Examples from Chapter 9

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Outline

Example

Consider the function

$$F = \begin{bmatrix} 11-100-1000 \\ 11-010-0100 \\ 11-001-0011 \\ 01-110-0001 \end{bmatrix}$$

Example

Since X_3 has four active columns, we expand F with respect to X_3. Let

$$\begin{aligned}
S_A &= \{0,1\}, S_B = \{2,3\}. \text{ We have} \\
c_1 &= (11-111-1100), \\
c_2 &= (11-111-0011), \\
F_1 &= F(|c_1) = \begin{bmatrix} 11-100-1011 \\ 11-101-0111 \end{bmatrix}, \text{and} \\
F_2 &= F(|c_2) = \begin{bmatrix} 11-001-1111 \\ 01-110-1101 \end{bmatrix}.
\end{aligned}$$

Example

Next, expand F_1 by variable X_2. We have

$$
\begin{aligned}
F_1 &= c_3 F_1(|c_3) \vee c_4 F_1(|c_4), \text{ where} \\
c_3 &= (11 - 100 - 1111), \\
c_4 &= (11 - 011 - 1111), \\
F_3 &= F_1(|c_3) = (11 - 111 - 1011), \text{ and} \\
F_4 &= F_1(|c_4) = (11 - 110 - 0111).
\end{aligned}
$$

Example

Similarly, expand F_2 by variable X_2. We have

$$
\begin{aligned}
F_2 &= c_5 F_2(|c_5) \vee c_6 F_2(|c_6), \text{ where} \\
c_5 &= (11-110-1111), \\
c_6 &= (11-001-1111), \\
F_5 &= F_2(|c_5) = (01-111-1101), \text{ and} \\
F_6 &= F_2(|c_6) = (11-111-1111).
\end{aligned}
$$

Example

F_3 - F_6 consist of single products. We have

$$
\begin{aligned}
\overline{F_3} &= (11-111-0100), \\
\overline{F_4} &= \begin{bmatrix} 11-001-1111 \\ 11-110-1000 \end{bmatrix}, \\
\overline{F_5} &= \begin{bmatrix} 10-111-1111 \\ 01-111-0010 \end{bmatrix}, \\
\overline{F_6} &= 0.
\end{aligned}
$$

Example

By combining all products, we have:

$$
\begin{aligned}
\overline{F} &= c_1\overline{F_1} \vee c_2\overline{F_2} \\
&= c_1(c_3\overline{F_3} \vee c_4\overline{F_4}) \vee c_2(c_5\overline{F_5} \vee c_6\overline{F_6}) \\
&= c_1c_3\overline{F_3} \vee c_1c_4\overline{F_4} \vee c_2c_5\overline{F_5} \vee c_2c_6\overline{F_6} \\
&= \begin{bmatrix} 11-100-0100 \\ 11-001-1100 \\ 11-010-1000 \\ 10-110-0011 \\ 01-110-0010 \end{bmatrix}.
\end{aligned}
$$

Note that $\overline{F}$ is disjoint.

Example

Consider the function

$$F = \begin{bmatrix} 1000 - 1100 \\ 1100 - 0010 \\ 0010 - 1111 \\ 1111 - 0010 \end{bmatrix}$$

Example

Expand F with respect to X_1. Let $S_A = \{0, 1\}$ and $S_B = \{2, 3\}$.

$$
\begin{aligned}
c_1 &= (1100 - 1111) \\
c_2 &= (0011 - 1111) \\
F_1 &= F(|c_1) = \left[\begin{array}{c} 1011 - 1100 \\ 1111 - 0010 \\ 1111 - 0010 \end{array} \right] \\
F_2 &= F(|c_2) = \left[\begin{array}{c} 1110 - 1111 \\ 1111 - 0010 \end{array} \right]
\end{aligned}
$$

Example

Expand F_1 with respect to X_2. Let $S_A = \{0, 1\}$ and $S_B = \{2, 3\}$.

$$\begin{aligned} c_3 &= (1111 - 1100), c_4 = (1111 - 0011) \\ F_3 &= F_1(|c_3) = (1011 - 1111) \\ F_4 &= F_1(|c_4) = \left[\begin{array}{c} 1111 - 1110 \\ 1111 - 1110 \end{array} \right] \end{aligned}$$

$PI(F_1)$ is obtained from $c_3 \cdot PI(F_3) \cup c_4 \cdot PI(F_4) \cup PI(F_3) \cdot PI(F_4)$.

$$\begin{aligned} c_3 \cdot PI(F_3) &= (1011 - 1100) \text{ deleted} \\ c_4 \cdot PI(F_4) &= (1111 - 0010) \\ PI(F_3) \cdot PI(F_4) &= (1011 - 1110) \end{aligned}$$

Example

F_2 is strongly unate. There is no cube that is contained in another cube.

$$PI(F_2) = \begin{bmatrix} 1110-1111 \\ 1111-0010 \end{bmatrix}$$

Example

$PI(F)$ is obtained from $c_1 \cdot PI(F_1) \cup c_2 \cdot PI(F_2) \cup PI(F_1) \cdot PI(F_2)$.

$$
\begin{aligned}
c_1 \cdot PI(F_1) &= \begin{bmatrix} 1100 - 0010 \\ 1000 - 1110 \end{bmatrix} \\
c_2 \cdot PI(F_2) &= \begin{bmatrix} 0010 - 1111 \\ 0011 - 0010 \end{bmatrix} \\
PI(F_1) \cdot PI(F_2) &= \begin{bmatrix} 1111 - 0010 \\ 1010 - 1110 \end{bmatrix}
\end{aligned}
$$

Example

After deleting cubes that are contained in other cubes, we have:

$$PI(F) = \begin{bmatrix} 0010-1111 \\ 1111-0010 \\ 1010-1110 \end{bmatrix}$$

Example

	0	1	2	3
0	1		1	
1	1		1	
2	1	1	1	1
3			1	