Nature Is Fundamentally Discrete but Our Basic Formalism Is Not*

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Abstract In physics, we have been seriously confronted with the question of continuous vs. discrete since the beginning of the last century. Why is it still with us, and increasingly so in the last sixty years? The title of the essay suggests the reason: so far, we have relied on the "continuous", or vector space, mathematics (*of spatial origin*), the only one we have, while the experiments suggest that, at the bottom, the nature is non-continuous, or *discrete*, albeit *in a sense unfamiliar to us*. We have tried to save the situation by "discretizing" our conventional models, but for the reasons I discuss here such desperate attempts to transform our basic formalism (by destroying its integrity) are not meaningful. We may have no other choice than to set aside for a while the millennia-old numeric, or spatial, forms of representation and the associated measurement processes and to begin *completely anew*, by shifting to a *non-numeric—relational, or temporal—representational formalism*, which should give the meaning to the nebulous concept of discreteness and, even more importantly, should *remove the enormous present gap between the physical and the mental.*

1. Introduction: How are we to understand the "discreteness" of nature?

We are facing a very peculiar situation concerning the term "discrete". Although it is quite popular in mathematics and natural sciences, its familiarity is quite deceptive, since we use it simply as a name by which we designate *all non-continuous models*—e.g. graphs (with nodes and edges), strings over a finite alphabet—or models obtained from a continuous model by simply "discretizing" it. So, "discrete" means anything that is not continuous, i.e. we are dealing with the *negation of a particular formalism* rather than with another, clearly delineated formalism, and when we ask if the nature is discrete, we are simply asking if it is not continuous. To the latter question we have already had the answer (which we don't like), and I quote, for example, Schrödinger [1]:

> If you envisage the development of physics in *the last half-century*, you get the impression that the discontinuous aspect of nature has been forced upon us *very much against our will*. We seemed to feel quite happy with the continuum. Max Planck was seriously frightened by the idea of a discontinuous exchange of energy ... Twenty-five years later the inventors of wave mechanics indulged for some time in the fond hope that they have paved the way of return to a classical continuous description, but again the hope was deceptive. Nature herself seemed to reject continuous description … [p. 158]
>
> The observed facts (about particles and light and all sorts of radiation and their mutual interaction) appear to be *repugnant* to the classical ideal of continuous description in space and time. ... So the facts of observation are irreconcilable with a continuous description in space and time … [pp. 143–44]

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Thus we actually have the answer to the essay contest question. The reason why we have not been satisfied with this answer is simple: having excluded the "standard" (continuous) formalism, we are left with *no concrete formalism* to take its place, hence *no adequate understanding of the "discreteness"*. Before discussing shortly a proposal for discrete formalism, let us first deal briefly with one important, but overlooked, point regarding *the spatial origin of our basic (continuous) formalism*.

2. Why are our mathematics and physics "spatial"?

As we trace the roots of our current mathematics, we see that its *earnest* development began with the geometric considerations: "The most ancient mathematical texts available are Plimpton 322 (Babylonian mathematics c. 1900 BC), ... the Rhind Mathematical Papyrus (Egyptian mathematics c. 2000-1800 BC) ... and the Moscow Mathematical Papyrus (Egyptian mathematics c. 1890 BC). All of these texts concern the so-called Pythagorean theorem ..." [2]. Indeed, it is not surprising that the spatial, or geometric, considerations are at the root of our mathematics: they were indispensable for various constructions, including the recovery of agricultural plots of various shapes after the yearly flooding of the Nile in ancient Egypt, as well as the constructions of temples and pyramids. In general, it is important to keep in mind that *the very concept of measurement emerged as that of spatial measurement*, of length in particular.

Much later, Descartes' "fusion" of geometry and algebra—via analytic geometry, leading eventually to the concept of vector space—provided the basis for the development of calculus, with the decisive contributions by Newton and Leibniz. Finally, what completed the "spatialization" of mathematics is the "consolidation of foundations of mathematics in the form of set theory at the end of the 19^{th} – beginning of the 20^{th} centuries. The concept of set—a collection of (*unstructured, point-like*) elements—abstracted and *entrenched the ubiquitous spatial form of representation, "the point"*; so that at present, a mathematical structure (e.g. a vector space) is defined axiomatically *starting with some underlying set of elements of unspecified structure*, and this structure is "revealed" only later, during the analysis of the formal consequences of the chosen axiomatic system (e.g. the algebraic structure of vectors)." [3, p. 3]

There are also perfectly natural, i.e. physiological, reasons for the dominance of spatial considerations in our mathematics: *vision is by far our dominant and most powerful sense*. However, we should not forget the above historical reality, notwithstanding the large number of mathematical abstractions that have been introduced over the last two centuries. Incidentally, in topology—which over the last century grew into one of, if not the most powerful branch of mathematics—spatial considerations are also dominant. No doubt it is the latter fact that prompted the comment of a leading mathematician of the last century Hermann Weyl: "the angel of topology and the devil of abstract algebra fight for the soul of every individual discipline of mathematics."

As far as the dominant role of geometric (spatial) considerations, *including symmetry*, in modern physics is concerned, the famous Einstein's comments on his field equation of general relativity are still quite representative, as David Gross observes [4, pp. 149–50, my emphasis] :

> Einstein used to say that he liked the left-hand side of the equation—that was beautiful, that was geometry, that was the curvature of space. But he didn't like the right-hand side, which referred to this "matter" that you had to put in an arbitrary way. So he used to say the left-hand of his equation is beautiful and the right-hand side is ugly. Much of what he was doing in the latter part of his career was trying to move the right-hand side to the left-hand side and understand matter as a geometrical structure. *To build matter itself from geometry—that in a sense is what string theory does*.

Finally, it is also important to keep in mind that the spatialization of time in physics, i.e. associating it with one-dimensional subspace of the corresponding vector space, is a consequence of the above entrenched practical experience of *approaching any concept of measurement via that of spatial measurement*. Thus I would say regrettably but *unavoidably*, the enigma of time has so far been tamed relying on our *most complex* sense, vision, rather than on the auditory sense, which gives us a *more direct access to the temporal side of reality* (see the last paragraph of Section 5.3).

3. Why we cannot "save" the continuous framework by discretizing it

Continuity, as a concept, has matured during the formalization of the real number system, which in turn has been guided by the considerations coming out of calculus, e.g. limits. Yet already in antiquity the necessity to *complete* the set of rational numbers by the set of irrational numbers — which are physically intangible and whose size, i.e. cardinality, is, paradoxically, incomparably larger than that of rational numbers— was realized.

However, as far as physics is concerned, a key difficulty arises from the following basic but important observation made by Riemann in his lectures on partial differential equations [5, pp. 166–67]:

> As is well known, physics became a science only after the invention of differential calculus. It was *after realizing* [rather postulating] *that natural phenomena are continuous* that attempts to construct abstract models were successful. …
>
> *True basic [physical] laws can only hold in the small and must be formulated as partial differential equations. Their integration provides the laws for extended parts of time and space.* [my italics]

At present, it appears that these laws may not "hold in the small" (e.g. Heisenberg's indeterminacy relations), so *the applicability of the logic of calculus—extending local laws to global laws—is in serious question*. This constitutes one of the fundamental issues where the very structure of the continuous formalism appears to be inconsistent with the physical reality.

Another serious difficulty arises in connection with some, I would say desperate, attempts to address the experimentally observed discreteness by simply "discretizing" continuous models, i.e. to avoid infinite divisibility by postulating some "indivisible" units of space and time, Planck units. Such attempts should not be taken seriously simply because our basic *formal* model of a (real or complex) vector space does not allow for any "discretization". Besides, by doing so one introduces an oxymoron concept of an "indivisible segment", since the boundaries of such segment are *points* which one is trying to eliminate.

Thus, regrettably, at the time when, I believe, we are actually faced with an *unprecedented, complete reconstruction of physics*, what prevents us from fully focusing on this task is a *widespread misunderstanding regarding the nature, the role, and the malleability of our basic mathematical formalism*. This formalism—which, guided by the spatial intuition, has gradually emerged over thousands of years and has served us quite well—with all its limitations, represents an *integral whole* that cannot be tinkered with without destroying its integrity[1] and therefore the logic of its applications. Similar observations were made by Heisenberg regarding the integrity of all basic physical theories [6, pp. 123–29].[2]

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[1] In desperation, we have already proceeded to do precisely this (plus heavy reliance on the statistical modeling).

[2] "Small improvements in these theories can no longer be undertaken" (since they represent integral wholes).

4. The need for a fundamentally new formalism to elucidate the "discreteness" and to guide the development of new physics

Together with a number of leading physicists (and philosophers) of the last and present centuries[i], I believe that in order to capture adequately the physical concept of discreteness we need to begin a very gradual, *unprecedented*, conceptual rethinking of physics, *from the ground up*. As was historically the case, *such complete reconstruction cannot productively begin without the guidance of a fundamentally new, discrete, formal foundation*, motivated, I believe, by *(non-spatial) temporal and relational* considerations, which must be addressed at *the most fundamental* yet still unexplored level, the *representational* level.

Representational formalism is responsible for *the form of data representation*, i.e. it specifies the abstract structure for representing data, *the data template* (which, once introduced, allows theoreticians to proceed with its refinement and elaboration). Such formalism is our scientific means of representing reality, the "spectacles" through which we see it. *So far, we have relied on the single representational formalism*, i.e. the real numbers, including its variations such as complex numbers, quaternions, octonions, etc. I believe that *we are poised to shift from the ubiquitous numeric representation and the associated measurement process to the evolving structural representation and the associated structural measurement process*. Thus I suggest the key to understanding the difference between the continuous and the discrete is this *transition from the point-based representation to the evolving structural representation* (see Figs. 2, 4, 5).

In contrast to our numeric formalism, this new formalism cannot be properly attempted without *greater commitment to the primacy of a particular (structural) side of physical reality*, which, as we will see (Section 5.3), *brings with it decisive advantages* compared to the more *promiscuous* numeric form of representation. I suggest this commitment to be as follows: we should both view and represent all "objects" in nature as (instantiations of irreversible) temporal processes comprised of temporally related events.

It is interesting to note that, for example, James Jeans (following Bertrand Russell) thought along related lines as early as in 1933 [9, my emphasis]:

> Thus the "world-line" of a particle is ... not a line at all, but a ... curved region, and must logically be separated into small curved spots—the particle resolves itself into events. Most of these events are unobservable; it is only when two particles meet or come near to one another that we have an observable event which can affect our senses. We have no knowledge of the existence of the particle between ... [events], so that *observation only warrants us in regarding its existence as a succession of isolated events*. [p. 293]
>
> *Thus the events must be treated as the fundamental objective constituents* and we must no longer think of the universe as consisting of solid pieces of matter which persist in time, and move about in space. ... We now begin to suspect that *events and not particles constitute the true objective reality* ... [pp. 294–5]

Some modern physicists, e.g. Smolin [10], also support the above view:

> From this new point of view, the universe consists of a large number of events. An event may be thought of as a smallest part of the process, a smallest unit of change. ... The universe of events is a relational universe. That is all its properties are described in term of relationships between the events. [p. 53]
>
> [The future quantum theory] will be reformulated as a theory about *the flow of information among events*. ... *The idea of "states" will have no place in the final theory, which will be framed around the*

idea about the processes and the information conveyed between them and modified within them. [pp. 210–11, my emphasis]

5. On the proposed concept of discrete representation

The representation we propose is a *structural* generalization of natural numbers, as defined by the Peano axioms [(ii)]. Generalizing the successor operation in these axioms to the fundamentally new concept of a (*structured*) *successor event*, we arrive at the basic representational idea of the *evolving transformations system* (ETS) formalism [11], developed by us during the last two decades and motivated by the problem of pattern recognition, or inductive learning. The proposed *informational structure of Nature* is outlined in my previous FQXi essay [3, Section 2.2], but one should mention that pattern recognition is concerned with the simulation of *inductive processes in nature*, responsible for an organism's orientation in the environment. I am convinced, however, that such orientation would not have been possible if the environment itself was not structured by means of classes: *any object in the Universe belongs to some class of (similarly structured) objects*, be it a star, a molecule, or an organism. So we postulate that *classes are the basic units in the informational organization of Nature*. Moreover, putting the inductive considerations (including classes) in the center allows one to bridge the enormous present gap between the physical and the mental created by the reliance on the numeric representation, within which one cannot deal effectively with the concept of class and hence with the induction.

5.1 The initial ETS concepts

The class of processes, i.e. similarly structured processes—with *a common generative* structure (Endnote (vi))— pervades all levels of consideration in ETS. In particular, the most basic concept, the primitive event, is defined via the *classes of primal processes* (Fig. 1). Only two, most basic, concepts are *outlined* here, while their formal definitions can be found in [11, Parts II, III], which is the principal exposition of the formalism (see also [12] – [15]). Again, the important point to keep in mind is that, in ETS, each "object" is viewed and represented as an instantiation of the corresponding (temporal) structural process, "struct" (Fig. 2), which is a stream of interconnected structured events, and so the concept of (instantaneous) state—represented in the conventional setting by a vector—is obviated (Fig. 4).

5.1.1 The basic structural units: Primitive events

The initial concept is that of a **primitive event**, or **primitive transformation**, or simply **primitive,** see Fig. 1 (also Fig. 2). Each **primitive** stands for an event of fixed structure, which, in ETS, is viewed as associated with the *transformation* of several **initial primal processes** (the lines just above an event in Fig. 2) into several **terminal primal processes** (the lines just below an event). *The formal structure of a primitive event*—i.e. an **abstract primitive** or the class of concrete primitives—is specified by the two fixed tuples: of initial and terminal *classes* (of processes). A *concrete* primitive depends, in addition, on the concrete initial and terminal processes where each *must* come from the corresponding class of processes. At the initial stage of representation—the only one discussed here—the structure of all initial and terminal processes is suppressed (hence the name "primal") as is the internal structure of the event itself (hence the name "primitive"). What is being captured by this basic concept is the "external" structure of an actual event. So a primitive event is the basic structural unit that stands for the junction at which the relational information, recorded *temporally*, (typically) gets modified.

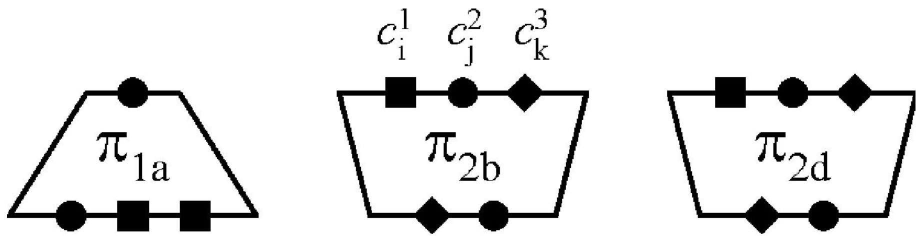

Figure 1: Pictorial depiction of three primitives. The first subscript in the primitive's name stands for the class of primitives sharing the same structure, i.e. for the abstract primitive (and hence the same *overall shape*), e.g. π_{2b} and π_{2d}. The i**nitial classes** of processes are shown as *small solid shapes* on the top and the **terminal classes** are those on the bottom of each primitive. The only labels of the individual processes shown in the figure, i.e. of the elements of these classes, are c_t^s $(s = 1, 2, 3)$ — the t^{th} process in the initial class C_s for primitive π_{2b}, where $b = < c_i^1, c_j^2, c_k^3 >$.

Despite its *relative* intricacy, the concept of primitive is more *transparent* than any other formal concept: it functions syntactically, or formally, exactly according to its semantic content, and, indeed, it is intended[3] to be a direct informational copy of the "real thing" (see Section 5.3).

Finally, as nature is comprised of various temporal processes composed of events, examples of the above events are all around us: all events in particle physics (the initial and terminal processes can easily be read off from the corresponding Feynman diagrams); formation of a two-cell blastula from a single cell (initial process is the original cell and the terminal processes are the resulting two cells); etc. The crucial hypothesis is that *the proposed structure of events is universal*.

5.1.2 The structural representation of a process: Struct

The second basic ETS concept is that of a **struct** (iii), which is an (irreversible temporal) stream of interconnected primitives, Fig. 2. It is easy to see how the classical Peano construction of natural numbers (see Endnote (ii)) was generalized to the construction of structs: *the single rudimentary structured unit out of which a number is built* (Fig. 3) *is replaced by several structural ones*, i.e. by ETS primitives. However, the decisive consequence of the non-trivial structure of primitives is that we can now see *which primitive is attached to which* and "*when*". In fact, the struct, as the new form of object representation—and consequently the output of the *structural measurement process* (iv)—for the first time, embodies both relational and temporal information in the form of the *formative, or generative, process history* recorded as the stream of the corresponding events.

Thus, on the one hand, the concept of struct should be viewed as the direct, event-based structural generalization of the natural number, and on the other hand, it embodies a fundamentally new, "non-linear" and "discrete", kind of temporality, which can be called structural or relational and which should clarify the nature of temporality in general. (v)

[3] Of course, such (main) postulate about the informational structure of actual events will have to be verified. However, what we currently know from particle physics and other fields supports such postulate.

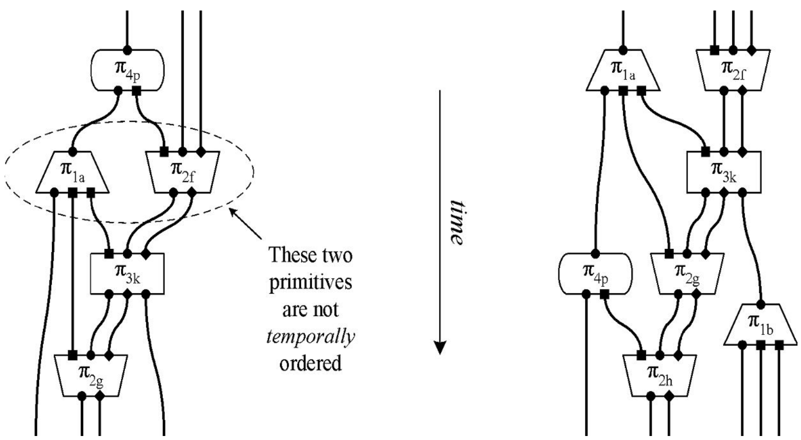

Figure 2: Pictorial depiction of two (short) structs.

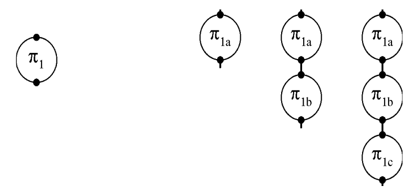

Figure 3: The single primitive for the ETS representation of natural numbers (left) and the three structs corresponding to numbers 1, 2, 3.

5.2 Temporal (ETS) representation as specifying spatial representation

The independent and informational nature of ETS representation suggests its primacy over the conventional (spatial) representation (see also Fig. 4), and it appears quite possible, given some local parameter(s), to instantiate the latter on the basis of the former. Indeed, a primitive's relational structure in a struct—interpreted as the spatial neighborhood information—allows one *to extend the already instantiated (up to this primitive) spatial structure*. In the context of computer science, we illustrated this in [11, Section 8] (also [13, Section 3]), and in the master's thesis [17].

5.3 On the unique structure of ETS formalism

The unique—among all known formal and spoken languages—feature of the proposed formalism is the congruence of its syntax and semantics [18]. Specifically, if the ETS hypothesis about the informational structure of reality—*that the basic structure of Nature is the event-based temporal structure as postulated by ETS*—turns out to be correct, then, obviously, there is no significant difference between the syntax and semantics of the ETS representation: "what you see is what you get." It goes without saying that *in any spoken language or any formalism in science the syntax is not related to the semantics*, e.g. the syntactic structure of the word "dog" has nothing to do with the semantic, or actual, structure of dog. So this feature of the proposed discrete formalism radically changes the nature of representation in science.

A closely related and also important observation is that *the struct*—as capturing both kinds of process representation, subjective and objective—ensures the agreement of their forms and hence *the removal of the exiting gap between the mental and the physical*. One kind of struct is constructed by an agent *during its interaction with the "object"*—the agent's representation, based on the agent's primitives—while the other kind is maintained by Nature and encapsulates the entire process of the object's formation, based on the original and complete set of primitives.

Another significant and unique feature of ETS—not introduced here, see [3, Section 2.3.5], [11, Part IV]—is that it offers a *seamless integration of several representational stages* within a single formalism. Transition to the next stage of representation is associated with a representational compression, in which certain recurring global patterns of process interactions, called transformations, are *compressed into the (new) primitives* for the next stage: each of the interacting processes/structs is *compressed* into a primal process for this new primitive.

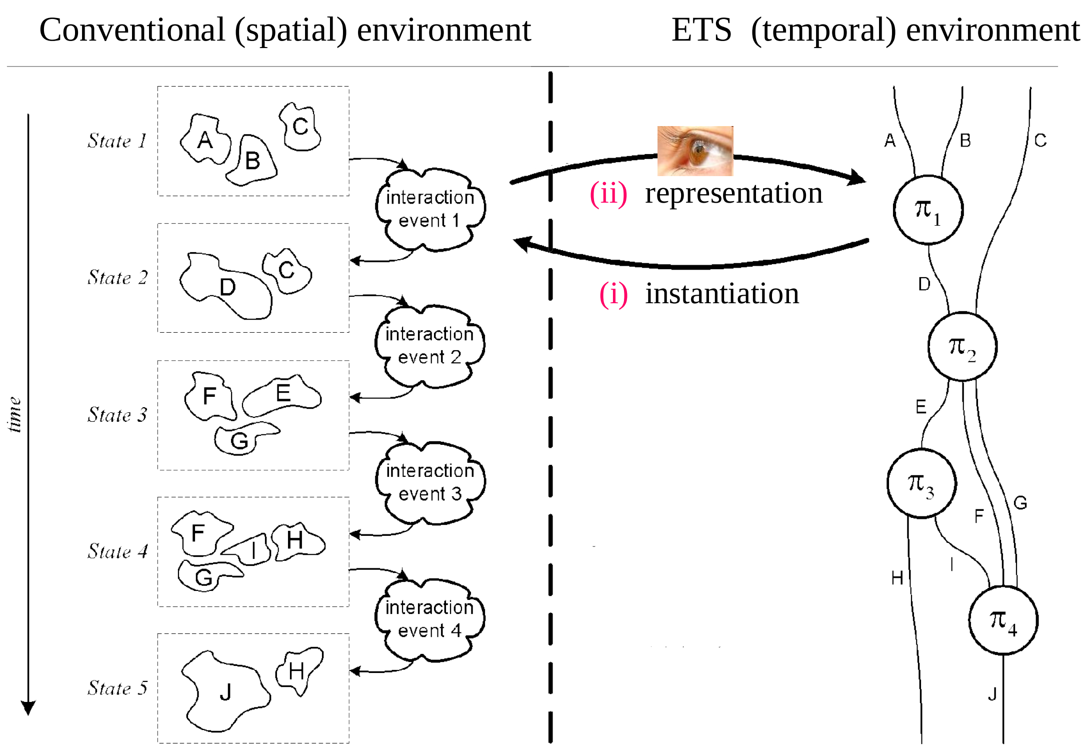

Figure 4: On the left (the conventional *state view*), objects A and B in State 1, as a result of the first event (denoted by primitive π_1 on the right), merge to form object D in State 2. Three subsequent state changes are also illustrated: D and C change into E, F, and G; E divides into I and H; finally F, G, and I merge to form J.

(i) This is the direction of precedence of temporal representation over the spatial one *in nature*.

(ii) This is the direction in which *an agent's sensory processing* operates.

Turning to the question of auditory vs. visual perception, one should note that the former gives us a *more direct* access to the temporal side of reality [8, pp. 371, 373–74, my underlining]:

> In the musical experience of melody ... the quality of a new tone is tinged by the whole antecedent musical context which, in turn, is retroactively changed by the emergence of a new musical quality.
>
> Two successive "specious presents" are not separated by imaginary durationless instants, but *by their qualitative differences*. The term "separation" is misleading; it suggests separation in a spatial sense. <u>We need to realize that the qualitative differences of successive moments of duration are untranslatable into spatial imagery</u>. To differ qualitatively and to be distinct in space are two different notions.

6. Some immediate implications of the proposed discrete formalism for physics

One of the most unexpected hypotheses suggested by the proposed formalism, as mentioned in Section 5.2, is that what we view as the basic (spatial) representation of reality might actually be secondary and instantiated on the basis of the evolving primary, temporal, representation.

Next, to contrast the proposed discrete description of a physical process with the conventional, analytical, or formula-based, description, I present in Fig. 5 a *rough* pictorial ETS description of the hydrogen atom process.

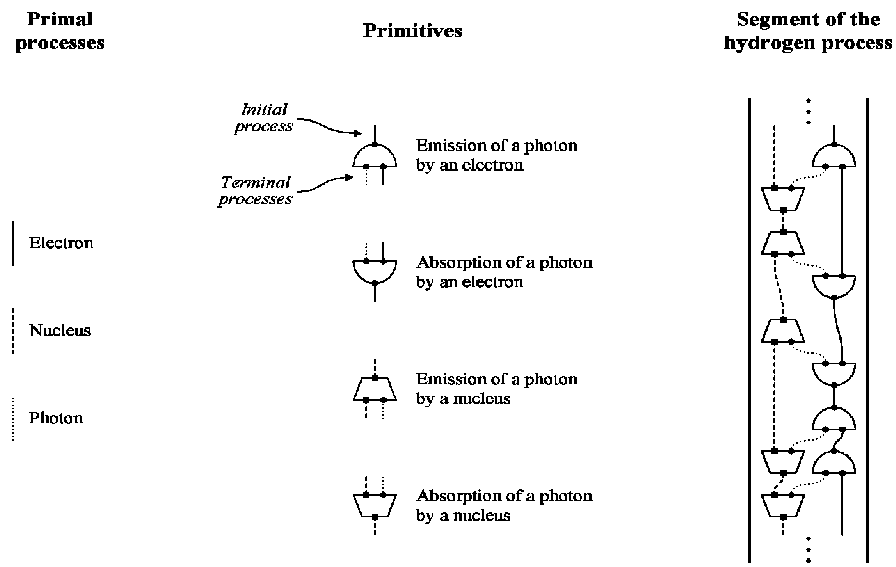

Figure 5: Pictorial depiction of the hydrogen atom process over an extremely short time interval: three classes of particles (1st col.), four primitive events (2nd col.), and a conceivable event scenario for the hydrogen process (3rd col.).

Of course, the new, "structural", mathematics —including *some analogues* of topology and algebra— is still to be developed, but even now, at the very beginning, one can already suggest how *the immediate features* of the proposed discrete representation make quite plain some *previously baffling* features of quantum processes. First, the quantum nature of these processes appears to be adequately captured: observed quanta as instantiated events. The "particle-wave duality" can probably be explained by the different forms of interaction between a quantum process and its environment: any process *always* incorporates *some* of the "external" events that are *allowed to intervene* in its construction during its generation. Similar, and in light of ETS *natural*, explanation may apply to "the most profound mystery" of quantum mechanics, quantum entanglement. Indeed, if several quantum processes belong to the same class, they are being composed by the same generating system, *the class generating system* (vi) (see [11]–[14]). When we are interacting with one of such processes, we might be modifying the class generating system itself, which should then modify all the quantum processes it generates.[4]

7. Conclusion

Today, in our "spatial", or numeric, physics, we have eventually settled on the basic *structural principles* in the form of (spatial) symmetry principles, via group theory. However, I would like to suggest that this has been a *reasonable solution only under the continuous model*, which, in view of the underlying "point" form of representation, i.e. vector representation, *does not allow us to approach reality in a structural form directly*. As far as this goal of structural description of nature is concerned, we can achieve it *much more fully* by shifting to an evolving structural/discrete representation, i.e. by approaching reality *directly in a structural form*. Such new form of representation allows one to develop more general, than via group theory, symmetry principles, based, for example, on the (*temporal*) matchings of the proposed structural representation, i.e. matchings of the struct. Moreover, the new (informational) symmetry principles would now apply to the physical and the mental, erasing the enormous present fissure between them.

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[4] Compare with the view expressed by John Bell: "For me, it is so reasonable to assume that the photons in those experiments carry with them programs that have been correlated in advance, telling them how to behave." [19, p. 84]

Endnotes

(i) Here are, for example, the views of Einstein and Schrödinger.

Einstein [7, p. 467]: in a 1941 letter to Infeld: "I tend more and more to the opinion that one cannot come further with a continuum theory."; in a 1954 letter to his friend Besso: "I consider it quite possible that physics cannot be based on the field concept, i.e., on continuous structures. In this case, *nothing* remains of my entire castle in the air, gravitation theory included, [and of] the rest of modern physics."

Schrödinger [8, pp. 301–302, my italics]: "The quantum mechanics of today commits the error of maintaining concepts of the classical mechanics of points—energy, impulse, place, etc.—at the cost of denying to a system in a precisely determined state any definite values for these magnitudes. This shows how inadequate these concepts are. *The concepts themselves must be given up, not their sharp definability*."

(ii) According to Peano axioms (the most basic axiomatic system in mathematics) each natural number *n* has a unique "successor" ***S(n)***—defined via the successor operation ***S***—and all natural numbers are thus *inductively* constructed starting from 0.

(iii) More accurately, this is a **level 0 struct**, implying that *even at the initial stage of representation* (the only one considering here) there are higher level structs, which are, basically, hierarchical partitions of the former; see [11, Part III].

(iv) See [16] (which is now outdated). Such structural measurement devices *can* actually be built within a 2-3 year period.

(v) It is hard to resist the feeling of universality of the event view of reality, including the common structure of physical and mental events and the integration of relational structure, temporality, and irreversibility in the representation of a process.

(vi) **Class generating system** relies on the **class representation** (the structural analogue of the equation), which delineates a stepwise mode of construction for the class elements. Each step is specified by the set of *structural constraints* restricting the kinds of *struct segments* admissible at this step in the construction of the class element. The constraints are always flexible enough to allow *some* "environmental" events to participate constructively in this generating process, in which case such events become a part of the resulting struct; see [11, Part III].

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