Representational formalisms: why we haven't had one

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Abstract. Currently, the *only* discipline that deals with *scientific* representations—albeit non-structural ones—is mathematics (as distinct from logic). I suggest that it is this discipline, only vastly expanded based on a new, structural, foundation, that will also be dealing with structural representations.

Logic (including computability theory) is not concerned with the issues of various representations useful in natural sciences. Artificial intelligence was supposed to address these issues but has, in fact, hardly advanced them at all.

How do we, then, approach the development of representational formalisms? It appears that the only reasonable starting point is the primordial point at which *all* of mathematics began, i.e. we should start with the generalization of the process of construction of natural numbers, replacing the identical *structureless* units, out of which numbers are built, by structural ones, each signifying an elementary "transforming" event.

> *Mathematics is the science of the infinite*, its goal is the symbolic comprehension of the infinite with human, that is finite, means.
>
> Hermann Weyl

> The definition of mathematics accepted in the 20$^{\text{th}}$ century, as a science of the infinite, should be replaced by another one, which more accurately captures its nature: it is the science of the relationship between the finite and the infinite.
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> N. N. Yanenko

1 Introduction: the situation in artificial intelligence

Why is the problem of representation so critical? Mainly because, once we discover the "right" representational formalism, we will be able "to know the mind of god", to use a popular phrase coined by modern physicist Stephen Hawking, i.e. we will truly understand what the nature of intelligence in the Universe is. “The problem [of representation] persists because it is extraordinarily difficult, perhaps the most difficult one in all of science. It is essentially the question of knowledge and of thought, and all that these imply…” [1, p. 3].

The scientific area that was supposed to focus on various representational formalisms is artificial intelligence (AI), and almost from the very beginning of AI a very paradoxical situation arose: representation was indeed considered to be a key issue but practically no work *at a basic scientific level* has been directed toward it. In other words, this area has evolved along the lines of least resistance: after some minor explorations, the researchers have chosen not to embark on the development of radically new representational formalisms.

The name “artificial intelligence” was proposed by John McCarthy at the 1956 Dartmouth workshop on “thinking machines”, and later widely accepted as the name of the area dealing with the (computer) modeling of various “intelligent” processes [2, pp. 39–40, 48–50]. Unfortunately for the development of the field, “Dartmouth indeed defined the AI establishment: for almost two decades afterwards all ‘significant AI advances’ were made by the original group members or their students” [2, p. 49; the single quotation marks are added]. In particular, two historical facts have exacerbated this state of affairs: the dominance of logical formalisms in AI[1] and the exclusion of pattern recognition from AI[2].

The issue of why logical formalisms are not relevant to the development of representational formalisms, and are therefore not central to the development of AI, will be addressed briefly in section 7. Here it suffices to quote Bertrand Russell, one of the prominent logicians of the last century: “Nature herself cannot err, because *she makes no statements*. It is men who may fall into error, when they formulate propositions” [7, p. 311; emphasis is added]. At the same time, it is not difficult to see why the choice of the logical formalism by McCarthy is, to some extent, not really surprising: at that time it was the only widely known “qualitative”, as opposed to quantitative/mathematical, formalism. (In fact, this characterization is superficial.) Although many AI conferences as well as topics in AI conferences have the popular but nebulous name of “knowledge representation”, they have not addressed representational formalisms in the sense that all natural sciences would find useful: *for the most part* they have been dealing with logical formalisms.

The separation of pattern recognition and AI had a very negative effect on the development of both fields but particularly on AI, since it put on the back burner the modeling of inductive learning, the process I believe to be the *central* intelligent process. Nevertheless, as stated in [8, p. 18], “in the late ‘80s, … once political events intervened (in the form of the recent rise of connectionism…), the situation has begun to change as can be seen from the content of recent leading AI textbooks” (such as, for example, [9]). But this had no substantive effect on the investigation of representational formalisms, except by introducing various hybrids of classical formalisms. Thus, for example inductive logic programming is still one of the leading machine learning paradigms. What has recently changed is that the relative role of the vector-space-based statistical learning methods has increased in AI, i.e. only a very small part of the damage caused by the above exclusion of pattern recognition from AI during the last 30 years has been patched up. It is quite clear that this exclusion

[1] See some of the popular AI texts [3], [4], [5]. The dominating role of logical formalisms in AI is mainly due to the influence of McCarthy and his Stanford AI school.

[2] “In 1973 … pattern recognition was explicitly excluded causing much sound and fury, and wailing and gnashing of teeth….” [6, p. 8].

was motivated by the incompatibility of logical and vector-spaced-based formalisms.

Primarily, because AI researchers avoided dealing with the representational issues at a more fundamental level, I believe that, when all said and done, no truly lasting ideas have emerged within this field so far, an opinion echoed in the following quote from Hilary Putnam, one of the leading philosophers of mind of the second half of the 20th century [10, pp. 182–83]:

> I am disturbed by the following, which is an undeniable sociological fact of Artificial Intelligence: no branch or sub-branch of science in the twentieth century has engaged in the kind of salesmanship that Artificial Intelligence has engaged in. A scientist in any other field who tried to sell his accomplishments in the way Artificial Intelligence people have consistently done since the sixties would be thrown out. This is something very disturbing. There are valuable engineering achievements, but the claim to have made something like a breakthrough or even a really new approach in thinking about the mind and psychology, seems to me fraudulent.

I want to emphasize one general point about the practice of AI that is largely responsible for its current regrettable state of affairs: the reliance on the language of existing (especially logical) formalisms to formalize the relevant concepts, instead of relying on the "logic" of the relevant intuitive concepts to drive the development of the corresponding formalisms.

The reliance on basic logical and mathematical formalisms suggests that the failure of AI can be explained by the state of affairs in mathematics and logic, which will be discussed in sections 3, 7, and 8.

The situation in pattern recognition (and machine learning) will be addressed in several sections below. Again, as previously mentioned in regards to AI, the basic methodological point to keep in mind is an understandable but inappropriate reliance on the existing basic formalisms to lead the way.

In light of the above, the main lesson from the development of AI and the basic point of the paper is this: we need to focus our efforts on the development of radically new representational formalisms that are "structural" generalizations of the classical numeric formalism and *allow them to lead the way.* I believe that, at present, we have *no other reasonable option.* I am also convinced that these representational formalisms must shed new light on the nature of mathematics, as it is clarified in the epigraphs. As far as I know, there is only one such formalism being developed—the evolving transformation system (ETS) [11], [12][3]—which is, for convenience, briefly summarized in the next section.

Finally, embarking on the development of a representational formalism, we must be prepared, to an even greater extent than Einstein was suggesting to physicists, that "In order further to approach this goal, we must resign to the fact that the logical basis departs more and more from the facts of experience, and that the path of our thought from the fundamental basis to those derived propositions, which correlate with sense experiences, becomes continually harder and longer" [13, p. 322]. I would like to add that while this "path" had quite some time to become "harder and longer" in physics, in AI, due to the more abstract nature of the field, it *must* be "harder and longer" from the very beginning.

[3] A part of [12] is not covered in [11].

2 Representational formalism through the eyes of ETS: a high level summary

In this section, I briefly summarize the lessons learned from the development of the ETS formalism (for details see [11]). Let me start by characterizing this representational formalism as one designed to support the view of the **environment** as formed by a hierarchically evolved—and therefore **hierarchically organized**—dynamic system of **classes** (of entities). Thus, the environment should be viewed as being **multileveled**, with the objects at each level composed of structural **primitives**, or **primitive transformations**, each of which (except from the initial level) stands for a *class*[4] from the previous level. A structural primitive must be defined in such a way that **its syntax and semantics are inseparable**, and this is what makes it radically different from similar concepts in all previous formalisms. Moreover, the classes, levels, and the number of levels are not fixed, i.e. they can be modified by the **inductive learning process**, *which is the basic process supporting the "reconstruction" of the multilevel environment by an intelligent agent.* It is understood that the formalism must provide the same formal language for dealing with each level, i.e. the language must be "level independent".

For example, "water molecule" and "mammal's ear" are names (but not representations) of corresponding classes of entities separated by several levels. Obviously, during the evolution of the universe, the former class has participated, at a much later time, in the formation of the latter. A very good, suggestive example for modeling a *single level* representational structure is provided by basic chemical *structures*, i.e. molecules. Of course, this is not to suggest that scientists already know how to formalize these observable structures in a satisfactory manner.

For each class, the (structural) **class representation** is defined as consisting of several same-level weighted (structural) **transformations**. A (non-primitive) transformation is composed of its context and its body, where each one, in turn, is composed of primitives. The class representation must be introduced in such a way that, when "plugged" into the generating process (which is possibly common to all classes), class elements are generated in a systematic manner. Hence, *class elements are built out of structural components supplied by the class representation.* In other words, having fixed the generating process, any class must be adequately reconstructable from its representation. Also, the class representation must be efficiently learnable from a small training sample, which cannot be assumed to be noise free. The last two statements point to a powerful and most general *connection between the finite (class representation) and the infinite (the set of class elements).* At the same time, the introduction of levels allows for an exponential reduction of representational complexity: each primitive stands for a set of transformations from the previous level.

3 The situation in mathematics: numbers as the sole foundation of, and directing force for, the entire current scientific edifice

The story of numbers goes back in history *at least* many tens of millennia, predating the development of civilization itself. In [14, section 3], I briefly outlined the inductive origin

[4] Or, more accurately, the *representation* of this class.

of natural numbers, including four stages in their emergence, and tried to justify why "the decisive and irreplaceable role of the induction axiom in the well-known Peano axiomatization of natural numbers is a reflection of the inductive cognitive origin of numbers".

A lesser-known fact about the much more recent role of numbers in shaping Western civilization is researched by the noted historian Alfred Crosby.

> Europeans of the late middle ages inherited a profound change in *mentalité* that had been fermenting for centuries, a change from the ancient qualitative way of comprehending the world—what Crosby calls the "venerable model"—to a quantitative model that would soon dominate Western society and provide Europe with the power to dominate the world.... In the space of less than a century just before and after 1300, Europe produced its first mechanical clock (which quantized time), marine charts and perspective painting (which quantized space), and double-entry bookkeeping (which quantized financial accounts).
>
> ... In 1300 everyone thought of nature as heterogeneous, each quality with its own measure. Yet 250 years later Pieter Bruegel (in his 1560 painting *Temperance*) portrayed people engaged in visualizing reality as aggregates of uniform units (or quanta): leagues, miles, degrees, letters, guilders, hours, minutes, musical notes. [15, p. 1] (based on [16])

> In practical terms, the new approach was simply this: reduce what you are trying to think about to the minimum required by its definition; visualize it on paper, or at least in your mind, be it the fluctuation of wool prices at the Champagne fairs or the course of Mars through the heavens, and divide it, either in fact or in imagination, into equal quanta. Then you can measure it, that is, count the quanta.
>
> Then you possess a *quantitative representation* of your subject that is, however simplified, even in its errors and omissions, precise. You can think about it rigorously. [16, pp. 228–29; emphasis added]

In fact, in the preface of his book [16], Alfred Crosby attributes to the above events the later "amazing success of European imperialism" around the beginning of the 14^{th} century.

Why is there, then, a (relatively recent) perception, particularly among some mathematicians and theoreticians, that "mathematics is not about numbers"? There is no doubt that, *partly*, the latter is true, but *fundamentally* it is only an illusion. The main reason for this recent perception, and misconception, is the nature of the axiomatic form of one of the main areas of mathematics—algebra—as it gradually emerged during the last 100–150 years. During the last century, algebra became a general "repository" for many useful abstract mathematical concepts/structures, e.g. group, ring, field, vector space, algebra, category, morphism. Simultaneously a tradition[5] emerged in mathematics for organizing these various "abstract" (axiomatic) structures in a hierarchical manner. As a result, many (but not all) texts, including some authoritative ones, have propagated the illusion of the primacy of abstract structures themselves, without explaining where these structures came from, i.e. what motivated them in the first place or by tracing their historical roots. For example, one should know that the concept of group has emerged gradually from the research on the solutions of higher degree (numeric) polynomials by Lagrange, Ruffini, Abel, and Galois around the start of the 19^{th} century.

Indeed, the concept of group is one of the most general concepts in algebra. In science, particularly in physics and chemistry, it gradually began to play the role of a concept that is supposed to algebraically characterize the nature of a particular "invariant" of a phenomenon,

[5] The major protagonist of this tradition was a French group of mathematicians under the pseudonym of Nicolas Bourbaki.

i.e. the nature of the set of all transformations under which the corresponding "feature", or structure, is preserved (see, for example, [17], [18]). In particular, the group can be used to characterize the nature, or "degree", of symmetry of a given geometric figure. I would like to emphasize here that, although the group concept appears to be independent of numeric forms of representation, this is a misapprehension: currently, the (formal) understanding of geometric as well as non-geometric objects is based on their numeric representations. It is interesting to note that the group concept can be viewed as a concept of "*class description*" when the set of admissible "object" classes is substantially restricted, i.e. when dealing with much more "regular" classes of phenomena. With the introduction of a structural (non-numeric) representational formalism, e.g. the ETS formalism, the description of much more general classes of phenomena becomes possible.

Thus, to be sure, abstractions are present in mathematics, but their origin is in numeric structures/representations. Moreover, since they emerged in the course of the investigation of numeric structures, they *must* carry at the very least *some* limitations associated with this origin. Another example is what is probably the most central mathematical concept, the concept of a "space" of (numeric) functions. Although this set could potentially be endowed with many abstract (and non-linear) structures, what we see in mathematics is a particular dominating structure—the linear, or vector space, structure—inspired by corresponding "numeric" considerations. And, of course, it couldn't be any other way: generalizations are always inspired by some concrete structure.

I would like to make one final point regarding a future, "structural", mathematics, to which I will return in section 5. Although the concept of set is an auxiliary one in mathematics[6], it still occupies a very prominent, foundational, place. It appears that in a "structural" mathematics its place will be occupied by the concept of a *class* (of related objects).

4 Our starting point: generalization of Peano axioms via structural transformations that model actual object transformations

What should guide us in the construction of a formalism for structural representation? Here we are entering *absolutely* unfamiliar territory: we don't have any examples of formalisms on which our (inductively trained) esthetical selection principles can be based [19]. So what should our starting point be?

We do have *one example* of a "representational formalism", albeit a non-structural one: the set of natural numbers, which are the abstract entities on top of which the edifices of mathematics and natural sciences are built. So, since this is all we have, we should try to learn *as much as we can* from this example. To this end, we should look very carefully at the *process* of construction of natural numbers. The well-known Peano axioms that define natural numbers also perfectly capture their process of construction via *one simple operation*, which is applied successively to "objects", starting from some initial "object" (see [20] or [21]). I will not discuss here the overriding importance of this construction process to the development of mathematics (see [22, chapter 1], [23]).

The temporal nature of this construction process strongly suggests that, when general-

[6] The central concepts are those related to the basic mathematical structures, e.g. in algebra, the ones mentioned above.

izing it, we must also proceed in a *temporal manner* (with a possible allowance for atemporal/parallel application of a few operations at a time)[7]. The critical point to note, however, is this: if one simple, i.e. non-structural, operation in the construction of natural numbers is now replaced by at most a few "parallel" structural operations, one can now "see", in contrast to the non-structural case, *which structural operation was applied and when it was applied.*

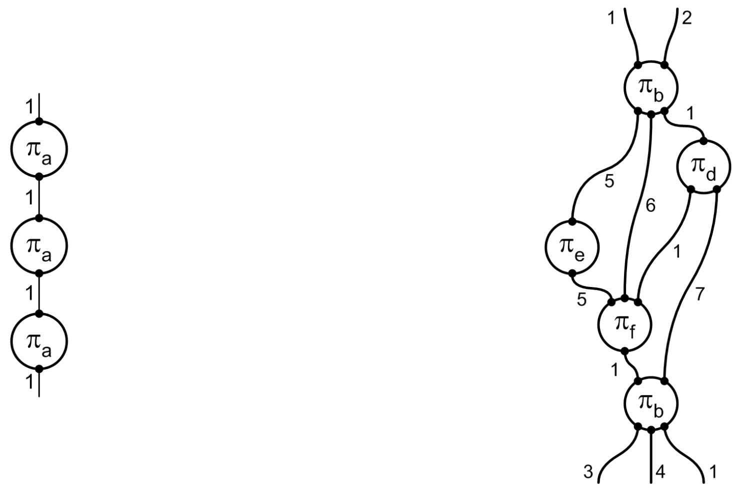

Figure 1: Representations of the natural number three (left) and a more general structural object (right), where each π_i denotes an ETS primitive transformation.

Such representational power, mainly due to the transparency/explicitness of the representation, has *never* before been available in any mathematical formalism, and it is this newly discovered power that we will have to learn to garner and exploit, starting literally from scratch. The full practical implications of such a representation is overwhelming since it includes, in particular, the concept of a "structural measurement process" [24]. But even outside the considerations of structural measurement, we are still faced with the unprecedented challenge of gradually developing the mathematics of such generative entities, which no doubt will require the development of completely new "tools" as compared to those developed within the classical numeric-based mathematics. (By perusing [11] it is not difficult to get a feeling why this is so.) However, for incremental progress, it is important to keep in mind that the development of any formalism proceeds much more effectively and efficiently if, at least initially, we separate the issues related to formalism construction from the purely implementational ones when necessary. Of course, one of the more immediate promises of such a powerful representation is the incredible reliability it affords in the solution of problems in pattern recognition and other "information retrieval" areas.

Returning to the above structural operations—let us call them primitive transformations (see section 2)—I note that these (syntactic) primitives can also be considered as semantic primitives, which is not the case with the prototype they generalized, i.e. with the single non-structured, numeric or Peano, primitive. Indeed, a structural primitive transformation

[7] Thus, in the most general case, we are still dealing with temporal order but extended now to small *sets of operations*, which are atemporal within each such set but whose (i.e. sets) relation to each other is temporal: the linear order of operations becomes a partial order.

is supposed to mimic/model the corresponding actual object transformation, which is the key feature of structural, as compared to numeric, representation.

The question of whether the information processes are irreversible is also resolved. Obviously, in view of the temporal nature of such processes, the *appearance* of a transformation in the representation cannot be undone, simply because all other transformations appear after it.

One of the most interesting questions raised by such an "irreversible" representation is this: is the formative/generative history that is captured by this representation necessary? From biology (more specifically, from developmental biology), we already have a clear affirmative answer: in order to "produce" an organism Mother Nature apparently relies on some kind of formative history of the species, which the organism must recapitulate during the early stages of development. But is this "wisdom" applicable to pattern recognition? Yes, it is, and it could not be otherwise. For example, in computer vision, if, as a part of a structural face representation, along with conventional face features, we were able to incorporate some of its contour lines and its expressions[8], this would dramatically reduce the size of the class of "similar" faces. Moreover, when trying to recognize a face, the recognition process would terminate earlier if it does not encounter some particular structural transformation involved in the representation of the given face.

In any case, to answer the above question, one should proceed in the usual scientific manner, which was succinctly summarized by well-known physicist Richard Feynman [25, p. 19]: "How do we *know* that there are atoms? By one of the tricks mentioned earlier: we make the hypothesis that there are atoms, and one after the other results come out the way we predict, as they ought to if things *are* made of atoms." So, how will we know that an object's generative history is important? We postulate the above representation and then see if the predictions of "new" class elements based on this representation are valid.

I should also mention one quite obvious but critical advantage of the above structural representation: given a small training pattern set from a class, in contrast to the numeric representation, we can directly "observe" the larger structural operations, macro operations, that delineate this class. This correlates, for example, with our visual experience in a familiar environment, when we can fairly quickly see all "interesting" class features.

Finally, I draw your attention to an important philosophical observation. Although a number of past philosophers, starting with Heraclitus, insisted on "change" as the main "reality", as compared to the "visible object" reality, these philosophical observations have never been realized—except in ETS, which postulates transformations as the basis of representation—as the foundation of a representational formalism.

5 Finite class representation of a possibly infinite class

Assuming that the concept of class is primary, the concept of a naturally evolving *class representation* must be the central concept in a representational formalism. I should note that, from the very beginning, it was this vision that inspired and directed the development of the ETS formalism.

The term *class representation* was introduced in [26], and although it refers to a par-

[8] These are not features in the conventional sense.

ticular form of *class description*, it may also (more loosely) be interpreted as implying a particular structure of the underlying formalism, within which this form of class description is a *natural* one. By introducing this term, I wanted to draw greater attention to the *form* of class description used in *any* representational formalism, given that very little attention has been paid to it: the (inductive) adequacy of the class representation plays the central role in assessing the adequacy of a formalism for pattern recognition. I believe that this lack of attention is not accidental, but is a consequence of an important aspect of the structure of basic (mathematical) formalisms: they lack a satisfactory concept of class description/representation. In other words, it turns out that their (underlying) *formal structure* cannot support a satisfactory concept of class, and therefore *the situation simply cannot be remedied within existing formalisms by developing "more powerful" classification algorithms.*

In connection with this, one of the most remarkable telltale signs is the lack of both the concept of class *as well as* the concept of class representation in AI—including machine learning—and the avoidance of both via the introduction of "a boolean-valued function from training examples" [27, p. 21] (see also [9, p. 651]). I find this remarkable because a boolean-valued mapping is a very unnatural/roundabout way of dealing with the concept of class and highlights the intrinsic limitations of the underlying formalisms.

Moreover, the substitution of the concept of class representation by the class indicator, or characteristic, mapping has a number of serious consequences, one of which is the concealment/removal from scientific attention of the question of the relationship between a (finite) class representation and the (possibly infinite) set of its elements. I believe that the resolution of this *most central scientific question* is of vital importance to the development of AI (including pattern recognition) and that the *quality* of the answer to this question, within a particular representational formalism, is the decisive factor in judging the adequacy of that formalism. In particular, if a representational formalism offers a scientifically satisfactory and experimentally supported solution to the nature of this relationship, it can be taken seriously. Not surprisingly, the great riddle of induction is closely associated with this scientific question. Also, in light of the above epigraphs addressing the nature of mathematics, I suggest that the concept of class representation, rather than the concept of set (with the subsequent development of *non-mathematical* logical languages to support it, see section 8), should be in the center of attention for mathematicians.

What is the relationship between a class and its representation in the ETS formalism? Very briefly (see Def. 29 in [11] and Def. 34 in [12]): since class representation is specified by a class super-transformation (consisting of a finite set of related transformations) with its weight scheme, the objects of the class can be generated in a systematic generative manner by applying the constituent transformations and simultaneously calculating the resulting object typicality. In other words, a possibly infinite set of class objects is generated by a finite set of weighted transformations. As the class evolves, the corresponding set of weighted transformations also evolves in a *very natural* manner, i.e. incrementally: the main, structural, changes are associated with changes in the constituent transformations, while auxiliary ones are those associated with the modifications of weights. This hypothesis about the nature of class representation, as was mentioned in section 4, can and should be experimentally verified. Another important point regarding this form of class representation is that we expect it to be efficiently learnable from a small training set and also to be stable with respect to various kinds of "legitimate" noise present in the object representations.

6 A new phase in applications: construction of structural representation

In this section, I want to briefly address a major shift in the relative importance of the representation phase as compared to other implementation phases.

First, following conventional scientific experience, it is quite natural to hypothesize the existence of a *structural measurement* process, which is a far-reaching generalization of the classical, numeric, measurement process. While the numeric process involves a systematic procedure (built into the measurement device) for "dismantling" the structural information present in the original object and encoding it into identical non-structural units, the structural measurement process involves the transduction of one kind of structural information into another via much more complex and dynamic interactions. Although we touched on this issue in [24], it should be quite clear that this paper is not the place to address this vast topic.

Next, I want to address a radical shift in the relative importance of various constructive stages associated with the application of the ETS formalism as compared to the application of the vector-space-based formalism. In short, within the latter framework, it appears that representation is "easy" but learning is "difficult", while within the ETS framework, the expectation is that *representation is difficult but learning is easy.*

Indeed, in the implementation of a *representational* formalism, especially in the initial stages of their development, it is only natural to expect the development of the basic representation to be of central importance. (Of course, once it is satisfactorily developed, it can be used in a routine manner.) At the same time, once this implementation phase is complete, the learning stage is relatively "simple", in the sense that it can now rely on the rich structural information present in the constructed representation. I would also like to draw your attention to the observation that human experience seems to support this "division of labor": when we find ourselves in a new environment, we spend most of our time building necessary representations.

On the other hand, within the vector-space-based formalism, which I consider separately in the next section, the main theoretical and practical efforts are directed towards the development of "good" classification algorithms, while the construction of "good" representations, of necessity, is accomplished in an ad hoc manner. As a result, we have no reliable object and class representations, and further, at the end of the learning process, we are also no better off with respect to our ability to utilize the results of learning for various other processing needs, e.g. discovering the connections between classes and gaining insight into the nature of data, including its main "features" (as are the needs of data mining and information retrieval). In light of this *basic* deficiency, I am suggesting, in particular, that *it is very misleading to call such statistical algorithms "learning" algorithms.*

7 The vector space as a formalism and the applied role of formal operations

I will focus here on two key points, both of which concern the role of the formal operations by means of which a formal model is axiomatically defined: their conceptual and applied roles.

The conceptual role of such operations in mathematics has been clarified by the work of the Bourbaki group (see section 3) but, apparently, is not widely understood. Basically, this point can be summarized as follows: the formal operations involved in the *axiomatic definition* of a particular mathematical structure (and the compositions of those operations) are the only *legitimate/admissible* operations within that structure. In case of the vector space, the two basic operations are vector addition and multiplication of a vector by a scalar. Their compositions specify what are called "linear" operations, which are the only legitimate operations in a vector space, also called a *linear space*. *No non-linear operation* is a part of this mathematical structure. Incidentally, there are uncountably many such operations, and so *even if* we wanted to choose one of them over the other, we have no *legitimate* way of doing so. Thus, if we decide to ignore this point, we are no longer dealing with a bona fide mathematical structure, and the many benefits that come from working within a mathematical structure are lost. Unfortunately, many statisticians and applied mathematicians are not aware of this.

The second point is related to the applied role of the defining operations, and has not, to my knowledge, been addressed (except briefly in several of my papers), but it is of even greater importance for us. Let us consider an "intelligent" agent operating in some environment. In order for this agent to be able to "communicate" effectively with the environment, the agent must possess a finite set of *basic representational operations* whose compositions should be able to capture all critical "features" in the environment. Moreover, since many of these features are *compositionally* related to one another in a very particular way, the corresponding compositions of the agent's operations must be related in a *similar way*[9]. Otherwise the agent will not be able to capture the relationships within a class as well as between classes of objects (and between features). For example, the agent will not be able to understand what a dog's ear is (which is both a feature and a class) and that ears of dogs of various breeds play the same compositional "role", independent of their shape, size, texture, etc. The common role of the ears can be hypothesized on the basis of the fact that they form a class with a fixed compositional role in the structure of the head. Without such a capability, no autonomous functioning in a non-trivial environment is possible. This observation implies that if we ignore the basic representational operations and their interrelations, we will not be able to build a truly autonomous/intelligent system or agent, since, in this case, such a system would completely depend on the implementer to hardcode the necessary features in the environment. Furthermore, in many areas, such as AI, data mining, and information retrieval, we would also need to hardcode the meanings of all the features. However, both of these tasks are practically impossible. And even in the case that we were to hardcode most of the relevant features, the learning and classification processes will be quite brittle, as has been experienced so far.

Thus, I want to draw attention to the "applied" role of the basic operations (in an axiomatic specification) of a mathematical formalism: **they must be "isomorphic" to the basic object operations in the modeled environment**. In other words, one should choose that representational formalism whose basic operations "mimic" most accurately those of the environment. The latter can be verified by observing how well the agent can

[9] During the last century, this point was made a number of times in the context of "symbolic computation", tracing back to [28] at least.

autonomously learn various classes in the environment.

In this respect, the representational capability of the vector space model is incredibly weak, were it not sanctioned by the millennia-old tradition supported/entrenched by numeric measurement devices (see section 2): there are only two basic operations and both are modeled on numeric operations rather than on structural ones. Moreover, their compositions do not offer sufficient richness, and this is precisely the main reason why various non-linear operations, including kernels, are being (inevitably) introduced. But, as explained above, this will not save the situation: non-linear operations may somewhat improve classification, but since *the representational question is not being addressed at all*, the whole construction becomes very brittle, with no benefits to various postclassification processings, which are increasingly in demand, for example, in data mining and information retrieval.

One final but important point relates to a number of recent developments in pattern recognition and machine learning, in particular to various kernel- and dissimilarity-based approaches. The main observation I want to make is, in a way, quite obvious: to obtain good kernels or dissimilarity measures one needs to rely on some good "representation", since this is how they are computed, i.e. without such an (implicit, at least) representation, no kernel or dissimilarity can be introduced. In other words, without some operations transforming one object into another, these concepts cannot be implemented. In fact, this is how I was led to the development of transformation-based, or structural, representations, when I realized that, in order to be successful in a sufficiently rich environment, the agent must be able to evolve a *wide variety* of new dissimilarities without the help of a human designer [29].

8 Logical formalisms

As mentioned above, the key to the success of a representational formalism is the *quality* of the class representation it provides. In the case of a logical formalism, although the situation may initially appear to be more promising than in the case of the vector space formalism[10], in reality, this turns out to be an illusion.

Logical formalisms emerged from relative obscurity early in the 20th century, when researchers working on the foundations of mathematics—mainly trying to clean up the "language" of set theory introduced by Georg Cantor in the last quarter of the previous century—discovered a number of serious antinomies, e.g. Russell's, Burali-Forti's, Richard's, and Berry's. Their rise to prominence in the '20s and '30s was fueled by David Hilbert, the most influential mathematician of the time (after Poincare's death in 1912), who became involved in the program to secure the foundations of mathematics[11]. At that time, the work of George Boole, Gottlob Frege, Bertrand Russell, and several other researchers on the *foundations of mathematics*—and definitely not on "modeling the mind"—suddenly became of interest to many other *mathematicians.*

But what was this logical formalism (whose main ideas are attributed to Frege) motivated by? Here are some relevant points regarding Frege's motivation, which shaped, and is fully reflected in, the formalism [30, pp. 14–18; emphasis added]:

[10] This is simply because logical formalisms appear to be more "qualitative" as compared to quantitative mathematical formalisms.

[11] It was Hilbert who insisted "No one shall be able to drive us from the paradise that Cantor created for us."

> Although ... some ... have been tempted to claim that mathematical truths in general are ultimately justified on the basis of our experience, Frege *rejects the view that the contents of our experience are at all relevant* to the truth of arithmetical claims.
>
> ....
>
> Logic, according to Frege, limns the laws of thought. *These must not be confused with the laws of thinking*, the psychological processes that might take place as one reasons. Rather, they are laws that constrain what can be rationally thought. *Logic does not aim to describe how humans think*, but instead to characterize how they must think if their thought is to remain within the bounds of reason.
>
> ....
>
> The laws of logic govern all rational thought, regardless of its particular content: *logic '[disregards] the particular characteristics of objects*, [and] depends solely on those laws upon which all knowledge rests.'

Thus, the basic logical formalism (developed for needs of the foundations of mathematics) is deliberately structured in such a way that the structure of inductive experience, as well as the structure of the corresponding biological processes, are irrelevant, while these are precisely the processes that should be of central importance to AI.

In connection with the development of logic at the beginning of the century, it is *very instructive* to read the scathing reaction of the great Henri Poincare to those developments, which he gave in chapters 3, 5 of [31]. Poincare also criticized the introduction in mathematics of *actual* infinity, as opposed to potential infinity. Most importantly, he was convinced of the central role of inductive constructions in mathematics and that

> All the efforts that have been made to upset this order, and to reduce mathematical induction to the rules of logic, have ended in failure, [and this failure is] but poorly disguised by the use of a language inaccessible to the uninitiated [31, last section "General Conclusions"].

It is also interesting to note that, as his quotes in section 1 and below clearly indicate, later on in his life Bertrand Russell changed his mind about the role of logical formalisms:

> Inference is supposed to be a mark of intelligence and to show the superiority of men to machines. At the same time, the treatment of inference in traditional logic is so stupid as to throw doubt on this claim .... I have never come across any ... case of new knowledge obtained by means of a syllogism. It must be admitted that, for a method which dominated logic for two thousand years, this contribution to the world's stock of information cannot be considered very weighty. [32, p. 63]
>
> I had become increasingly aware of the very limited scope of deductive inference as practiced in logic and pure mathematics. I realized that all the inferences used both in common sense and in science are of a different sort from those in deductive logic. [33, p. 141]

To the above, I want to add the following, more formal, reasons why logical formalisms cannot be used as representational ones. Basically, all main features in the characterization of a representational formalism, as proposed in section 2, are missing: logical primitives cannot at all model structural primitives (and *they were not intended* for this purpose)[12], no multilevel representational structure can be supported[13], and no inductive class representation is possible. (For additional discussion see also [29, section 5].) Thus, in view of the above, it was a gross misunderstanding to choose the logical formalism, even in an expanded form, as a basic *representational* formalism for AI.

[12] That is why, in logical formalisms, one has to deal with syntax, semantics, and interpretations.

[13] At least one prominent logician has drawn attention to this: [34, part III].

9 Computational models

Turning our attention to computational models, I note that, as is generally accepted, computability theory is in fact part of logic[14], and since computational models do not have much claim on representational novelty, I want to touch on the following two issues only: Chomsky's generative grammar model and the inappropriate obsession with computational complexity.

First of all, one should acknowledge the historical importance of Chomsky's formal grammar model: it was the first model to emphasize the critical role of generativity, and in this sense its importance should not be underestimated. However, even though Chomsky stressed its central role in this model, a generative object history simply cannot be represented as part of a string (over the basic alphabet), and therefore this critical feature of the model *could not be incorporated into an object's representation*, i.e. into a string (see also the next section). Moreover, since Chomsky has been consistently opposed to putting this model into an inductive, i.e. more dynamic, context, no important issues related to the inductive nature of languages and grammars or their inductive connections have been addressed by his school. In particular, a *grammar is not viewed as an (inductive) class representation.* When the corresponding issues arose in pattern recognition in the '70s and '80s, it gradually became (reasonably) clear that the framework in its current form was inadequate for the goals of pattern recognition [29], [35]. In essence, given a training sample, since no generative information is present in the training strings themselves, there is no reliable way to associate a class grammar with the training strings (even under reasonable restrictions on the class grammars). Of course, the presence in the model of the second, somewhat ad hoc, alphabet does not help, either.

The second point I would like to draw attention to is the absolutely inappropriate obsession with computational complexity above all other considerations, including representational issues. I am convinced that we should follow the wisdom of, for example, a physicist who would be appalled if she was approached about complexity issues before a satisfactory physical model of the phenomenon was available. Of course, complexity issues have their place, but *one should not put the cart before the horse.* Moreover, a satisfactory representational formalism *should* allow for efficient learning algorithms, otherwise, as was mentioned above, it simply cannot be "satisfactory". (Again, when have physicists worried about computational issues?) Thus, for example, the presence of a generative object history as a part of an object's representation in the ETS formalism exponentially reduces the number of candidate classes from which a training set could have originated.

10 The inadequacy of popular discrete representations

First of all, it is important to note that all such "representations"—e.g. strings, trees, graphs—have emerged *outside the scope of any representational formalism* and became "representations" simply by default as the importance of "discrete" representations gradually grew with the advent of computers. As a result, there is no framework in which, for example, strings over a fixed finite alphabet can be reliably considered as elements of some

[14] One should not forget that the main tenets of computability theory were developed by logicians, i.e. Post, Gödel, Church, Turing, Kleene, and (more recently) Cook.

(formal/mathematical) "space", and there are actually some good reasons for this: in contrast to classical mathematical objects, there is no "universal space" that can *naturally* accommodate this set of strings. In my opinion, the latter situation is mainly due to the fact that the relationships between two strings are not defined well, and therefore there are no *well-defined* classes of strings. I believe that unless some temporal "frame of reference" is introduced, as was done in the ETS formalism, for example, we are not dealing here with a reliable form of representation. A simple example might clarify this point: How do I compose/construct a new, non-trivial sentence? The process does not proceed from left to right as the final written form may suggest, simply because the generation of a sentence in my mind follows a more complex "non-linear" order, not related to the linearly-written sequence of words. The representational situation with graphs is even more problematic, since a graph's formative history is more complex than that of a string.

Thus, the inadequacy of the popular discrete representations, e.g. strings, trees, and graphs, cannot be remedied in a simple manner: it requires radical rethinking. It is such rethinking that actually led to the ETS formalism. I am afraid that a "relational revolution", in which "statistical machine learning is [currently] in the midst of" [36], has nothing to do with the radical *representational* rethinking I am referring to.

11 Why conventional methodologies for the evaluation of performance of learning algorithms are inadequate for that purpose

First of all, it is very important to realize that, today, in the age of computing, almost any model could be "made to work" in a number of applications, especially when given enough resources. That does not at all mean that the model has a scientific value. What makes a new model *scientifically* attractive is its important "explanatory" value, i.e. it must hypothesize a qualitatively new, non-trivial feature of *reality* that one should be able to verify "experimentally" (otherwise, at best, it is not a "new" model). If no interesting feature of reality is being hypothesized, the model, accordingly, does not have much *scientific* value. In AI, one must insist on this criterion to an even greater extent, since there is a strong intuitive expectation that any natural environment is "meaningful", or "full of meaning". Thus, in pattern recognition (and machine learning), from a *scientific* point of view, a useful formalism is supposed to advance a "useful" hypothesis about the *structure of pattern classes* in the universe[15]. Again, as was mentioned in section 4, there is a longstanding scientific practice concerning the verification of such hypotheses. But, above all, one must *have* such a hypothesis. The ETS formalism advances such a hypothesis, while no other popular machine learning model does so.

On the other hand, from an engineering, or applied, perspective, the situation looks quite different. This is because an engineering practice, in contrast to a scientific one, revolves around implementations. Of course, this is not to say that implementations do not require creativity, but rather that this kind of creativity is typically confined to a different "level": scientific creativity is about advancing a new hypothesis concerning the nature of reality, while engineering creativity is about finding original and efficient ways of implementing the

[15] I don't believe that by restricting the environment we can expect the corresponding hypothesis to be much simpler.

experimental part of the enterprise, including the verification of scientific hypotheses.

So what is the situation in pattern recognition? Since pattern recognition (PR) is an applied field, engineering methodologies and practices apply, including the adoption of conventional statistical methodology. As a result, we have the following situation. Suppose you and I preprocess the data independently, and my PR algorithm performs at the level of 98% and yours performs at 88%. Does that mean that my algorithm is more satisfactory than yours? No, of course not. For example, no one knows which kinds of "talents" were put to work in the preprocessing stage. However, if we use completely different pattern representations, the situation becomes even more confusing. For example, how do we know that my algorithm is as robust as yours when we substantially increase the number of classes or slightly change the original setting?

Thus my main point is this: the statistical methodology *could* be made more meaningful, *provided* the underlying representational formalism is fixed. What should be clear from the above is that, at this stage in the development of AI (including PR), it is the representational formalism that should be of main concern, the development of which is much more a scientific, rather than an engineering, matter. Once a satisfactory representational formalism is developed, *only then* will it be safe to proceed with the development of learning algorithms, as well as the corresponding statistical methodology for the verification of their performance.

12 Conclusion

We need a representational formalism that offers powerful class representation capabilities. Since the development of mathematics and logic (including computability theory) were not motivated by, and their existing models are not appropriate for, such a goal[16], a radically new formalism is called for. Of course, I am not the first to suggest the need for a fundamentally new formalism. In fact, a number of leading scientists during the last century have suggested this; see, for example, Schrödinger [37, pp. 143–162] and, more relevantly, von Neumann [38, especially pp. 80–82].

Over the last 50 years, we have witnessed too many "non-radical attempts" in AI and PR, including the appearance in the '80s of such "new" areas as machine learning and neural networks. I believe that, by now, a healthy scientific sense should strongly suggest that we can't squeeze out of the conventional formalisms (e.g. out of decision surfaces or internal nodes of ANNs) any representational miracles. Moreover, the same scientific sense suggests that we should not expect any other miracles from conventional formalisms, or that "one should not expect to find a lost key, at night, only under an existing light".

We are at a *very important* crossroads in the history of science: we will not make substantive progress unless we embark on the development of a fundamentally new (structural) representational formalism. In fact, there appears to be no way around this undertaking. It is a very big—indeed *unprecedented*—scientific step, precipitating an attendant restructuring of science. To some of us, however, it is also a very exciting journey, one which actually began more than ten millennia ago with a small collection of clay shapes (spheres, disks, cones, tetrahedrons, ovoids, cylinders, rectangles, and others) [39, p. 11] and which, after a "numeric" detour, is once again leading to new, much more powerful, structural entities.

[16] In fact, they have not been dealing with *any* (structural) representational formalisms.

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