

Faculty of Computer Science
CS 3113
Fall 2003

Assignment # 2

Due: 8:30am, Oct. 10, 2003

1. Population growth over a short period of time can be modeled using Malthusian growth. The solution of the Malthusian growth model is:

$$P(t) = P_0 e^{rt}$$

where P_0 is the initial population, $P(t)$ is the population at time t , and r is the rate of growth. For example, from the population of the U.S.A. in 1950 ($P_0 = 151.3$ Million) and 1960 ($P(t = 10) = 179.3$ Million) we can easily find that this model gives a growth rate of about $r = 1.7\%$.

However, this growth model simply gives the net growth rate. A major factor in the growth of human populations has been the inflow of immigrants. The Malthusian growth model can be modified to include an immigration rate m , and solved to give the following equation:

$$P(t) = (P_0 + \frac{m}{r})e^{rt} - \frac{m}{r} \quad (1)$$

We would like to determine how much of the 1.7% growth rate is due to immigration and how much is the growth rate r (births - deaths), between 1950 and 1960 in the U.S.A., given an immigration rate of $m = 250,000$ immigrants per year.

Solve Equation (1) for r using

- (a) fixed-point iteration
- (b) bisection
- (c) Newton's method
 - i. using a good initial guess.
 - ii. Demonstrate that your results exhibit quadratic convergence.
 - iii. Redo using a guess of $r = 2$; comment on the difference in convergence rate.

Comment on the different behaviour of the three methods. Hand in all MATLAB .m files that you create, as well as printouts showing the estimated root and estimated relative error (see comment to last question below) for each iteration.

2. Recktenwald, Numerical Methods with MATLAB (NMM), Chapter 6, exercise 7 (note the error in the question: the inequality should be $|g'(x)| < 1$)
3. NMM, Chapter 6, exercise 22
4. NMM, Chapter 6, exercise 29, **with the following modification:** use $|\frac{x_{k+1} - x_k}{x_{k+1}}| < xtol$ to determine convergence. Therefore, $ftol$ is not required.