Solutions:

1. a. $\lambda = 5(0.4) = 2$ per five minute period

   b. $P(x) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{2^x e^{-2}}{x!}$

x	$P(x)$
0	0.1353
1	0.2707
2	0.2707
3	0.1804

   c. $P(\text{Delay Problems}) = P(x > 3) = 1 - P(x \leq 3) = 1 - 0.8571 = 0.1429$

2. a. $\mu = 0.6$ customers per minute

   $P(\text{service time} \leq 1) = 1 - e^{-(0.6)1} = 0.4512$

   b. $P(\text{service time} \leq 2) = 1 - e^{-(0.6)2} = 0.6988$

   c. $P(\text{service time} > 2) = 1 - 0.6988 = 0.3012$

3. a. $P_0 = 1 - \dfrac{\lambda}{\mu} = 1 - \dfrac{0.4}{0.6} = 0.3333$

   b. $L_q = \dfrac{\lambda^2}{\mu(\mu - \lambda)} = \dfrac{(0.4)^2}{0.6\,(0.6 - 0.4)} = 1.3333$

   c. $L = L_q + \dfrac{\lambda}{\mu} = 1.3333 + \dfrac{0.4}{0.6} = 2$

   d. $W_q = \dfrac{L_q}{\lambda} = \dfrac{1.3333}{0.4} = 3.3333$ min.

   e. $W = W_q + \dfrac{1}{\mu} = 3.3333 + \dfrac{1}{0.6} = 5$ min.

   f. $P_w = \dfrac{\lambda}{\mu} = \dfrac{0.4}{0.6} = 0.6667$

4. $P_n = \left(\dfrac{\lambda}{\mu}\right)^n P_0 = \left(\dfrac{0.4}{0.6}\right)^n (0.3333)$

n	P_n
0	0.3333
1	0.2222
2	0.1481
3	0.0988

   $P(n > 3) = 1 - P(n \leq 3) = 1 - 0.8024 = 0.1976$

5. a. $P_0 = 1 - \dfrac{\lambda}{\mu} = 1 - \dfrac{10}{12} = 0.1667$

   b. $L_q = \dfrac{\lambda^2}{\mu(\mu - \lambda)} = \dfrac{10^2}{12\,(12 - 10)} = 4.1667$

c. $W_q = \dfrac{L_q}{\lambda}$ = 0.4167 hours (25 minutes)

d. $W = W_q + \dfrac{1}{\mu}$ = .5 hours (30 minutes)

e. $P_w = \dfrac{\lambda}{\mu} = \dfrac{10}{12} = 0.8333$

6. a. $P_0 = 1 - \dfrac{\lambda}{\mu} = 1 - \dfrac{1.25}{2} = 0.375$

b. $L_q = \dfrac{\lambda^2}{\mu(\mu - \lambda)} = \dfrac{1.25^2}{2(2 - 1.25)} = 1.0417$

c. $W_q = \dfrac{L_q}{\lambda} = \dfrac{1.0417}{1.25}$ = 0.8333 minutes (50 seconds)

d. $P_w = \dfrac{\lambda}{\mu} = \dfrac{1.25}{2} = 0.625$

e. Average one customer in line with a 50 second average wait appears reasonable.

7. a. $L_q = \dfrac{\lambda^2}{\mu(\mu - \lambda)} = \dfrac{(2.5)^2}{5(5 - 2.5)} = 0.5000$

$L = L_q + \dfrac{\lambda}{\mu} = 0.5000 + \dfrac{2.5}{5} = 1$

b. $W_q = \dfrac{L_q}{\lambda} = \dfrac{0.5000}{2.5}$ = 0.20 hours (12 minutes)

c. $W = W_q + \dfrac{1}{\mu} = 0.20 + \dfrac{1}{5}$ = 0.40 hours (24 minutes)

d. $P_w = \dfrac{\lambda}{\mu} = \dfrac{2.5}{5} = 0.50$

8. $\lambda = 1$ and $\mu = 1.25$

$$P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{1}{1.25} = 0.20$$

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{1}{1.25(0.25)} = 3.2$$

$$L = L_q + \frac{\lambda}{\mu} = 3.2 + \frac{1}{1.25} = 4$$

$$W_q = \frac{L_q}{\lambda} = \frac{3.2}{1} = 3.2 \text{ minutes}$$

$$W = W_q + \frac{1}{\mu} = 3.2 + \frac{1}{1.25} = 4 \text{ minutes}$$

$$P_w = \frac{\lambda}{\mu} = \frac{1}{1.25} = 0.80$$

Even though the services rate is increased to $\mu = 1.25$, this system provides slightly poorer service due to the fact that arrivals are occurring at a higher rate. The average waiting times are identical, but there is a higher probability of waiting and the number waiting increases with the new system.

9. a. $P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{2.2}{5} = 0.56$

b. $P_1 = \left(\frac{\lambda}{\mu}\right) P_0 = \frac{2.2}{5}(0.56) = 0.2464$

c. $P_2 = \left(\frac{\lambda}{\mu}\right)^2 P_0 = \left(\frac{2.2}{5}\right)^2 (0.56) = 0.1084$

d. $P_3 = \left(\frac{\lambda}{\mu}\right)^3 P_0 = \left(\frac{2.2}{5}\right)^3 (0.56) = 0.0477$

e. P(More than 2 waiting) = P(More than 3 are in system)
$= 1 - (P_0 + P_1 + P_2 + P_3) = 1 - 0.9625 = 0.0375$

f. $$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{2.2^2}{5(5 - 2.2)} = 0.3457$$

$$W_q = \frac{L_q}{\lambda} = 0.157 \text{ hours} \quad (9.43 \text{ minutes})$$

10. a.

$\lambda = 2$	$\mu = 3$	$\mu = 4$
Average number waiting (L_q)	1.3333	0.5000
Average number in system (L)	2.0000	1.0000
Average time waiting (W_q)	0.6667	0.2500
Average time in system (W)	1.0000	0.5000
Probability of waiting (P_w)	0.6667	0.5000

b. New mechanic $= \$30(L) + \$14$
$= 30(2) + 14 = \$74$ per hour

Experienced mechanic $= \$30(L) + \$20$
$= 30(1) + 20 = \$50$ per hour

∴ Hire the experienced mechanic

11. a. $\lambda = 2.5 \quad \mu = 60/10 = 6$ customers per hour

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{(2.5)^2}{6\,(6 - 2.5)} = 0.2976$$

$$L = L_q + \frac{\lambda}{\mu} = 0.7143$$

$$W_q = \frac{L_q}{\lambda} = 0.1190 \text{ hours} \quad (7.14 \text{ minutes})$$

$$W = W_q + \frac{1}{\mu} = 0.2857 \text{ hours}$$

$$P_w = \frac{\lambda}{\mu} = \frac{2.5}{6} = 0.4167$$

b. No; $W_q = 7.14$ minutes. Firm should increase the mean service rate (μ) for the consultant or hire a second consultant.

c. $\mu = 60/8 = 7.5$ customers per hour

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{(2.5)^2}{7.5\,(7.5 - 2.5)} = 0.1667$$

$$W_q = \frac{L_q}{\lambda} = 0.0667 \text{ hours} \quad (4 \text{ minutes})$$

The service goal is being met.