi(\mathbb{R})$, and so $\mathcal{O}^+_{\mathbb{R}} \models \varphi(\{v\}) \sqcap \neg\varphi(\mathbb{R}) \sqsubseteq \bot$.

Case 3.3 There exists $\{v'\} \in h^+$ and $v^{\mathcal{D}} \neq v'^{\mathcal{D}}$. Lines 17 to 18 ensures $\mathcal{O}^+_{\mathbb{R}} \models \varphi(\{v\}) \sqcap \varphi(\{v'\}) \sqsubseteq \bot$.

4 Transformation for Inverse Roles

In this section, we discuss how we transform an $\mathcal{ALCHI}$ ontology $\mathcal{O}^-_{\mathcal{D}}$ into an $\mathcal{ALCH}$ ontology $\mathcal{O}^-_{\mathcal{ID}}$, such that $\mathcal{O}^-_{\mathcal{D}} \models A \sqsubseteq B$ iff $\mathcal{O}^-_{\mathcal{ID}} \models A \sqsubseteq B$. To simplify our analysis, we assume that the ontology $\mathcal{O}^-_{\mathcal{D}}$ has been normalized to contain only axioms of the forms $\bigsqcap A_i \sqsubseteq \bigsqcup B_j$, $A \sqsubseteq \exists r.B$, $\exists r.A \sqsubseteq B$, $A \sqsubseteq \forall r.B$, $r \sqsubseteq s$ and $r^- = r'$, where r denotes an atomic role in this section. The normalization procedure extends structural transformation in previous work [13] by introducing an atomic role r' for each r^- adding an inverse properties axiom $r^- = r'$.

An inverse role axiom $r^- = r'$ can affect inference of $\mathcal{ALCH}$ [13] through:

- The role hierarchy, where (1) $r^- = r'$ and $r^- = r^*$ imply $r' = r^*$; (2) $r \sqsubseteq s$, $r^- = r'$ and $s^- = s'$ imply $r' \sqsubseteq s'$.
- The well-known equivalence $A \sqsubseteq \forall R.B \Leftrightarrow \exists R^-.A \sqsubseteq B$, i.e., $r^- = r'$ and $A \sqsubseteq \forall r.B$ imply $\exists r'.A \sqsubseteq B$; and $r^- = r'$ and $\exists r.A \sqsubseteq B$ imply $A \sqsubseteq \forall r'.B$.

Algorithm 3 shows the details of transformation for inverse roles. In the procedure Inv_r contains the set of atomic roles which are inverses of r. Line 1 initializes $\mathcal{O}^-_{\mathcal{ID}}$ with $\mathcal{ALCH}$ axioms in $\mathcal{O}^-_{\mathcal{D}}$. Lines 2 to 6 initializes Inv_r and put all r where $\mathsf{Inv}_r \neq \emptyset$ into $\mathsf{RolesToBeProcessed}$. Lines 7 to 16 processes each role in $\mathsf{RolesToBeProcessed}$ and adds axioms into $\mathcal{O}^-_{\mathcal{ID}}$ to address the effect of inverse role axioms mentioned above. Line 9 ensures all roles in Inv_r are equivalent. Lines 10 to 14 ensure that if a role in $\mathsf{RolesToBeProcessed}$ has a role subsumption relationship with another role, then their corresponding inverse roles also have role subsumption relation. Concretely, adds $r' \sqsubseteq s'$ if $r \sqsubseteq s$,

Algorithm 3: Transformation for inverse roles

Input: Normalized ontology $\mathcal{ALCHI}$ ontology $\mathcal{O}_{\mathcal{D}}^{-}$
Output: An $\mathcal{ALCH}$ ontology $\mathcal{O}_{\mathcal{ID}}^{-}$ having the same classification result as $\mathcal{O}_{\mathcal{D}}^{-}$

1 Initialize $\mathcal{O}_{\mathcal{ID}}^{-}$ with all $\mathcal{ALCH}$ axioms in $\mathcal{O}_{\mathcal{D}}^{-}$, excluding inverse role axioms;
2 **foreach** $r \in N_R \cap \mathsf{Sig}(\mathcal{O})$ **do** $\mathsf{Inv}_r \leftarrow \emptyset$;
3 $\mathsf{RolesToBeProcessed} \leftarrow \emptyset$;
4 **foreach** $r' = r^- \in \mathcal{O}_{\mathcal{D}}^{-}$ **do**
5     $\mathsf{Inv}_r \leftarrow \mathsf{Inv}_r \cup \{r'\}$; $\mathsf{Inv}_{r'} \leftarrow \mathsf{Inv}_{r'} \cup \{r\}$;
6     $\mathsf{RolesToBeProcessed} \leftarrow \mathsf{RolesToBeProcessed} \cup \{r, r'\}$;
7 **while** $\mathsf{RolesToBeProcessed} \neq \emptyset$ **do**
8   remove a role r from $\mathsf{RolesToBeProcessed}$ and pick a role r' from Inv_r;
9   **foreach** $r^* \in \mathsf{Inv}_r$ *where* r^* *is not* r' **do** add $r' \equiv r^*$ to $\mathcal{O}_{\mathcal{ID}}^{-}$;
10   **foreach** $r \sqsubseteq s \in \mathcal{O}_{\mathcal{D}}^{-}$ **do**
11   **if** $\mathsf{Inv}_s = \emptyset$ **then**
12   add a fresh atomic role s' to Inv_s;
13             $\mathsf{RolesToBeProcessed} \leftarrow \mathsf{RolesToBeProcessed} \cup \{s\}$;
14   pick a role s' from Inv_s and add $r' \sqsubseteq s'$ to $\mathcal{O}_{\mathcal{ID}}^{-}$;
15   **foreach** $\exists r.A \sqsubseteq B \in \mathcal{O}_{\mathcal{D}}^{-}$ **do** add $A \sqsubseteq \forall r'.B$ to $\mathcal{O}_{\mathcal{ID}}^{-}$;
16   **foreach** $A \sqsubseteq \forall r.B \in \mathcal{O}_{\mathcal{D}}^{-}$ **do** add $\exists r'.A \sqsubseteq B$ to $\mathcal{O}_{\mathcal{ID}}^{-}$;
17 **return** $\mathcal{O}_{\mathcal{ID}}^{-}$

$r' \in \mathsf{Inv}_r$ and $s' \in \mathsf{Inv}_s$. Note that if $r' \sqsubseteq s^-$ where $\mathsf{Inv}_s = \emptyset$, we need to introduce a new atomic role s' for s^-, because an axiom $\exists s.A \sqsubseteq B$, equivalent to $A \sqsubseteq \forall s'.B$, can interact with $A' \sqsubseteq \exists r'.B$ in the inference. Lines 15 and 16 add $A \sqsubseteq \forall r'.B$ or $\exists r'.A \sqsubseteq B$.

The following theorem shows our transformation preserves the subsumptions in $\mathcal{O}_{\mathcal{D}}^{-}$.

Theorem 6 *For every* $A, B \in N_C^{\top,\bot} \cap \mathsf{Sig}(\mathcal{O}_{\mathcal{D}}^{-})$, $\mathcal{O}_{\mathcal{D}}^{-} \models A \sqsubseteq B$ *iff* $\mathcal{O}_{\mathcal{ID}}^{-} \models A \sqsubseteq B$.

Proof. Let $\mathcal{O}' = \mathcal{O}_{\mathcal{D}}^{-} \cup \{s' = s^- \mid s'$ is a fresh role instroduced for s in line 12$\}$. Since the new axioms only define fresh roles, it is trivial that $\mathcal{O}_{\mathcal{D}}^{-} \models A \sqsubseteq B$ iff $\mathcal{O}' \models A \sqsubseteq B$. Hence we only need to prove $\mathcal{O}' \models A \sqsubseteq B$ iff $\mathcal{O}_{\mathcal{ID}}^{-} \models A \sqsubseteq B$.

According to Algorithm 3, it is easy to see $\mathcal{O}' \models \mathcal{O}_{\mathcal{ID}}^{-}$, thus $\mathcal{O}_{\mathcal{ID}}^{-} \models A \sqsubseteq B$ implies $\mathcal{O}' \models A \sqsubseteq B$. Next we will prove if $\mathcal{O}' \models A \sqsubseteq B$ then $\mathcal{O}_{\mathcal{ID}}^{-} \models A \sqsubseteq B$ by its contraposition: if $\mathcal{O}_{\mathcal{ID}}^{-} \not\models A \sqsubseteq B$, then $\mathcal{O}' \not\models A \sqsubseteq B$. Let $\mathcal{I}$ be a model of $\mathcal{O}_{\mathcal{ID}}^{-}$ where $(A \sqcap \neg B)^{\mathcal{I}} \neq \emptyset$, and $\mathcal{I}'$ be an interpretation of $\mathcal{O}'$ such that:

- For every atomic concept A, $A^{\mathcal{I}'} = A^{\mathcal{I}}$.
- For every atomic role r, $r^{\mathcal{I}'} = r^{\mathcal{I}} \cup (\bigcup_{r' \in \mathsf{Inv}_r} (r'^-)^{\mathcal{I}})$.

Now we show $\mathcal{I}'$ is a model of $\mathcal{O}'$. Since the only change from $\mathcal{I}$ to $\mathcal{I}'$ are the interpretations of roles r where $\mathsf{Inv}_r \neq \emptyset$, we only need to prove $\mathcal{I}'$ satisfies every axiom that has such a role. Note that Inv_r is made nonempty by an operation in line 5 or 12 of Algorithm 3, and r is added to $\mathsf{RolesToBeProcessed}$ by line 6 or 13 respectively in both cases. So r will be processed in the loop from lines 7 to 16, and there exists a representative role r_i which is picked from Inv_r in line 8.

Note that according to line 9 of Algorithm 3, all roles in Inv_r are equivalent to r_i in $\mathcal{O}^-_{\mathcal{ID}}$. Thus the following statement (*) holds:

$$\text{If } \mathsf{Inv}_r \neq \emptyset,\ r'^{\mathcal{I}} = r_i^{\mathcal{I}} \text{ for every } r' \in \mathsf{Inv}_r, \text{ and } r^{\mathcal{I}'} = r^{\mathcal{I}} \cup (r_i^-)^{\mathcal{I}} \quad (*)$$

Now we prove $\mathcal{I}'$ satisfies every axiom that has a role r s.t. $\mathsf{Inv}_r \neq \emptyset$, on a case-by-case analysis.

- $r' = r^-$ According to line 5, $r \in \mathsf{Inv}_{r'}$ and $r' \in \mathsf{Inv}_r$. Let r'_i be the representative role in $\mathsf{Inv}_{r'}$. By (*), we have

$$(r^-)^{\mathcal{I}'} = (r^-)^{\mathcal{I}} \cup ((r_i^-)^-)^{\mathcal{I}} = (r^-)^{\mathcal{I}} \cup r_i^{\mathcal{I}} = (r_i'^-)^{\mathcal{I}} \cup r'^{\mathcal{I}} = r'^{\mathcal{I}'}$$

- $r \sqsubseteq s$ Since $r \sqsubseteq s \in \mathcal{O}^-_{\mathcal{ID}}$, $r^{\mathcal{I}} \subseteq s^{\mathcal{I}}$. If $\mathsf{Inv}_s \neq \emptyset$ but $\mathsf{Inv}_r = \emptyset$, we have $r^{\mathcal{I}'} = r^{\mathcal{I}} \subseteq s^{\mathcal{I}} \subseteq s^{\mathcal{I}'}$. Otherwise $\mathsf{Inv}_r \neq \emptyset$. By line 10 to 14 we know that there exists some $s' \in \mathsf{Inv}_s$ such that $r_i \sqsubseteq s' \in \mathcal{O}^-_{\mathcal{ID}}$. Let s_i be the representative role in Inv_s, by (*) we have

$$r^{\mathcal{I}'} = r^{\mathcal{I}} \cup (r_i^-)^{\mathcal{I}} \subseteq s^{\mathcal{I}} \cup (s'^-)^{\mathcal{I}} = s^{\mathcal{I}} \cup (s_i^-)^{\mathcal{I}} = s^{\mathcal{I}'}$$

- $A \sqsubseteq \exists r.B$ By (*) and $B^{\mathcal{I}'} = B^{\mathcal{I}}$, we have $(\exists r.B)^{\mathcal{I}'} = (\exists r.B)^{\mathcal{I}} \cup (\exists r_i^-.B)^{\mathcal{I}}$. Thus $A^{\mathcal{I}'} = A^{\mathcal{I}} \subseteq (\exists r.B)^{\mathcal{I}} \subseteq (\exists r.B)^{\mathcal{I}'}$.

- $\exists r.A \sqsubseteq B$ By (*) and $A^{\mathcal{I}'} = A^{\mathcal{I}}$, we have $(\exists r.A)^{\mathcal{I}'} = (\exists r.A)^{\mathcal{I}} \cup (\exists r_i^-.A)^{\mathcal{I}}$. Since $\exists r.A \sqsubseteq B \in \mathcal{O}^-_{\mathcal{ID}}$, $(\exists r.A)^{\mathcal{I}} \subseteq B^{\mathcal{I}}$. By line 15, $A \sqsubseteq \forall r_i.B \in \mathcal{O}^-_{\mathcal{ID}}$, thus $\mathcal{O}^-_{\mathcal{ID}} \models \exists r_i^-.A \sqsubseteq B$, and so $(\exists r_i^-.A)^{\mathcal{I}} \subseteq B^{\mathcal{I}}$. Hence $(\exists r.A)^{\mathcal{I}'} \subseteq B^{\mathcal{I}} = B^{\mathcal{I}'}$.

- $A \sqsubseteq \forall r.B$ By (*) and $B^{\mathcal{I}'} = B^{\mathcal{I}}$, we have $(\forall r.B)^{\mathcal{I}'} = (\forall r.B)^{\mathcal{I}} \cap (\forall r_i^-.B)^{\mathcal{I}}$. Since $A \sqsubseteq \forall r.B \in \mathcal{O}^-_{\mathcal{ID}}$, $A^{\mathcal{I}} \subseteq (\forall r.B)^{\mathcal{I}}$. By line 16, $\exists r_i.A \sqsubseteq B \in \mathcal{O}^-_{\mathcal{ID}}$, thus $\mathcal{O}^-_{\mathcal{ID}} \models A \sqsubseteq \forall r_i^-.B$, and so $A^{\mathcal{I}} \subseteq (\forall r_i^-.B)^{\mathcal{I}}$. Hence $A^{\mathcal{I}'} = A^{\mathcal{I}} \subseteq (\forall r.B)^{\mathcal{I}'}$.

Hence $\mathcal{I}'$ is a model of $\mathcal{O}'$, and $(A \sqcap \neg B)^{\mathcal{I}'} = (A \sqcap \neg B)^{\mathcal{I}} \neq \emptyset$. So $\mathcal{O}' \not\models A \sqsubseteq B$. Thus for every $A, B \in N_C^{\top,\bot} \cap \mathsf{Sig}(\mathcal{O}^-_{\mathcal{D}})$, $\mathcal{O}' \models A \sqsubseteq B$ implies $\mathcal{O}^-_{\mathcal{ID}} \models A \sqsubseteq B$, and the theorem is proved. □

5 Experiment and Conclusion

In experiments we compare the runtime of our WSClassifier with all other available $\mathcal{ALCHI}(\mathcal{D})$ reasoners HermiT, FaCT++ and Pellet, which all happen to be tableau-based reasoners. We use all large and highly cyclic ontologies we can access to. FMA-constitutionalPartForNS(FMA-C) is the only large and highly cyclic ontology that contains $\mathcal{ALCHI}(\mathcal{D})$ constructors. We remove 7 axioms using $xsd:float$. For Full-Galen which language is $\mathcal{ALEHIF}+$ without "D", we introduce some new data type axioms by converting some axioms using roles hasNumber and hasMagnitude into axioms with new features hasNumberDT and hasMagnitudeDT. Some concepts which should be modeled as data ranges are also converted to data ranges. For Galen-Heart, we did not introduce "D" to it, but it contains inverse roles. Wine is a small but cyclic ontology. We also include two commonly used ontoloiges ACGT and OBI which are not highly cyclic. For Wine, ACGT and OBI, we change xsd:int, xsd:positiveInteger, xsd:nonNegativeInteger to xsd:integer, xsd:float to owl:rational, and remove xsd:dateTime if applicable. For all the ontologies, we reduce their language to $\mathcal{ALCHI}(\mathcal{D})$. The ontologies are available from our website.[3].The experiments were conducted on a laptop with Intel Core i7-2670QM 2.20GHz quad core CPU and 16GB RAM. We set the Java heap space to 12GB and the time limit to 24 hours.

Table 3: Comparison of classification performance of $\mathcal{ALCHI}(\mathcal{D})$ ontologies

	HermiT	**Pellet**	**FaCT++**	**WSClassifier**
Wine	1.160 sec	0.430 sec	0.005 sec	0.400 sec
ACGT	9.603 sec	2.955 sec	*	1.945 sec
OBI	3.166 sec	45.261 sec	*	8.835 sec
Galen-Heart	123.628 sec	–	–	2.779 sec
Full-Galen	–	–	–	16.774 sec
FMA-C	–	–	–	32.74 sec

Note: "–": out of time or memory "*": some datatypes are not supported

Table 3 summarizes the result. HermiT is set to configuration with simple core blocking and individual reuse. WSClassifier is significantly faster than the tableau-based reasoners on the three highly cyclic large ontologies Galen-Heart, Full-Galen and FMA-C. ACGT is not highly cyclic, but WSClassifier is still faster. For the other two ontologies where WSClassifier is not the fastest, Wine is cyclic but small, OBI is not highly cyclic. The classification time for them on all reasoners are significantly shorter comparing with the time on large highly cyclic ontologies. Then WSClassifier took a larger percentage of time on the overhead to transmit the ontology to and from ConDOR. FaCT++ does not support owl:rational, so it can not classify ACGT and OBI, but it is much faster on Wine.

We have transformed some commonly used OWL 2 datatypes and facets and inverse role axioms in an $\mathcal{ALCHI}(\mathcal{D})$ ontology to $\mathcal{ALCH}$ and classified it on an $\mathcal{ALCH}$ reasoner with soundness and completeness of classification preserved. WSClassifier greatly outperforms tableau-based reasoners when the ontologies

[3]`http://isel.cs.unb.ca/~wsong/ORE2013WSClassifierOntologies.zip`

are large and highly cyclic. Future work includes extension to other data types and facets, and further optimization, e.g. adapting the idea of Magka *et al.* [9] to WSClassifier to distinguish positive and negative occurrences of data ranges, in order to reduce the number of axioms to be added.

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