CS3383 Unit 3: Dynamic Programming

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Outline

Dynamic Programming

Shortest path in DAG

March Break Hotels

Scenario

Wanted	Cheap holiday
Costs	Hotel + Taxi, no charge for inconvenience

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Taxi Cost

	a	b	c	aprt
a	0	10	30	50
b	10	0	30	50
c	30	30	0	50
aprt	50	50	50	0

Hotel Price

	1	2	3	4
a	100	100	100	100
b	80	40	120	120
c	50	80	80	80

It's a trap!

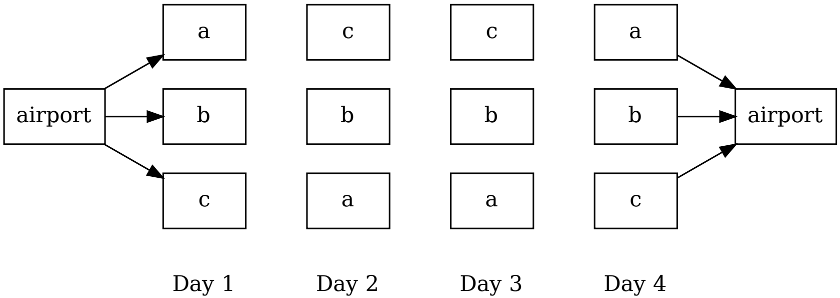
Hotel Price

	1	2	3	4
a	100	100	100	100
b	80	40	120	120
c	50	80	80	80

Taxi Cost

	a	b	c	airport
a	0	10	30	50
b	10	0	30	50
c	1000	1000	0	500
airport	50	50	50	0

Let's get graphical



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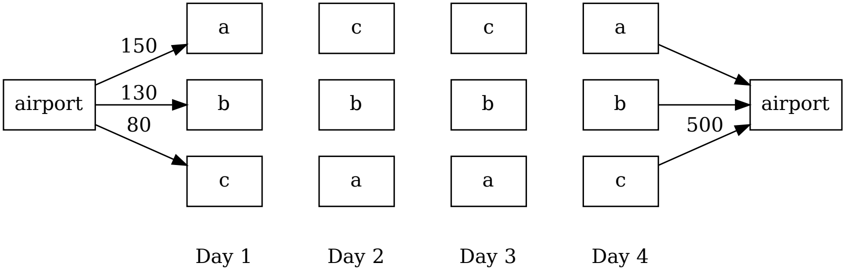

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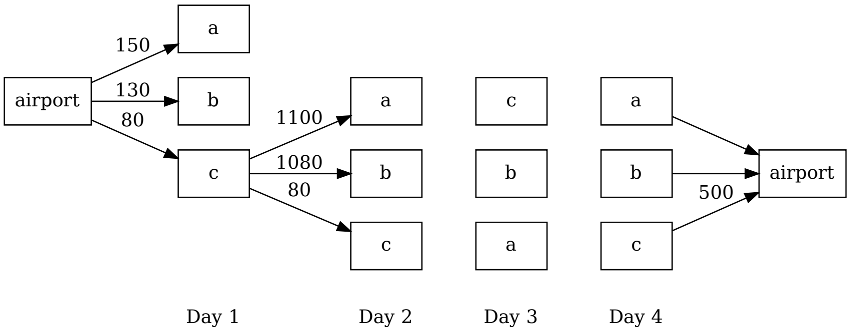

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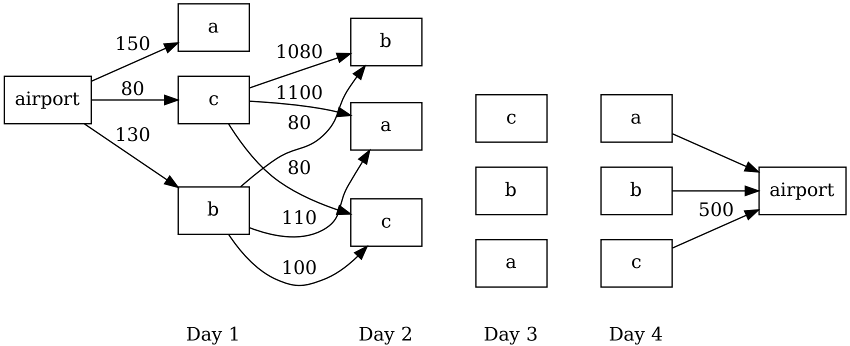

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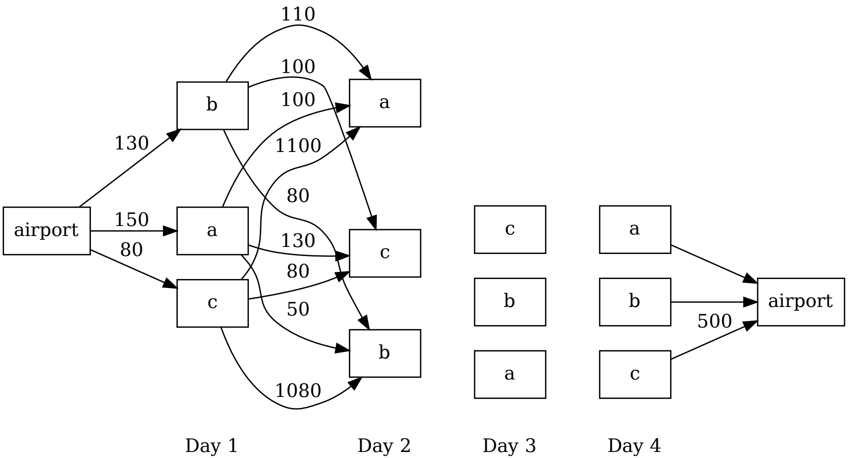

Djikstra considered overkill

- There are no negative edge weights, so shortest path is tractable.
- Even better, we have an *acyclic* graph (why?)
- So we find a shortest path in linear time after *topological sorting*.

“Recursive” topological sort

Recursive ”algorithm”

1. Remove a *source* from the DAG, and put it first.
2. Topologically sort the remaining graph.

- how to quickly find a source?

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- how to quickly find a source?
- Use some auxilary data structure to track sources across iterations

Using a Queue

BFS-like topological sort

```
function TopSort(G)
    Q ← All Sources
    while !empty(Q) do
        v ← deq(Q)


    end while
end function
```

Using a Queue

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        Remove v, add new sources to Q
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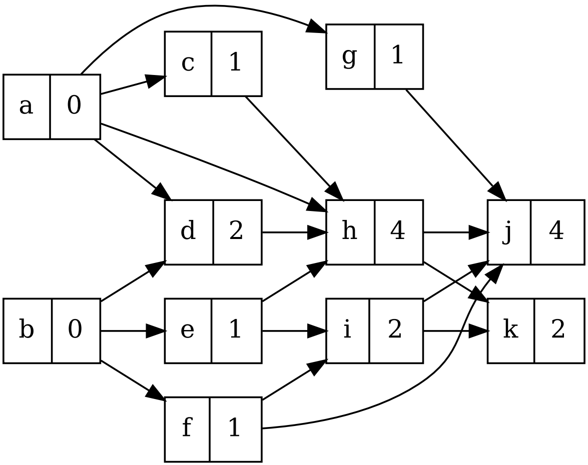
Using a Queue

BFS-like topological sort

1: **function** TopSort(G)
2: $Q \leftarrow$ All Sources
3: **while** !empty(Q) **do**
4: $v \leftarrow \mathrm{deq}(Q)$
5: Output v
6: Remove v, add new sources to Q
7: **end while**
8: **end function**

- What is the complexity of step 6?

Topological sort with counters



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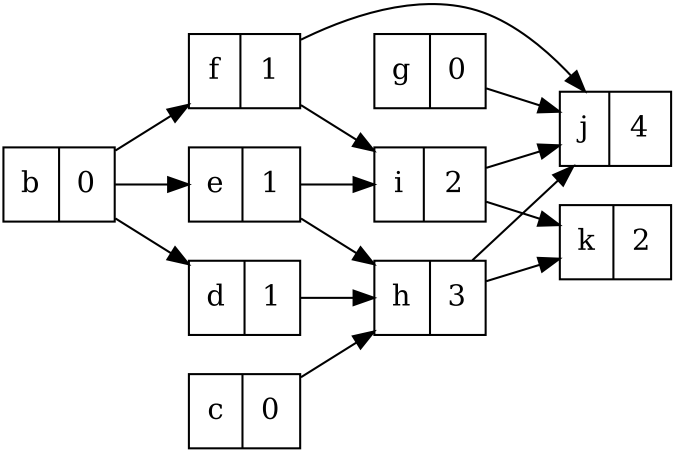

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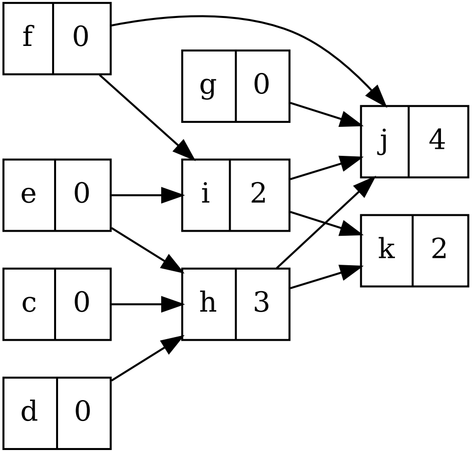

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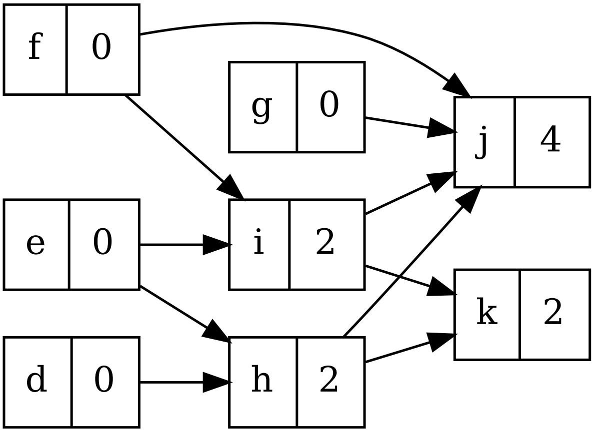
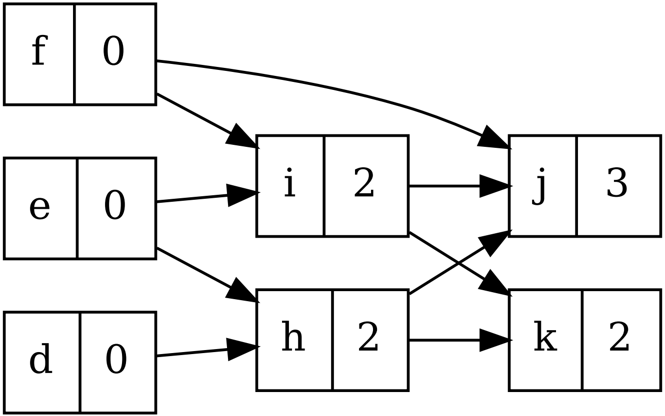


Topological sort with counters



No priority queue needed

```
while len(Q) > 0:
    v = Q.popleft()
    rank[v]=len(output)
    output.append(v)
    for (u,_) in G[v]:
        count[u] -= 1
        if count[u] == 0:
            Q.append(u)
```

Shortest Paths in DAGs

- Every path in a DAG goes through nodes in linearized (topological sort) order.

```
for j in range(rank[root]+1,n):
    v = order[j]
    for (prev,w) in In[v]:
        if w+dist[prev] < dist[v]:
            dist[v]=w+dist[prev]
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Shortest Paths in DAGs

- Every path in a DAG goes through nodes in linearized (topological sort) order.
- *every node is reached via its predecessors*
- So we need a single loop after sorting.

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Ordered Subproblems

In order to solve our problem in a single pass, we need

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Ordered Subproblems

In order to solve our problem in a single pass, we need

- An ordered set of subproblems $L(i)$
- Each subproblem $L(i)$ can be solved using only the answers for $L(j)$, for $j < i$.