Homomorphic MAC

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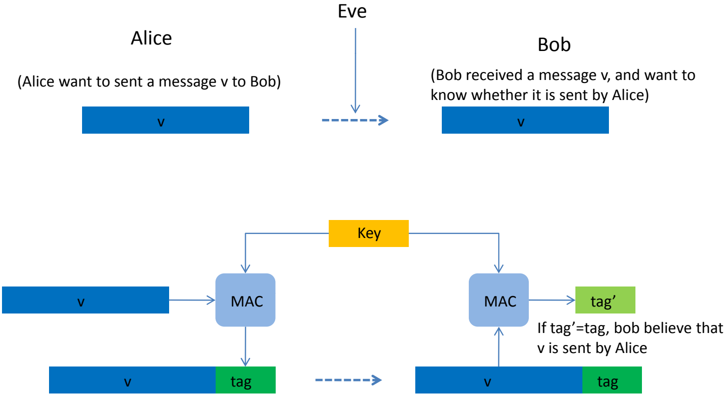
Outline

References

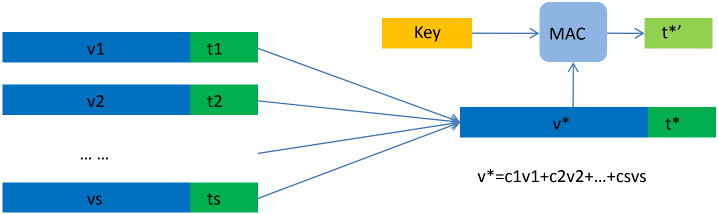
[1] Chi Cheng and Tao Jiang. An efficient homomorphic mac with small key size for authentication in network coding. *IEEE Trans. Computers*, 62(10):2096–2100, Oct 2013.

1 Communication Model

1.1 MAC



1.2 Homomorphic MAC



$v^* = c_1 v_1 + c_2 v_2 + \ldots + c_s v_s$

$t^* = c_1 t_1 + c_2 t_2 + \ldots + c_s t_s$

2 Algorithm Model

A homomorphic MAC scheme includes the following PPT algorithm.

- MAC: takes as input a secret key k and a message vector v, outputs a tag t for v.
- Verify: takes as input a 3-tuple (v, k, t), where k is the secret key, v is a message vector, and t is the corresponding tag, output 1 or 0 according to the tag is accepted or not.
- Combine: takes as input a sequence of 3-tuple $(v^{(1)}, t^{(1)}, c_1), (v^{(2)}, t^{(2)}, c_2), \cdots, (v^{(r)}, t^{(r)}, c_r)$, where $v^{(i)}$ is the message vector, $t^{(i)}$ is the corresponding tag, and $c_i \in \mathbb{F}_q$ is the combination coefficient. Output a tag t for the vector $v = \sum_{i=1}^{r} c_i v^{(i)}$, satisfying

$$\mathsf{Verify}\Big(\sum_{i=1}^{r} c_i v^{(i)}, k, \mathsf{Combine}((v^{(1)}, t^{(1)}, c_1), \cdots, (v^{(r)}, t^{(r)}, c_r))\Big) = 1$$

3 Homomorphic MAC Scheme I [1]

3.1 Basic Construction

- MAC: for a n dimension vector $v = (v_1, \cdots, v_n) \in \mathbb{F}_q^n$, and a $n+l$ dimension secret key $k = (k_1, \cdots, k_{n+l}) \in \mathbb{F}_q^{n+l}$, compute

$$t_j = -\Big(\sum_{i=1}^{n} v_i k_i\Big)/k_{n+j}$$

for $j = 1, \cdots, l$. Output the corresponding tag $t = (t_1, \cdots, t_l) \in \mathbb{F}_q^l$.

- Verify: for a input (v, k, t), check whether

$$t_j = -\Big(\sum_{i=1}^{n} v_i k_i\Big)/k_{n+j}$$

hold for every $j \in [1, l]$. If do, output 1, otherwise output 0.

- Combine: for the input sequence $(v^{(1)}, t^{(1)}, c_1), \cdots, (v^{(r)}, t^{(r)}, c_r)$, output a tag $t = \sum_{i=1}^{r} c_i t^{(i)}$.

Correctness: Let $x^{(i)} = (x_1^{(i)}, \cdots, x_n^{(i)}), i = 1, \cdots, m$ are message vectors, $t^{(i)} = (t_1^{(i)}, \cdots, t_l^{(i)})$ is the tag corresponding to $x^{(i)}$. By the algorithm MAC, we have

$$t_j^{(i)} = -\Big(\sum_{h=1}^{n} x_h^{(i)} k_i\Big)/k_{n+j}$$

which is equivalent to

$$\sum_{h=1}^{n} x_h^{(i)} k_i + t_j^{(i)} k_{n+j} = 0$$

it follows that

$$\sum_{i=1}^{m} c_i \Big(\sum_{h=1}^{n} x_h^{(i)} k_i\Big) + \sum_{i=1}^{m} c_i \Big(t_j^{(i)} k_{n+j}\Big) = 0$$

Security: Suppose that an adversary can at most enquire m message vectors $y^{(1)}, \cdots, y^{(m)}$, and obtain their tags $t^{(1)}, \cdots, t^{(m)}$, let $y^{(i)} = (y_1^{(i)}, \cdots, y_n^{(i)})$, and $t^{(i)} = (t_1^{(i)}, \cdots, t_l^{(i)})$, and let $y^{(*)}, t^{(*)}$ are successful forged message vector and tags, then we have the following equations:

$$\begin{pmatrix} y^{(1)} & t_1^{(1)} & 0 & \cdots & 0 \\ y^{(2)} & t_1^{(2)} & 0 & \cdots & 0 \\ \cdots & & \cdots & & \cdots \\ y^{(m)} & t_1^{(m)} & 0 & \cdots & 0 \end{pmatrix} \cdot k = 0 \qquad \begin{pmatrix} y^{(1)} & 0 & t_2^{(1)} & \cdots & 0 \\ y^{(2)} & 0 & t_2^{(2)} & \cdots & 0 \\ \cdots & & \cdots & & \cdots \\ y^{(m)} & 0 & t_2^{(m)} & \cdots & 0 \end{pmatrix} \cdot k = 0$$

$$\begin{pmatrix} y^{(1)} & 0 & \cdots & 0 & t_l^{(1)} \\ y^{(2)} & 0 & \cdots & 0 & t_l^{(2)} \\ \cdots & & \cdots & & \cdots \\ y^{(m)} & 0 & \cdots & 0 & t_l^{(m)} \end{pmatrix} \cdot k = 0 \qquad \begin{pmatrix} y^{(*)} & t_1^{(*)} & 0 & \cdots & 0 \\ y^{(*)} & 0 & t_2^{(*)} & \cdots & 0 \\ \cdots & & \cdots & & \cdots \\ y^{(*)} & 0 & \cdots & 0 & t_l^{(*)} \end{pmatrix} \cdot k = 0$$

there are $n + l$ variables $k_1, \cdots, k_{N+l}$, let then rank of the system of the pervious ml equations is R, then the rank of the system of the total equations is $R + l$. Therefore, the probability of a successful forging is $\frac{1}{q^l}$.

Thanks! & Questions?