A Uniform Method for Quantum Costs Optimization of Reversible Circuits

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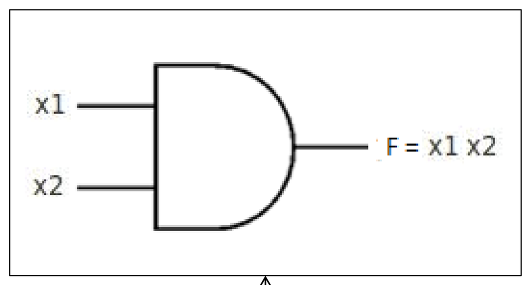
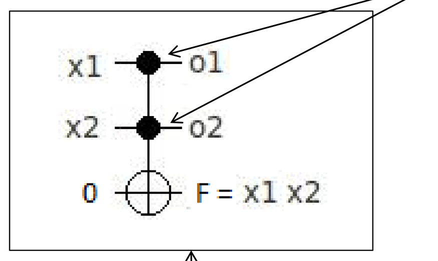
Reusable



- Classical Design
- Multiple-inputs and single-output
- Energy loss



- Reversible Design
- Multiple-inputs and multiple- outputs
- Potentially no energy loss

Problems

- Transformation of reversible logic into quantum circuits
- Quantum costs optimization of reversible circuits

Reversible Logic

- Multiple-inputs and multiple-outputs function.
- One to one correspondence between input vectors and output vectors.
- Quantum circuits inherently reversible.

Input	Output
000	000
001	001
010	010
011	111
100	110
101	011
110	100
111	101

Table 1. Reversible logic function

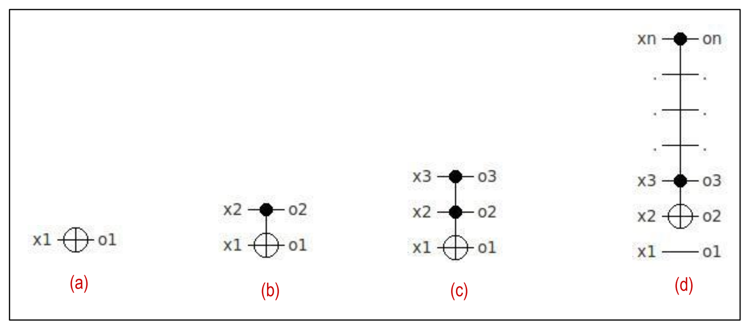


Figure 1. Reversible gates (a) Not, (b) Controlled Not, (c) Toffoli and (d) Generalized Toffoli

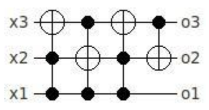


Figure 2. Reversible circuit that realizes function in table 1

- Bit in classical computation.
- Qubit in quantum computation a state vector .

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$|\alpha|^2 + |\beta|^2 = 1.$$

$\alpha_1|0\rangle + \beta_1|1\rangle$ $\alpha_2|0\rangle + \beta_2|1\rangle$

$\otimes$

$\lambda_1|00\rangle + \lambda_2|01\rangle + \lambda_3|10\rangle + \lambda_4|11\rangle$

Entangled state if
$\lambda_1\lambda_4 \neq \lambda_2\lambda_3$

Name	Symbol	Matrix
Not	x1 ⊕ o1	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
CNOT	x2 ● o2, x1 ⊕ o1	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$
Controlled –V	x2 ● o2, x1 V o1	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & .5(1+i) & .5(1-i) \\ 0 & 0 & .5(1-i) & .5(1+i) \end{pmatrix}$
Controlled –V+	x2 ● o2, x1 V+ o1	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & .5(1-i) & .5(1+i) \\ 0 & 0 & .5(1+i) & .5(1-i) \end{pmatrix}$

Table 2. Quantum gates and their corresponding matrices

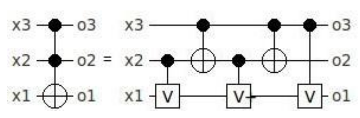


Figure 3. Quantum implementation of reversible Toffoli gate

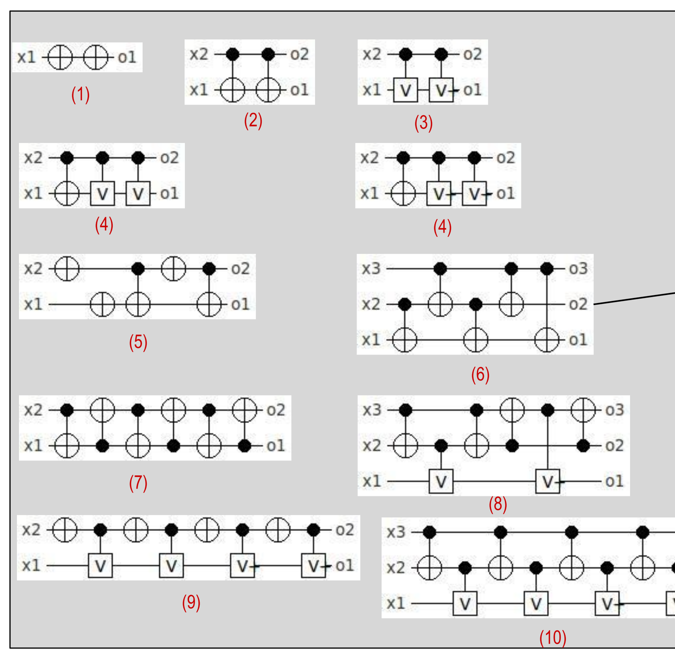


Figure 4. Sets of existing quantum templates.

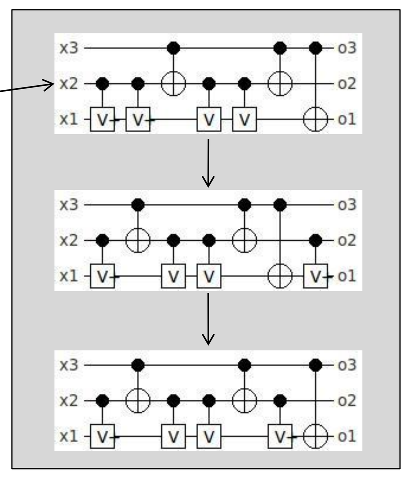
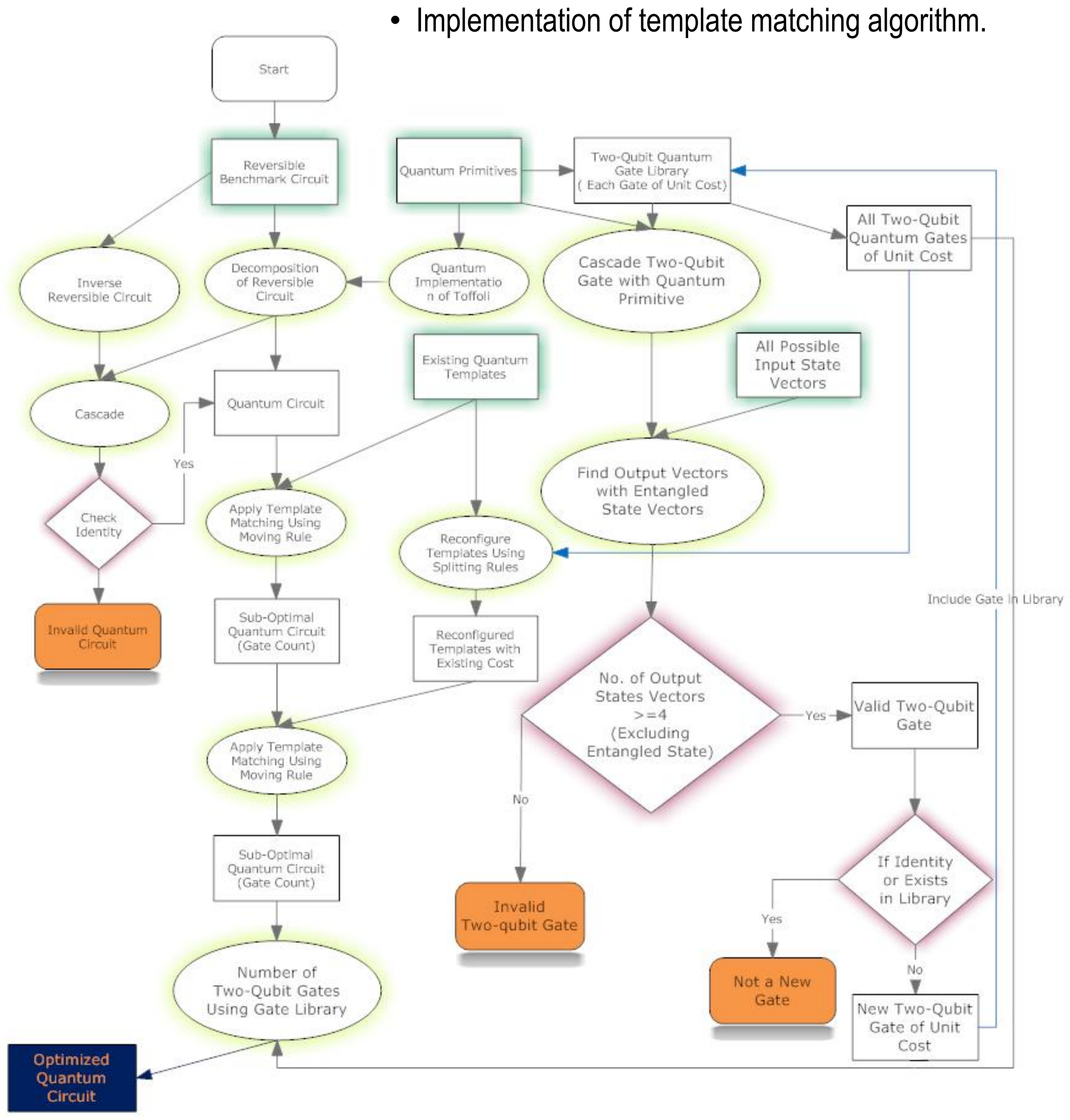


Figure 5. Reconfigured template.

Tools and Methods

- MATLAB programming language.
- RevKit-1.0 tools for quantum circuits simulation.
- RevLib: online resource for benchmark circuits.
- C/C++ programming language.

- Implementation of two-qubit quantum gate library.
- Decomposition of MCT-circuits into quantum circuits.
- Implementation of template matching algorithm.



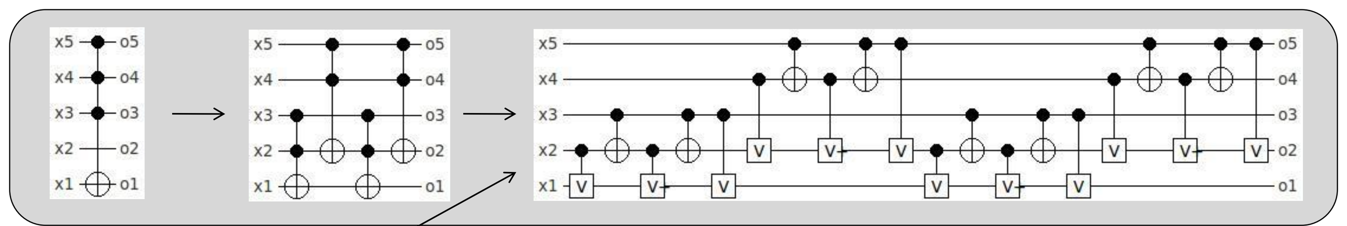


Figure 6. Decomposition of reversible into quantum circuit of quantum cost 20 .

- Decomposition a recursive transformation of higher order Toffoli into Toffoli-3 and then its quantum implementation.
- Quantum implementation of Toffoli -4 quantum costs 20.

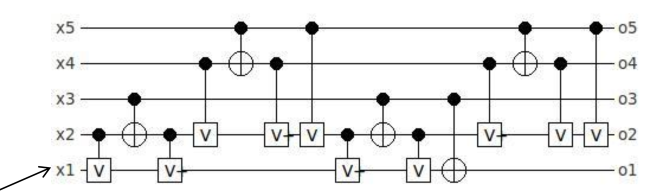


Figure 7. Optimized quantum circuit using existing templates, Quantum cost 15.

- Template (Identity of quantum circuit) matching a technique to reduce the number of quantum primitives of quantum circuits using moving rule.
- Significant reduction (not optimum) of number of quantum primitives (also called cost) using existing templates can be noticed in the figure 7.

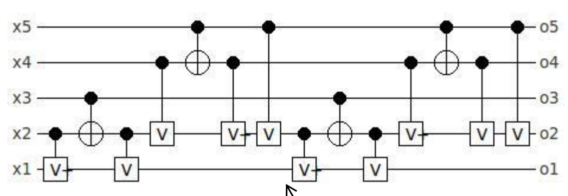


Figure 8. Optimized quantum circuit using reconfigurable template, Quantum cost 14.

- Reconfigured template(using splitting rules) does not exceed the number of single operations(cost) in template.
- Optimize the quantum cost of circuits as shown in figure 8 which is the optimum circuit of cost 14.

- Two-qubit operations in gates sequence a unit cost in quantum computation.
- Quantum entanglement restricts invalid two-qubit sequence of quantum primitives.
- The number of valid two-qubit gates is finite (783) due to the exotic phenomenon of quantum entanglement.
- The sequence of gates indicating by a circle a two-qubit quantum gate of single operation as well as unit cost , total 14 two-qubit operations.

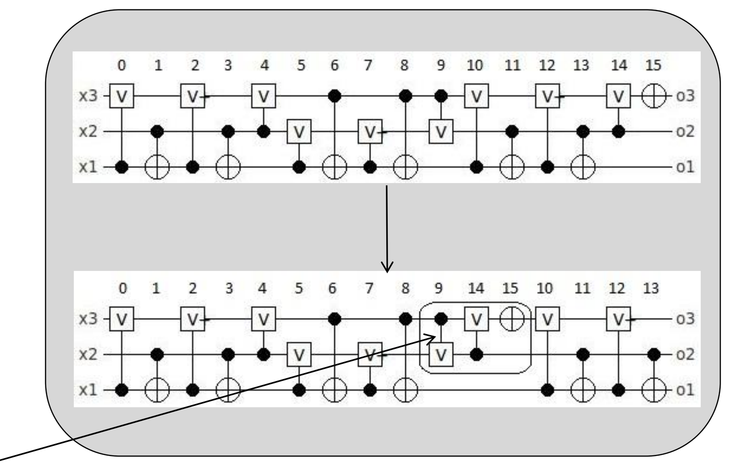


Figure 9. Total number of two-qubit operations 14.

Experimental Results

- Reversible benchmark MCT-circuits provided in RevLib pages as shown in the first column of table 4.
- The total costs reduction in percentage after applying existing templates, reconfigured template (only one) as well as two-qubit operations in last column.
- Significant costs reduction up to 64% by applying template matching algorithm as well as two-qubit operations without using optimum quantum implementation of high ordered Toffoli.

Benchmark	#Line	#Qgatet (T- Decomposition)	#Qgate (Existing Templates)	#Qgate (Recon. Template)	#Two-Qubit Qgate (Quantum Cost)	QC Red(%)
0410184_169	14	90	78	78	70	22.22
3_17_13	3	14	13	13	9	35.71
4_49_16	5	74	61	60	56	24.32
4_49_17	4	32	31	31	27	15.63
4gt10-v1_81	5	48	44	44	42	12.5
4gt11_82	5	16	14	14	9	43.75
4gt4-v0_79	6	77	71	71	67	12.99
4gt4-v1_74	6	85	75	73	72	15.29
4gt5_76	5	36	30	30	26	27.78
4gt5_77	5	42	36	36	36	14.29
4mod5-v0_18	5	25	14	11	9	64
4mod5-v0_19	5	13	10	10	9	30.77
4mod5-v0_20	5	9	7	7	6	33.33
4mod5-v1_22	5	9	6	6	6	33.33
4mod5-v1_23	5	24	17	17	17	29.17
alu-v2_31	5	143	122	122	118	17.48
alu-v2_33	5	15	15	15	11	20
miller_12	3	8	8	8	6	25
alu-v2_32	5	53	47	47	45	15.09
alu-v4_36	5	38	36	36	33	13.16
cnt3-5_179	16	65	54	54	54	16.92
ham15_109	15	248	228	228	214	13.71
ham3_102	3	9	7	7	5	44.44
ham7_104	7	111	98	98	94	15.32
hwb4_49	5	79	62	60	56	29.11
mini-alu_167	5	90	70	66	64	28.89
mod10_171	5	79	59	57	54	31.65
mod10_176	5	57	43	43	43	24.56
mod5d2_70	5	16	13	12	10	37.5
rd32-v1_68	4	13	7	7	6	53.85
rd53_135	7	98	79	77	77	21.43
rd53_137	7	86	72	70	68	20.93
rd53_138	8	44	31	31	31	29.55
rd73_140	10	76	55	55	55	27.63
rd84_142	15	112	86	86	86	23.21
graycode6_48	6	5	5	5	5	0

Table 4. Quantum cost reduction using template matching and two-qubit gate library.

No costs reduction by template matching

Total Costs reduction 0%

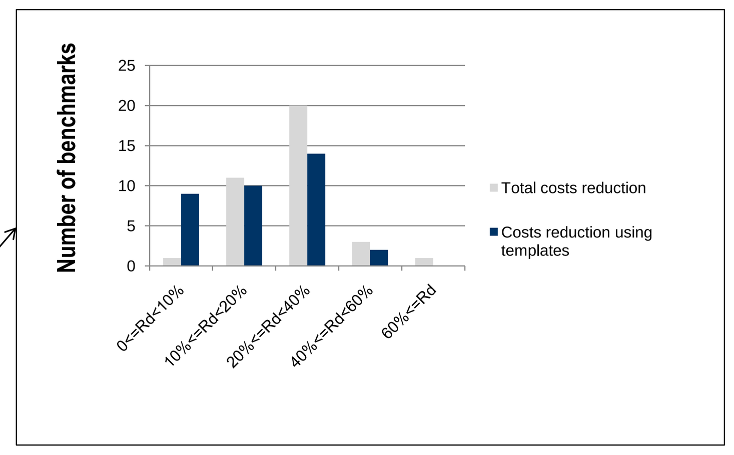


Figure 10. Analysis of costs reduction.

- Total 36 benchmarks.
- Average 20% to 40% costs reduction of 20 benchmarks.
- Significant cost reduction 64% of 1 benchmark.

Conclusion

- Template matching takes full advantage of costs reduction in quantum circuits.
- The best solution can be achieved by reconfigured templates.
- Uniform method of costs optimization in quantum circuits helps to reduce the complexity of finding optimal reversible circuits.

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