# A Randomized Reverse Auction for Cost-Constrained D2D Content Distribution

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***Abstract*—Device-to-device (D2D) communications are not only featured by high spectral and energy efficiency, but also offer appealing benefits for applications such as content distribution, traffic offloading and coverage expansion. In this paper, we consider a content distribution scenario where a base station (BS) can divert message requests to some source devices to be fulfilled via D2D communications and thereby save the resource cost. To maximize the BS's gain in cost saving, we need to properly assign a broadcast message for each source and decide the payment to incentivize participation. The message selection is an NP-hard problem, while the payment determination is also nontrivial. The payment should be sufficient to compensate for a source device's resource cost and meanwhile, incentivize the source to truthfully declare its private cost. Modeling the problem as a reverse auction, we develop a randomized mechanism which is truthful in expectation, individually rational, and subject to a polynomial computation time. Also, it maintains an approximation guarantee with respect to the fractional optimal solution. The numerical results show the performance of the randomized mechanism in the D2D content distribution scenario.**

***Index Terms*—D2D communications, content distribution, truthful in expectation, reverse auction, randomized mechanism.**

## I. Introduction and Related Works

The past decade has witnessed an explosive growth of mobile traffic, smart devices, and assorted applications. To accommodate the surging demands, a variety of new techniques are actively studied to aggregate resources and boost network capacity, such as multipath transport, user cooperation, and device-to-device (D2D) communications [1,2]. In particular, D2D enables direct communications between mobile devices in a peer-to-peer (P2P) fashion bypassing the base station (BS), and thereby offers various benefits such as expanding coverage, offloading traffic, improving energy efficiency, and facilitating proximity-based services.

In the literature, there have been many studies on resource allocation for D2D communications with underlay spectrum sharing, which allows D2D and cellular users share the same spectrum simultaneously. The main challenge is to limit the co-channel interference while achieving high spectral efficiency. With effective spectrum sharing, D2D can support promising applications such as content distribution and traffic offloading. In [3], a coalitional game is formulated for content distribution in vehicular ad hoc networks, and the analysis mainly focuses on characteristics such as convergence, stability, and efficiency. In [4], a randomized auction mechanism is proposed for data offloading with D2D communications, where the BS hires helper devices to share their messages with nearby requesting devices and get reimbursed for their costs. Auction is a popular trading form that can efficiently distribute resources at competitive prices. An auction mechanism is expected to hold desirable properties, such as *truthfulness*, *individual rationality*, *economic efficiency*, and *computational efficiency*.

In this paper, we consider a D2D content distribution scenario similar to that of [4] but with a specific resource cost model. We formulate a reverse auction from a different perspective, and further take into account a cost constraint which can be adjusted by the BS based on its affordable cost. Though the key problem of assigning D2D broadcast messages to helper devices is still NP-hard as the problem in [4], it has a fully polynomial-time approximation scheme (FPTAS). Thereby, we are able to develop a randomized mechanism by combining the mechanism design technique via linear programming (LP) in [5] and the fast convex decomposition method proposed in [6]. The randomized mechanism is extended from the *generalized* Vickrey-Clarke-Groves (VCG) mechanism, and guaranteed to be truthful in expectation and individually rational. In addition, the mechanism is subject to a polynomial computation time, and thus satisfies computational efficiency. As an expense, the mechanism fails to hold economic efficiency with respect to the optimal objective of the message assignment problem, but it maintains an approximation guarantee for the worst case.

The remainder of this paper is structured as follows. Section II describes the D2D content distribution scenario and our problem formulation as a reverse auction. In Section III, we introduce the preliminary techniques and details of the randomized mechanism. Numerical results are presented in Section IV, followed by conclusion in Section V.

## II. System Modeling and Problem Formulation

### A. D2D Content Distribution

Consider a content distribution scenario depicted in Fig. 1. Here, a set of destination devices, $D$, are requesting a set of $m$ messages, $M$. If all requests (denoted by $R$) were to be fulfilled through the BS, the BS had to unicast the

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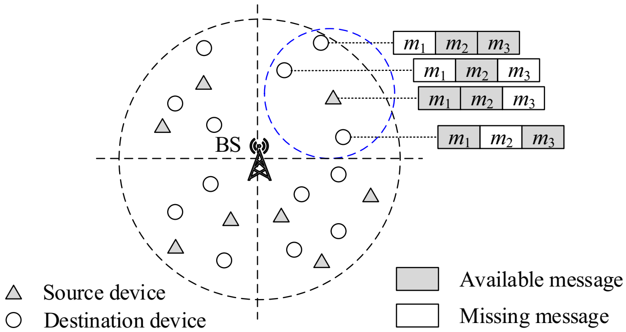


Fig. 1. Content distribution scenario.

requested messages to each device and thus introduce a high cost. Suppose there are $n$ source devices, $S$, in which each $s_i \in S$ already possesses a subset of messages $M_i \subseteq M$. It is potentially acceptable for a source device $s_i$ to broadcast a message $m_k \in M_i$ to multiple destination devices $d_j \in D$ which request message $m_k$ and stay in the close proximity (transmission range) of $s_i$. Thereby, certain traffic can be offloaded from the BS and the content is distributed in a more cost-effective manner. Assume that the content distribution task is executed periodically. In each time interval, at most one message is selected for a source device such that as many message requests as possible are served by the source devices and the BS is relieved of a high cost.

Given source device $s_i \in S$ which possesses message $m_k \in M_i$, let $D_{ik}$ denote the set of destination devices which request message $m_k$ and fall within the transmission range of $s_i$. Assume that the co-channel interference between D2D and cellular users can be approximated by a Gaussian process similar to additive thermal noise with mean $I_0$ and $N_0$, respectively. Considering log-distance path loss and Rayleigh fading, we characterize the signal to interference plus noise ratio (SINR) at the *worst* receiver $d_j \in D_{ik}$ by

$$\gamma_{ij_k} = \frac{P_0}{N_0 + I_0} \|s_i - d_j\|^{-\varphi} h_{ij} \tag{1}$$

where $P_0$ is the transmit power, $\|s_i - d_j\|$ is the distance between $s_i$ and $d_j$, $\varphi$ is the path-loss exponent, and $h_{ij}$ denotes the small-scale channel fading, which is exponentially distributed with unit mean. Similar to [7], we consider that a receiver is able to successfully decode the received message only when the local SINR is not less than a threshold $T_0$. Therefore, the probability that message $m_k$ is received successfully by all destination devices in $D_{ik}$ is given by

$$P_{ij_k} = \mathrm{P}\big[\gamma_{ij_k} \geq T_0\big] = e^{-\frac{T_0(N_0+I_0)}{P_0}\|s_i - d_j\|^{\varphi}}. \tag{2}$$

Assume that the resource cost of source device $s_i$ is proportional to the required SINR to achieve a minimum transmission success probability $P_s$, *i.e.*,

$$c_{ik} = z\Big(-T_0 \|s_i - d_j\|^{\varphi} / \log(P_s)\Big) \tag{3}$$

where $z(\cdot)$ is a monotonic non-decreasing function which maps the required resource to a monetary cost, *e.g.*, a logarithmic function for the numerical experiments in Section IV.

### *B. Problem Formulation*

Fig. 2 gives a concrete example for the content distribution problem described in Section II-A. The graph in Fig. 2 shows two source devices in $S$, three messages in $M$, and four destination devices in $D$. Three destination devices fall within the transmission range of each source device. To present the auction model for the content distribution problem, we extend the graph in Fig. 2 to a new graph in Fig. 3. Assume that the resource cost for a source device to broadcast a message is specified as above. Then, a *reverse auction* can be formulated from the perspective of source devices as sellers, while the buyers are the destination devices which want to purchase their missing messages. The sellers submit their asks (not be more than their resource costs) to an auctioneer to determine the allocation of broadcast messages to source devices and the clearing payments (not be less than their asks). Accordingly, we can formulate the message allocation problem as follows:

$$\text{maximize} \quad \sum_{i=1}^{n} \sum_{m_k \in M_i} (c_{Bk_i} - c_{ik}) x_{ik} \tag{4a}$$

$$\text{subject to} \quad \sum_{i=1}^{n} \sum_{m_k \in M_i} c_{ik} x_{ik} \leq C \tag{4b}$$

$$\sum_{m_k \in M_i} x_{ik} \leq 1, \forall s_i \in S \tag{4c}$$

$$x_{ik} \in \{0,1\}, \forall s_i \in S, m_k \in M_i. \tag{4d}$$

Here, $c_{Bk_i}$ is the BS's total cost to satisfy the requests that source $s_i$ is able to serve by broadcasting message $m_k \in M_i$, so $(c_{Bk_i} - c_{ik})$ is the gain by diverting requests from the BS to $s_i$. Also, the BS limits the total cost for all source devices assigned with a broadcast message by a cost budget $C$. The solution variable $x_{ik}$ is binary, indicating whether source $s_i$ is assigned to broadcast message $m_k \in M_i$. Thus, the problem dimension is $|\sqcup_i M_i|$, where $\sqcup_i M_i$ is the *disjoint* union of the subsets of messages of all sources, *i.e.*, the union of the elements in $M_i$'s by retaining the original set membership. Considering the example in Fig. 3, $M_1 = \{m_1, m_2\}$ and $M_2 = \{m_1, m_3\}$, which gives $M_1 \sqcup M_2 = \{s_1m_1, s_1m_2, s_2m_1, s_2m_3\}$. Assume that $c_{ik} \leq c_{Bk_i}$ and $0 < c_{ik} \leq C$, $\forall s_i \in S, m_k \in M_i$. Thus, the feasible solution domain $\Pi$ holds the *packing property* that, given $x \in \Pi$ and $x \geq y$, it must be $y \in \Pi$ as well. In particular, the origin vector $\mathbf{0}$ and unit vectors $\{e_\kappa\}$ all fall within $\Pi$. For every dimension $\kappa$, the unit vector $e_\kappa$ is all zeros except that the $\kappa^{th}$ component is equal to 1. It represents a feasible solution that one available message is selected for only one source, while the other sources are not assigned any message.

In (4), the resource costs of source devices are *private* and unknown to the BS. If the sources did not disclose their true information, the optimization result based on incorrect information would be non-optimal and even non-predictable.

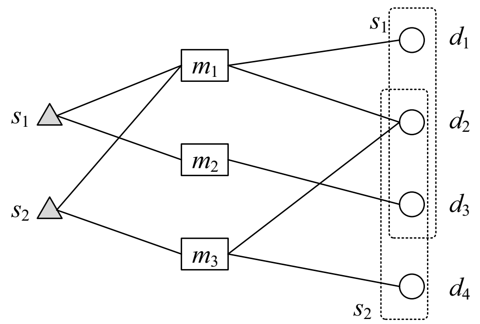


Fig. 2. A graph model for a concrete content distribution scenario, where $s_1$ can transmit to $\{d_1, d_2, d_3\}$ while $s_2$ can reach $\{d_2, d_3, d_4\}$.

Hence, we aim to design a *truthful* mechanism, which selects broadcast messages for the sources such that every source always maximizes its utility by revealing its true information regardless of other sources' declarations.

## III. A Randomized Reverse Auction Mechanism

### A. Design Rationale

The VCG mechanism is the most important truthful mechanism for agents with *quasilinear preferences*, in which the set of outcomes is given by $O = X \times \mathbb{R}^n$, where $X$ represents a finite set of nonmonetary choices, *e.g.*, assignment of broadcast messages for source devices in our reverse auction, and $\mathbb{R}^n$ represents the monetary transfer each agent gives (if positive) or receives (if negative). The choice rule and pricing rule of VCG are defined as follows:

$$\mathcal{X}(v) = \arg\max_{x \in X} \sum_i v_i(x) \tag{5}$$

$$\wp_i(v) = \sum_{i' \neq i} v_{i'}(\mathcal{X}(v_{-i})) - \sum_{i' \neq i} v_{i'}(\mathcal{X}(v)) \tag{6}$$

where $v_i(x)$ denotes the valuation of agent $i$ for choice $x \in X$, $v$ is the vector of all agents' valuations, and $v_{-i}$ is the valuation vector of all agents excluding $i$. The total valuation of all agents with respect to choice $x$ is also known as *social welfare*, denoted by $w(x) = \sum_i v_i(x)$. Note that the optimality in the choice rule is critical to ensure truthfulness. The VCG mechanism can be generalized to an *affine maximizer* for the choice rule in the form

$$\mathcal{X}(v) = \arg\max_{x \in X} \left(\beta_x + \sum_i \omega_i v_i(x)\right) \tag{7}$$

where $\omega_i$ is a non-negative weight and $\beta_x$ is an arbitrary constant. Correspondingly, given the choice $\mathcal{X}(v) = x$, the pricing rule can be extended to

$$\wp_i(v) = -\frac{1}{\omega_i}\left[\beta_x + \sum_{i' \neq i} \omega_{i'} v_{i'}(x)\right]. \tag{8}$$

The problem in (4) is the social choice function we aim to implement by a truthful mechanism. Here, the private valuation of source $s_i \in S$ is $v_{ik} = -c_{ik}, \forall m_k \in M_i$. The optimization objective is exactly an affine maximizer with $\omega_i = 1$ and $\beta_x = \sum_{i=1}^n \sum_{m_k \in M_i} c_{Bk_i} x_{ik}$. For a reverse auction, the (generalized) VCG pricing rule determines a non-positive price for agent $i$, $\wp_i(v)$. For notation simplicity, we write $\wp_i(v)$ as $-p_i$, where $p_i \geq 0$. Thus, if message $m_{k'}$ is selected for source $s_i$, the utility function of source $s_i$ is given by $u_{ik'} = (-c_{ik'}) - (-p_i) = p_i - c_{ik'}$. Unfortunately, problem (4) is NP-hard, which can be seen in the following reformulation. However, the optimality of the choice rule with respect to the optimization problem is essential to maintain truthfulness for the VCG mechanism. Hence, in Section III, we resort to a weaker notion of truthfulness. Also, we reformulate (4) to a slightly different but equivalent problem.

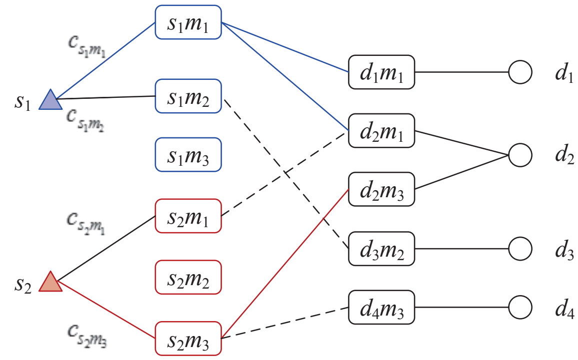


Fig. 3. An extended graph model for a concrete content distribution scenario.

For each source $s_i$, we can add a dummy message $m_d$ with a zero cost and zero gain to the subset of available messages at $s_i$, which gives $M'_i = M_i \cup \{m_d\}$ and extends $c_{ik}$ and $c_{Bk_i}$ correspondingly. For simplicity, we use the same notation for $c_{ik}$ and $c_{Bk_i}$ without confusion. Then, the inequality in constraint (4c) becomes equality and the problem is translated into the multiple-choice knapsack problem (MCKP) [8]:

$$\text{maximize} \quad \sum_{i=1}^{n} \sum_{m_k \in M'_i} (c_{Bk_i} - c_{ik}) x_{ik} \tag{9a}$$

$$\text{subject to} \quad \sum_{i=1}^{n} \sum_{m_k \in M'_i} c_{ik} x_{ik} \leq C \tag{9b}$$

$$\sum_{m_k \in M'_i} x_{ik} = 1, \forall s_i \in S \tag{9c}$$

$$x_{ik} \in \{0, 1\}, \forall s_i \in S, m_k \in M'_i. \tag{9d}$$

Here, the classes and items in MCKP correspond to the sources in $S$ and messages in $M' = \sqcup M'_i$, respectively. The new problem size is denoted by $\eta = |M'|$. For reference convenience, we denote $(c_{Bk_i} - c_{ik})$ by a gain parameter $g_{ik}$ and represent the objective function in (9a) by $g(x)$. Though MCKP is NP-hard, fortunately, a FPTAS [9] gives a $(1 + \epsilon)$-approximation for any arbitrary $\epsilon > 0$ in running time polynomial in both the problem size $\eta$ and $1/\epsilon$.

In [5], Lavi and Swamy propose a general technique via LP to construct *randomized* mechanisms which are *truthful in expectation* and different from truthfulness for *determin-*

*istic* mechanisms. A randomized mechanism can be viewed as a probability distribution over deterministic mechanisms and selects the outcome as flipping coins according to the probability distribution. A randomized mechanism is truthful in expectation if an agent always maximizes its *expected utility* by announcing its true valuation regardless of other agents' declarations. Lavi and Swamy's technique constructs a randomized truthful mechanism in polynomial time, which circumvents the computational complexity of solving NP-hard problems. As an expense, the social welfare of the randomly selected outcome is not maximized but only near-optimal with an approximation guarantee. This technique can be summarized in the following main steps:

1) Obtain a truthful *fractional* VCG mechanism via LP relaxation, which converts an NP-hard integer linear program (ILP) to an LP solvable in polynomial time. Given the input valuation $v$, let $x^{F^*}$ and $p^{F^*}$ denote the output choice and price, respectively.
2) Scale down both $x^{F^*}$ and $p^{F^*}$ by some constant $\alpha$ ($\alpha \geq 1$) to $\frac{x^{F^*}}{\alpha}$ and $\frac{p^{F^*}}{\alpha}$, respectively, which preserves truthfulness since this is in the fractional domain. Here, $\alpha$ is the *integrality gap* of the LP relaxation, which guarantees $w(x^{I^*}) \geq \frac{w(x^{F^*})}{\alpha}$, where $x^{I^*}$ denotes an optimal solution to the original ILP.
3) Decompose $\frac{x^{F^*}}{\alpha}$ *exactly* into a convex combination of polynomially many *integral* solutions $\{x^l\}$ such that $\frac{x^{F^*}}{\alpha} = \sum_l \lambda_l x^l \triangleq \sigma(\lambda)$, where $\lambda$ is the vector of all $\lambda_l$'s, and $\sum_l \lambda_l = 1, \lambda_l \geq 0$.
4) Select choice $x^I$ randomly among $\{x^l\}$ according to the probability distribution $\lambda$. For any agent $i$, define $\overline{v}_i(\lambda) \triangleq \sum_l \lambda_l v_i(x^l)$. Then, for the selected choice $x^I$, set the price for agent $i$ to $\frac{p^{F^*}}{\alpha} \cdot \frac{v_i(x^I)}{\overline{v}_i(\lambda)}$ if $\overline{v}_i(\lambda) > 0$, and set the price to 0 if $v_i(x^I) = \overline{v}_i(\lambda) = 0$. This pricing rule maintains *individual rationality*, which ensures that the utility of each agent is non-negative.

Here, the convex decomposition in Step 3) is the most important and time-consuming. To guarantee truthfulness, the decomposition cannot be approximate. Suppose that a polynomial-time approximation algorithm, $\mathcal{A}$, *verifies* the integrality gap $\alpha$, which means $w(\mathcal{A}(v)) \geq \frac{w(x^{F^*})}{\alpha}$. It is worth noting that this is an $\alpha$-approximation with respect to the LP-relaxed optimum instead of the ILP optimum. Based on the $\alpha$-approximation algorithm, Lavi and Swamy's technique uses the ellipsoid method with a proposed separation oracle. However, the ellipsoid method can be quite inefficient in practice. Since $\mathcal{A}$ verifies the integrality gap $\alpha$, a convex decomposition of $\frac{x^{F^*}}{\alpha}$ must exist. The key challenge is to sample only few but still sufficient integer points [6]. Hence, we use the fast convex decomposition (FCD) method proposed in [6] which yields an $\alpha(1+\epsilon)$-approximation but only requires a quadratic number of calls to an integrality gap verifier.

### B. Mechanism Details

Combining Lavi and Swamy's technique [5] and the FCD method [6], we need an approximation algorithm which verifies the integrality gap $\alpha$ with respect to the LP-relaxed optimum. For a natural LP relaxation of MCKP, Dyer and Zemel develop a *linear*-time algorithm to efficiently compute a continuous optimal solution [8]. The Dyer-Zemel algorithm obtains an optimal solution $x^{F^*}$, which has *at most* two fractional variables and they must be adjacent within the same class if they exist. Consider a simple rounding algorithm, which returns an integer solution $\bar{x}$ that maximizes the gain among three alternative integer solutions [9]: 1) $x^{I_1}$ which discards all fractional variables in $x^{F^*}$ if any; 2) $x^{I_2}$ which only keeps the *full* selection corresponding to one fractional variable; and 3) the similar integer solution $x^{I_3}$ corresponding to the other fractional variable. Here, the second and third solutions are feasible since any $c_{ik} \leq C$. Denoting the optimal integer solution by $x^{I^*}$, we can easily see that the objective values with respect to (9a) satisfy

$$\begin{aligned} g(x^{F^*}) &\leq g(x^{I_1}) + g(x^{I_2}) + g(x^{I_3}) \\ &\leq 3 \cdot \max(g(x^{I_1}), g(x^{I_2}), g(x^{I_3})) = 3 \cdot g(\bar{x}) \\ &\leq 3 \cdot g(x^{I^*}). \end{aligned}$$

Based on the Dyer-Zemel algorithm, for any $\epsilon_a > 0$, a FPTAS of time $O(n\eta/\epsilon_a)$ (recall $\eta = |M'|$ is the problem size) for MCKP is proposed in [9]. Thus, it gives an approximation algorithm $\mathcal{A}$ that verifies an integrality gap $\alpha'(1+\epsilon_a)$, $\alpha' = 3$.

Applying $\mathcal{A}$ with Lavi and Swamy's technique and the FCD method, we can use Alg. 1 to construct a randomized reverse

**Algorithm 1:** The randomized reverse auction mechanism.

```
Input: S, D, R, {M'_i ⊆ M : s_i ∈ S}, α', ε_a, ε_d,
       {c_ik : s_i ∈ S, m_k ∈ M'_i}, {c_Bk_i : m_k ∈ M'_i}
Output: x^I, p^I
begin Obtain a fractional VCG mechanism with choice x^{F*} and pricing p^{F*}
    Use the Dyer-Zemel algorithm for problem (9) to obtain x^{F*} and p^{F*}
begin Scale down x^{F*} and decompose x^{F*} / (α'(1+ε_a)(1+ε_d))
    Obtain an approximate decomposition, represented by
      λ = {λ_1, ..., λ_q} and X = {x^1, ..., x^q}, within an
      ε-distance to x^{F*} / (α'(1+ε_a)), where ε = ε_d / √η
    Extend X to X' = X ∪ {e_1, ..., e_η, 0}, convert λ into λ'
      such that σ(λ') ≥ x^{F*} / (α'(1+ε_a)(1+√η ε))
    Convert λ' to λ'' to obtain an exact decomposition such
      that σ(λ'') = x^{F*} / (α'(1+ε_a)(1+√η ε))
begin Implement the exact decomposition as a randomized mechanism
    θ ← a random value within [0, 1]
    Randomly select choice x^I ∈ X' such that
      I ← arg min_{1≤ℓ≤q+η+1} Σ_{l=1}^{ℓ} λ''_l ≥ θ
    p^I ← 0
    for s_i ∈ S such that Σ_{m_k∈M_i} x^I_ik > 0 do
        c̄_i(λ'') ← Σ_{l=1}^{q+η+1} λ''_l Σ_{m_k∈M_i} c_ik x^l_ik
        if x^I_ik' = 1 then
            p^I_i ← p^{F*} / (α'(1+ε_a)(1+ε_d)) · c_ik' / c̄_i(λ'')
    Return x^I and p^I
```

auction mechanism. As seen, there are three main parts of the algorithm. First, we use the Dyer-Zemel algorithm for problem (9) to obtain a fractional VCG mechanism with choice $x^{F^*}$ and pricing $p^{F^*}$. This step is very fast since the Dyer-Zemel algorithm is subject to a linear computation time.

Second, the scaled-down choice $\frac{x^{F^*}}{\alpha'(1+\epsilon_a)(1+\epsilon_d)}$ is decomposed with the FCD method. Here, we first obtain an *approximate* decomposition $\lambda$ within an $\varepsilon$-distance to $\frac{x^{F^*}}{\alpha'(1+\epsilon_a)}$, where $\varepsilon = \frac{\epsilon_d}{\sqrt{\eta}}$. This step takes a maximum number of iterations, $\eta\varepsilon^{-2} = \eta^2\epsilon_d^{-2}$, which takes time $O(n\eta^3\epsilon_d^{-2}\epsilon_a^{-1})$ since the Dyer-Zemel algorithm in each iteration has a time complexity $O(n\eta/\epsilon_a)$. After that, the approximate decomposition is extended by including the origin and unit vectors, which are also feasible integer solutions to problem (9). The *extended* decomposition $\lambda'$ *dominates* the new target $\frac{x^{F^*}}{\alpha'(1+\epsilon_a)(1+\sqrt{\eta}\varepsilon)}$, which admits an additional scaling factor of $\sqrt{\eta}\varepsilon$ compared to the preceding approximate decomposition. Last, the dominant decomposition is turned into an *exact* decomposition $\lambda''$ via an iterative adjusting procedure. This step takes a time $O(\eta(q+\eta+1)+\frac{\eta(\eta+1)}{2})$, where $q$ is the number of integer points corresponding to $\lambda$ and there are $(q+\eta+1)$ integer points in the decompositions $\lambda'$ and $\lambda''$.

Third, we implement the decomposition of the scaled-down fractional choice $\frac{x^{F^*}}{\alpha'(1+\epsilon_a)(1+\epsilon_d)}$ as a randomized mechanism. An integer choice $x^I$ can be randomly selected among $X'$, the set of integer solutions in the decomposition, by viewing $\lambda''$ as the probability distribution over the integer points in $X'$. Another main function of this part is to correspondingly scale down the fractional price to $\frac{p^{F^*}}{\alpha'(1+\epsilon_a)(1+\epsilon_d)}$, and revise it to preserve individual rationality. For the randomly selected choice $x^I$, we set the payment to source $i$ with an assigned message $m_{k'}$ as $p_i^I = \frac{p^{F^*}}{\alpha'(1+\epsilon_a)(1+\epsilon_d)} \cdot \frac{c_{ik'}}{\bar{c}_i(\lambda'')}$. Here, $\bar{c}_i(\lambda'')$ can be interpreted as the *mean* cost to source $i$ over all possible choices in $X'$. Since $\lambda''$ gives an exact decomposition to the scaled-down fractional VCG mechanism, which is individually rational, we have $\frac{p^{F^*}}{\alpha'(1+\epsilon_a)(1+\epsilon_d)} \geq \bar{c}_i(\lambda'')$. Therefore, the payment to each source $i$ is not less than its cost, *i.e.*, $p_i^I \geq c_{ik'}$, which ensures individual rationality for this reverse auction. Combining the three parts of Alg. 1, we can see the overall time complexity is $O(n\eta^3\epsilon_d^{-2}\epsilon_a^{-1}+q\eta)$, which is polynomial in the problem size $\eta$.

## IV. Numerical Results

In this section, we present numerical results to examine the performance of Alg. 1 in a D2D content distribution scenario. Here, we consider that the BS intends to distribute 10 ($m$) messages to some devices within its coverage, which is a circular area of a radius 500m and centered at the BS. Assume that 30 devices are uniformly distributed within this area, among which 9 ($n$) are randomly selected as source devices and the other 21 as destination devices. Each source device holds a subset of messages, while each destination has a random set of message requests. The transmission range of source devices is set to 200m, and the path loss exponent ($\varphi$) is set to 3. According to the model in Section II-A, each source device determines its resource cost to broadcast a message to certain requesting devices within its transmission range, aiming at a success probability of 0.8 ($P_s$). The cost budget of the BS ($C$) is limited to be not more than 20% of the total cost if the BS were to serve all message requests by itself.

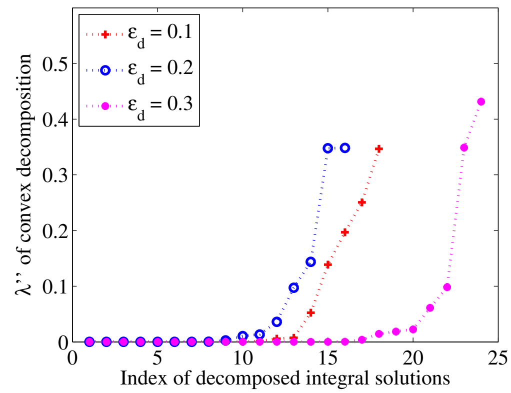


Fig. 4. Decomposition results with different values of $\epsilon_d$.

### A. Approximation Performance with Decomposition

Alg. 1 obtains an exact decomposition of the scaled-down fractional choice $\frac{x^{F^*}}{\alpha'(1+\epsilon_a)(1+\epsilon_d)}$ and implements it as a randomized mechanism. Fig. 4 shows $\lambda''$ of the decompositions with different approximation parameter $\epsilon_d$. For $\epsilon_d = 0.1, 0.2,$ and $0.3$, the decompositions include 18, 16, and 24 integer solutions, respectively. As seen, many components in $\lambda''$ are quite close to 0, which implies that the corresponding integer solutions are only chosen with small probabilities. However, these choices are essential to comprise an exact decomposition and ensure truthfulness.

According to Alg. 1, a decomposition of the scaled-down fractional choice $\frac{x^{F^*}}{\alpha'(1+\epsilon_a)(1+\epsilon_d)}$ can achieve an objective value of *at least* $\frac{1}{\alpha'(1+\epsilon_a)(1+\epsilon_d)}$ that of the fractional optimum for problem (9). One basis is the approximation algorithm $\mathcal{A}$, which is a FPTAS for MCKP with time complexity $O(n\eta/\epsilon_a)$. Fig. 5 shows the approximation ratios of the decompositions with different values of $\epsilon_a$. Here, we set $\epsilon_d$ to 0.1 and vary the value of $\epsilon_a$. Though an upper bound for the integrality gap is 3, a value less than 2 for $\alpha'$ is achievable with the experimental datasets. Hence, we set $\alpha' = 1.5$. As seen in Fig. 5, the overall approximation ratio ranges from 1.815 to 2.31, which exactly matches the scaling factor $\alpha'(1+\epsilon_a)(1+\epsilon_d)$.

Fig. 6 compares the fractional optimal solution $x^{F^*}$ with that of the integer decomposition, represented by $\lambda''$ and $X'$. Here, we set $\alpha'$ to 1.5, and both $\epsilon_a$ and $\epsilon_d$ to 0.1, which gives an overall scaling factor $\alpha'(1+\epsilon_a)(1+\epsilon_d) = 1.815$. The gain and cost are normalized by the corresponding maximum values for easy interpretation. As seen, when the number of messages varies from 10 to 20, the gain achieved by the integer decomposition, *i.e.*, $\sum_\ell \lambda''_\ell g(x^\ell)$, remains to be $\frac{1}{1.815}$ that of the fractional optimal solution, *i.e.*, $g(x^{F^*})$. Similarly, the cost ratio also keeps the theoretical scaling factor 1.815.

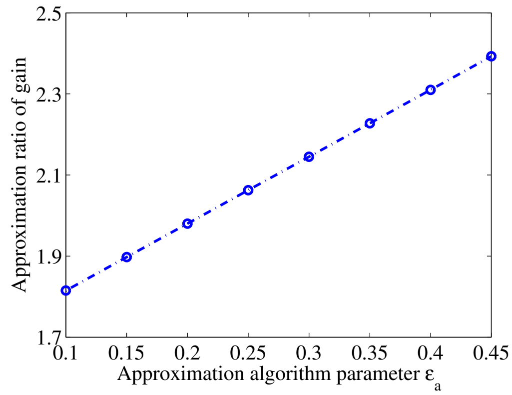


Fig. 5. Approximation ratios with different values of $\epsilon_a$.

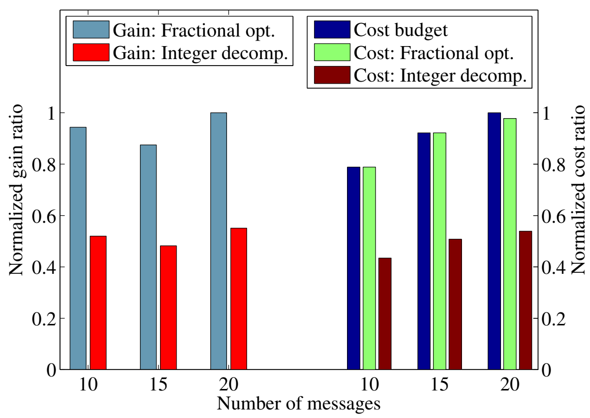


Fig. 6. Normalized gain and cost of the fractional optimal solution and the integer decomposition with different numbers of messages.

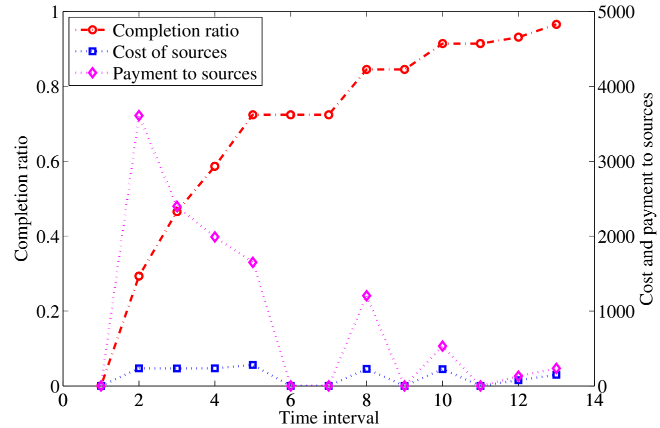


Fig. 7. Completion ratio and corresponding gain and payment with time.

In addition, the fractional optimal solution and the integer decomposition both satisfy the constraint of cost budget.

### *B. Content Distribution Performance*

Fig. 7 shows the cumulative completion ratio when the randomized mechanism based on the integer decomposition is applied over time. The completion ratio is the number of message requests diverted to source devices over the total number of messages. It also shows the corresponding payments to the sources, and the gain achieved by distributing message requests to the sources. As seen, the completion ratio steadily increases with time, though there are some short intervals with flat segments. This is because some destination devices receive duplicate messages broadcast by different sources. In addition, it can be seen that the payments to the sources are always not less than their corresponding costs, which confirms individual rationality of the randomized mechanism. The large gap between the cost and payment for some cases is because VCG is not *frugal*, *i.e.*, possibly with high overpayment.

## V. Conclusion and Future Work

In this paper, we consider a content distribution scenario via D2D communications. To fulfill the message requests from some destination devices, the BS can divert certain requests to some source devices which can satisfy these requests via D2D broadcast and thereby reduce the overall resource cost. Here, we model the problem that selects a broadcast message for each source as an MCKP. As the source devices should be incentivized by proper payments to compensate for their costs, it is essential to design a mechanism which can solicit truthful reports of private costs from the sources. Based on a FPTAS for the MCKP, we develop a randomized reverse auction mechanism. The mechanism is truthful in expectation, individually rational, and subject to a polynomial computation time. Also, it maintains an approximation guarantee with respect to the fractional optimal solution. The numerical results demonstrate the performance of the randomized mechanism. In the future, it would be interesting to further consider multiple classes of contents in a heterogeneous network environment [10].

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