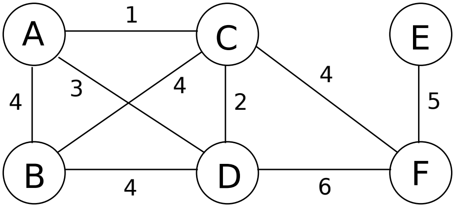
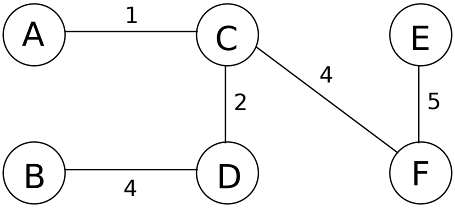
Outline

Greedy

- MST

Minimum spanning tree





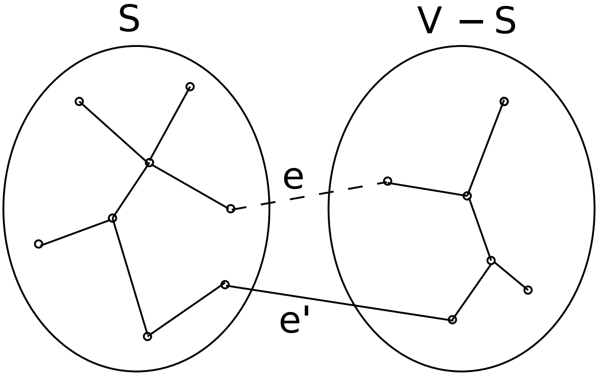
Minimum Spanning Tree

Given $G = (V, E)$, $w : E \to \mathbb{R}$, a *minimum spanning tree* T is a spanning tree (i.e. connecting all vertices) that minimizes $\mathrm{cost}(T) = \sum_{e \in T} w(e)$

Cut Property

Lemma

Let T be a minimum spanning tree, $X \subset T$ s.t. X does not connect $(S, V - S)$. Let e be the lightest edge from S to $V - S$. $X \cup e$ is part of some MST.



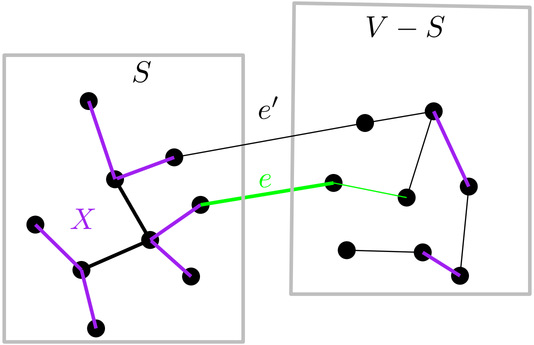
Cut Property Proof

Cut Property

Let T be an MST, $X \subset T$ s.t. X does not connect $(S, V - S)$. Let e be the lightest edge from S to $V - S$. $X \cup e$ is part of some MST.

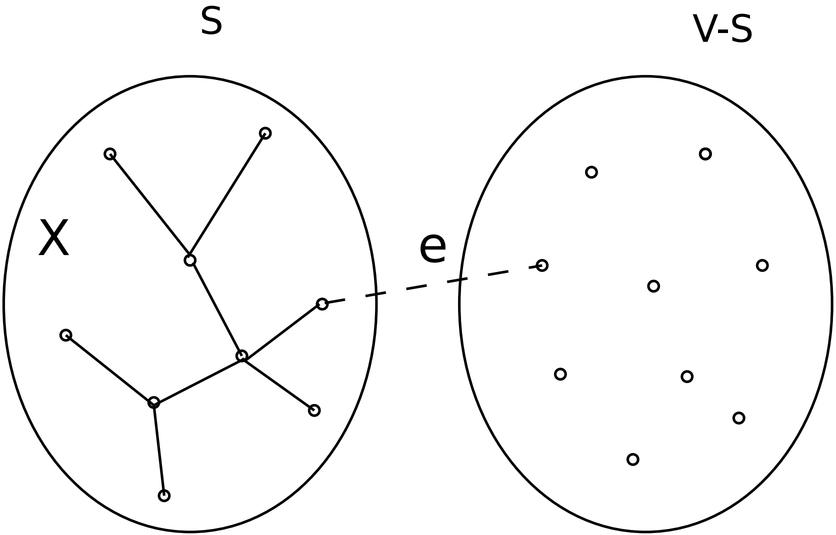
- Let $X \subseteq T$ where T is MST
- if $e \in T$, done
- add e to T, makes a cycle

Cut Property Proof



- Let $X \subseteq T$ where T is MST
- if $e \in T$, done
- add e to T, makes a cycle
- $\exists$ crossing $e' \in E - X$
- swap e and e'

Prim's Algorithm

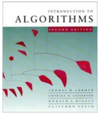


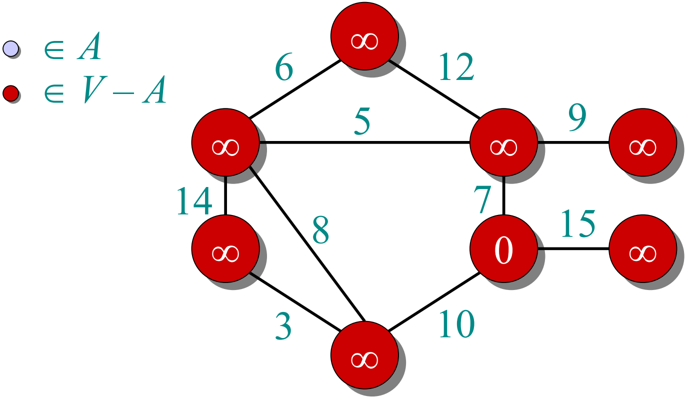
$S =$ nodes reached so far

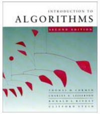
Prim's Algorithm

```python
def prim(G,root):
  pq = pqdict(); prev = {}
  for v in G.keys():
    pq.additem(v,inf)
  pq.updateitem(root,0)
  while len(pq)>0:
    v = pq.pop()
    for (z,weight) in G[v]:
      if z in pq and weight < pq[z]:
        prev[z]=v
        pq.updateitem(z,weight)
  return prev
```

Example of Prim's algorithm







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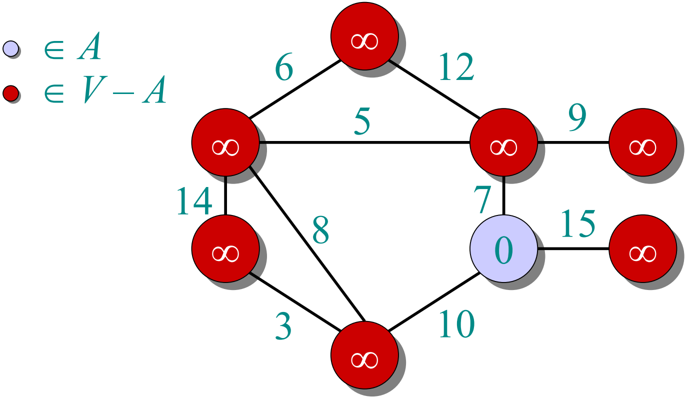

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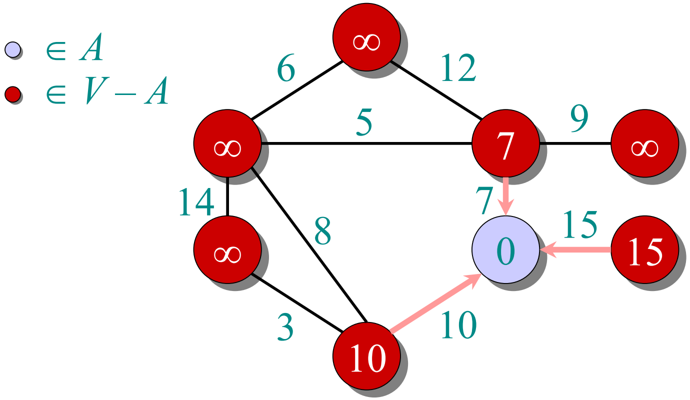

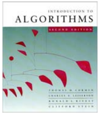

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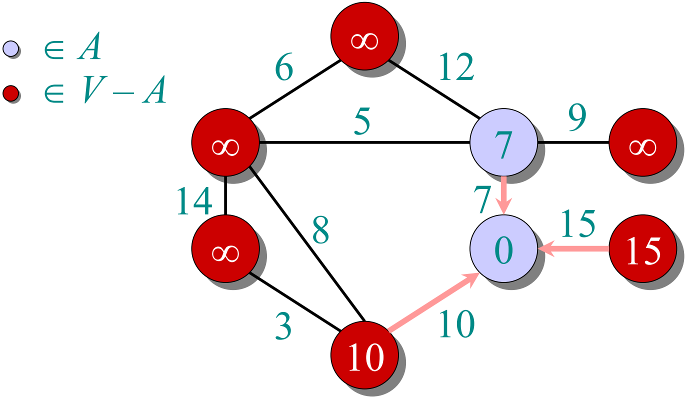

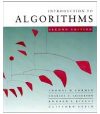

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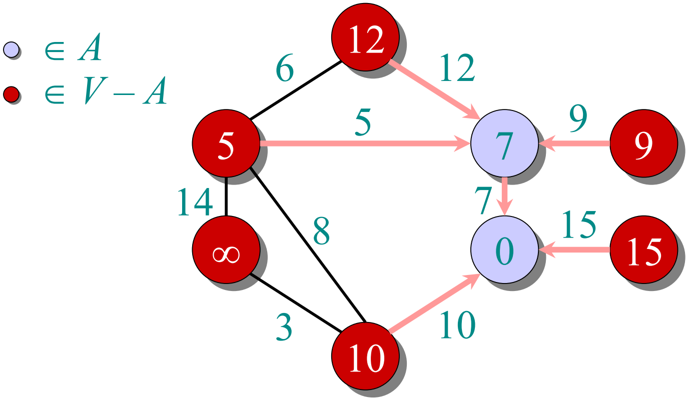

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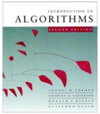

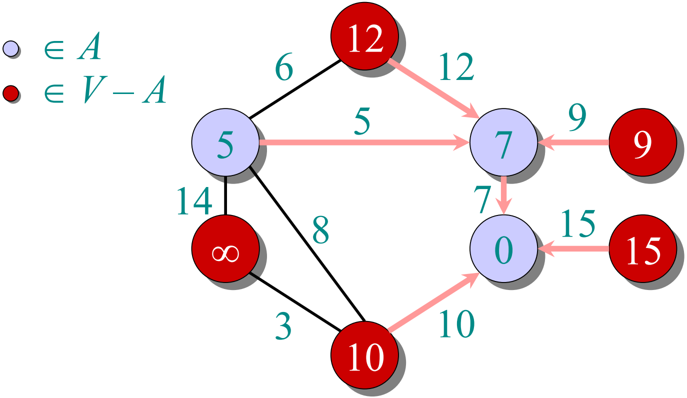

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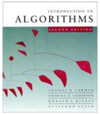

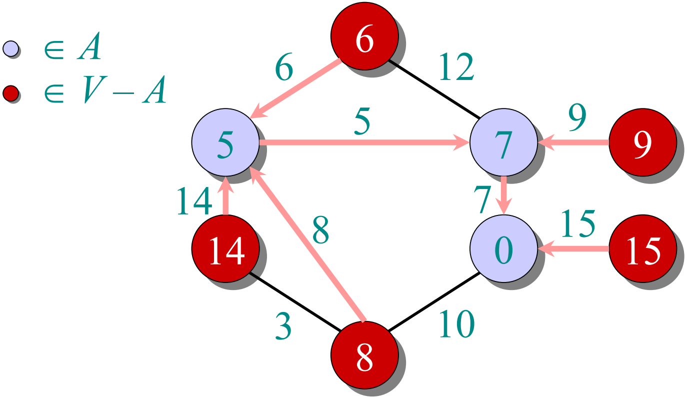

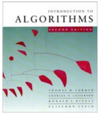

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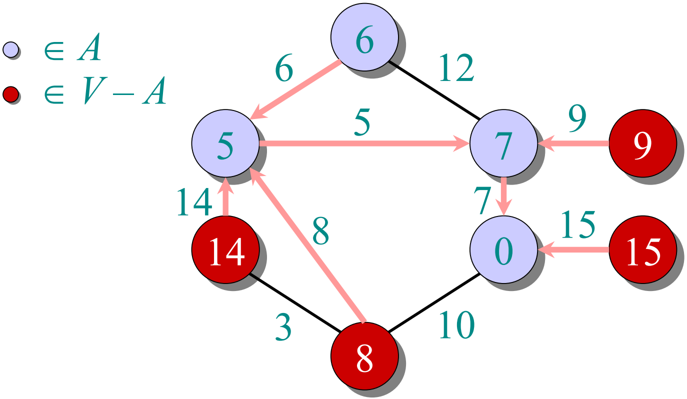
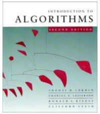
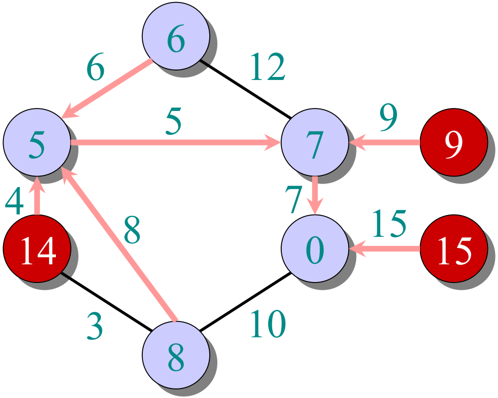




Example of Prim's algorithm

$\in A$

$\in V - A$



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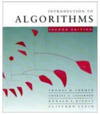

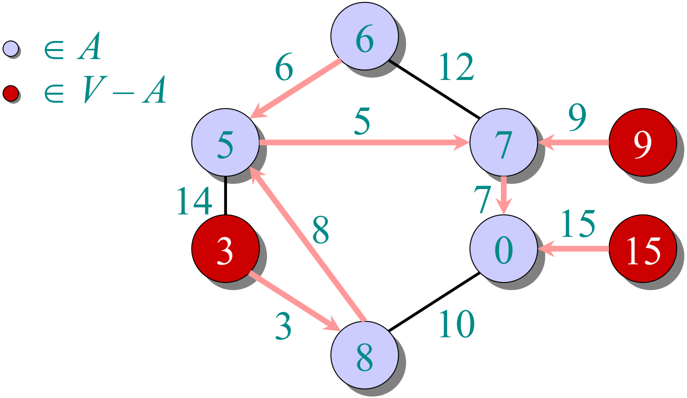

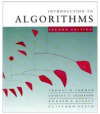

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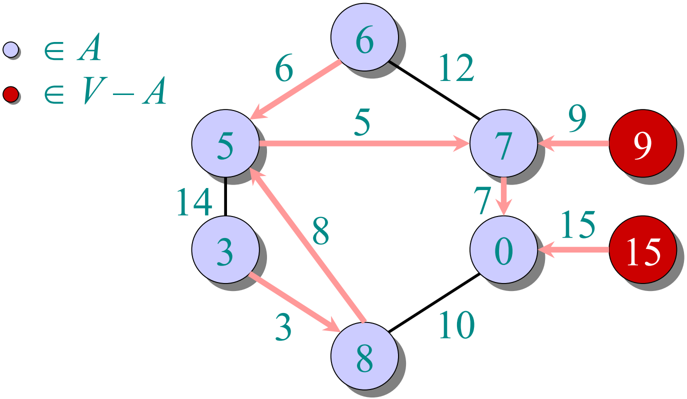

Example of Prim's algorithm