REASONING ABOUT PHYSICAL SYSTEMS IN ARTIFICIAL INTELLIGENCE

by

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This report, **Reasoning About Physical Systems in Artificial Intelligence,** consists of several files

1. *master.doc* is a file to print the whole documents.
2. *list_not.doc* is a list of notations used in this report.
3. *list_fig.doc* is a list of figures.
4. *acknow.doc* is acknowledgments of the author.
5. *toc.doc* is a list of table of contents.
6. *intro.doc* is an introduction chapter.
7. *dynamics.doc* is a discussion on dynamics systems.
8. *rigid.doc* is a discussion on solid systems.
9. *liquid.doc* is a discussion on liquid systems.
10. *nuclear.doc* is discussion on nuclear reactions.
11. *qsim_env.doc* is discussions on Qualitative Simulation and Envision physics reasoning systems.
12. *dy-analy.doc* is a discussion on reasoning about physical systems using dimensional analysis.
13. *qpt.doc* is a discussion on a physics reasoning system, Qualitative Process Theory.
14. *hpt.doc* is a discussion on a physics reasoning system, Hybrid Phenomena Theory.
15. *property.doc* is a comparison of physics reasoning systems.
16. *conclusi.doc* is conclusions of the discussions along with a brief discussion on commercial tool G2, and also open problems.
17. *biblio.doc* is a list of bibliographies.

These files are written using the Micrsoft Word version 2.0c and they are stored in Vermis under /home/grads/hartati/cstr directory. They are also stored in CORPUSC under /CORPUSC/HARTATI/CSTR directory.

List of notation

Notation	Description
A	: cross sectional area
A_C	: acceleration of the center of mass
A_m	: magnitude of amount
A_s	: sign of amount
a	: acceleration of a body
(a,b)	: (incident,product particles)
a_1	: bottom area associated with PAN
B	: width of open channel at water surface
B_t	: transverse width
$B_{\alpha v}$	: a quantity value of T=100 in QPT
C_d	: drag coefficient
C_f	: friction coefficient
C_p	: pressure drag coefficient
c_y	: specific heat constant
c_3	: heat capacity associated with HOT-PLATE
c_4	: heat capacity associated with CONTAINER-WITH-LIQUID-1
c_6	: heat capacity associated with PAN
D	: diameter of a pipe
D_m	: magnitude of a derivative
D_s	: sign of a derivative
E_k	: kinetic energy
E_p	: potential energy
$E_{p,i,j}$	: internal potential energy of a system.
F	: force acting on a body
$F_{B/F}$	: forces exerted by a bend on the fluid
F_f	: friction drag
F_p	: pressure drag
$F_{\alpha v}$	: a quantity value of T<0 in QPT
$(F_{R/F})_x$	: force exerted by a reducer on the fluid in the x direction
F_v	: force exerted by a vane on the fluid
G	: specific weight of fluid
g	: gravitational force
h	: maximum height
h_L	: fluid-friction energy loss per unit weight of fluid
h_M	: energy put into the flow by machine per unit weight of flowing fluid

I	: moment of inertia of a body relative to the axis of rotation Z
I_w	: internal energy per unit of weight
i	: internal energy per unit of mass
K_r	: radius of gyration of a body
K	: spring constant
k	: vector of state variables
k_5	: heat transfer per degree Kelvin associated with HEAT-BRIDGE-5
k_7	: heat transfer per degree Kelvin associated with HEAT-BRIDGE-3
k_9	: : heat transfer per degree Kelvin associated with HEAT-BRIDGE-1
L	: angular momentum of a body
L_S	: length of the surface parallel to the flow
l	: level of liquid
M	: total mass of a body
$\mathrm{M}_{\alpha v}$	: a quantity value of T=0 in QPT
m	: mass of a body
m_2	: mass associated with PAN
$\mathrm{N}_{\alpha v}$	: a quantity value of T=100 in QPT
N_R	: Reynold number
N_0, N_t	: the number of radioactive nuclei at time 0, at time t
p_m, P_m	: momentum of a particle, total momentum of particles
p	: pressure of fluid
Q	: volume flow rate of fluid
q	: flow rate
R	: radius of circular motion
$\boldsymbol{R}_F$	: resultant of several forces acting on a body
R_C	: center of mass of a system of particles
r	: position vector of a point in a rotating body
r_a	: air resistance
T	: temperature
S	: surface area
s	: vector of dependent variables
$\mathrm{t}_{\frac{1}{2}}$	: half life of a nuclear particle
U	: Uranium
U_p	: proper energy
u	: vector of independent or control variables
$\boldsymbol{u}_a$	: angular acceleration of rigid body
$\boldsymbol{u}_f$	: local velocity of fluid
$\boldsymbol{u}_N$	: unit vector normal to a curve
$\boldsymbol{u}_T$	: unit vector tangent to a curve
u_1	: mass associated with WATER
u_2	: power consumption associated with HOT-PLATED
V	: volume of a body
$\mathrm{V}_{\alpha v}$	: a quantity value of 100 < T in QPT
v	: velocity of a moving particle

v_f	: mean velocity of the fluid
vol	: unit volume
W	: weight of a particle
W_{ext}	: external work
X	: target nuclei
x	: repeating variable
x_2	: temperature associated with CONTAINER-WITH-LIQUID-1
x_3	: mass associated with WATER
x_4	: temperature associated with PAN
x_5	: temperature associated with WATER
y	: performance variable
Y	: product nuclei
z	: elevation of fluid
z_1	: bottom pressure associated with HEAT-FLOW-6
z_2	: liquid level associated with CONTAINER-WITH-LIQUID-1
z_3	: mass associated with CONTAINER-WITH-LIQUID-1
z_4	: heat flow from source to destination associated with HEAT-FLOW-6
z_5	: heat flow from source to destination associated with HEAT-FLOW-2
α	: an alpha particle, a stable nucleus of ${}_2^4He$
α_d	: an exponent of a basis variable
α_f	: a factor of a kinetic energy
$\propto$	: proportionality sign
β	: a beta particle
γ	: gamma ray
γ_f	: mass of fluid per unit volume
λ	: disintegration constant
μ	: viscosity coefficient
π	: dimensionless product
ϑ	: kinematic viscosity
ρ	: density of a body or a fluid
ρ_1	: density associated with WATER
ρ_c	: radius of a curvature
ω	: angular velocity of a particle moving circularity

List of figures

Acknowledgments

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Chapter 1

Introduction

The problem of modeling the behavior of physical systems had been studied for many years before the advent of qualitative physics. There are two components here: modeling processes and simulating processes. Qualitative reasoning focuses on the simulation process. The goal of a simulation process is the accurate prediction of the behavior of a physical system, thus it is important that the model system actually behave like the physical system.

A modeling system consists of two components [Zippel, 92]: (1) the combination of the physical laws and the scene description to produce a set of equations that describe the evolution in time of the system, and (2) the conversion of the state equation into a different form that is easier to manage and more readily answers the user's questions.

Reasoning about physical systems involves both modeling processes and simulation processes, each of which consists of some components mentioned above. Therefore, this report consists of two major parts. The first part is a review of physical systems along with the laws of physics that apply to these system. The second part gives an overview of the kind of representations and reasoning used in commonsense thinking about the physical world. Some discussions include Envision, QSIM, Dimensional Analysis, Qualitative Process Theory (QPT) and Hybrid Phenomena Theory (HPT) which are current theories of reasoning about physical processes.

This report will help readers understand the difficulties inherent in computer reasoning about physical systems. It is also intended to partially fulfill the requirement of a reading course in reasoning about physical systems, CS6341.

1.1 Physical Systems

Physics deals with laws governing physical systems. Many physical systems exist, for example:

- Dynamics, deal with causes of motion, interactions between particles.
- Rigid bodies, deal with laws governing solid systems.
- Fluid systems, deal with laws governing liquid system.
- Nuclear physical systems, deal with nuclear processes and their interaction.
- Kinematics, deal with geometric descriptions of physical systems.
- Electrical systems deal with laws governing electrical systems.
- Planet systems, deal with planet motions and forces in nature.

The first four of these physical systems are reviewed in this report along with the laws governing them and their interactions.

1.2 Reasoning about physical systems

Both reasoning about physical systems and physics attempt to characterize the physical world by formalizing knowledge about it in some language. Physics which elaborating new theory, does not bother to codify the knowledge. To develop machines capable of reasoning about physical systems, the knowledge must be codified in some degree.

Any computer program can be built to solve physics problems by solving some equations constituting the problems using a particular set of data. These solutions do not give causal explanations when the implicit assumptions under which it was written are violated. They are not capable of showing how the variables are related one another, for example a change in one variable can cause changes in others.

Reasoning about physical systems aims to expose the underlying institutions and make them sufficiently explicit, so that they can be directly reasoned with and about. It can give causal explanations of a change in one variable with respects to others. It can reason about when a process starts and stops. It gives causal explanations when the implicit assumptions under which it was written are violated. It can reason about the behavior of a system consisting of subsystems. The underlying and explicit knowledge enables us to develop machines capable of reasoning about physical systems.

Artificial Intelligence research on reasoning about physical system is important for several reasons [Weld, 90]

- allows robots to predict the effect of their actions on dynamic world.
- leads to powerful tools for automated design, diagnosis, and monitoring of complex systems such as electronic circuits or chemical plants.
- enables the construction of algorithms that generate causal descriptions explaining how physical systems work; these algorithms could be used in intelligence tutoring systems.

Since physics only deal with laws governing the physical systems and normally the intelligence of a human being is related to the ability to reason, hence reasoning about physical system is considered as part of Artificial Intelligence.

Chapter 2

Physical Systems

2.1 Dynamics

2.1.1 Forces and motion

An intuitive notion of force is derived from our everyday experience, such as the force needed to push or pull a given weight, the force exerted by certain tools, etc. Force is a vector quantity having magnitude and direction. Combining forces follows the rules of vector algebra. There two kind of forces - an external force and an internal force. The external force is a force exerted on the body by its surroundings, while an internal forces is a force exerted on one part of the body by another part of the body. If the forces are concurrent (i.e., if they are all applied at the same point), their resultant is their vector sum. Therefore, the resultant $\boldsymbol{R}_F$ of several concurrent forces $\boldsymbol{F}_1, \boldsymbol{F}_2$, and $\boldsymbol{F}_3, \ldots$ is

$$\boldsymbol{R}_F = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3 + \ldots. = \sum \boldsymbol{F}_i$$

If the forces are coplanar, say in the XY-plane, we have $\boldsymbol{R}_F = \boldsymbol{u}_x R_{Fx} + \boldsymbol{u}_y R_{Fy}$, where

$$R_{Fx} = \sum F_{ix} = F_i \cos\alpha$$

$$R_{Fy} = \sum F_{iy} = F_i \sin\alpha .$$

The magnitude of R_F is $\sqrt{R_{Fx^2} + R_{Fy^2}}$ and its direction is given by the angle α such that

$$\tan\alpha = \frac{R_y}{R_x} .$$

An object is in motion relative to another when its position, measured relative to the second body, is changing with time. On the other hand if this relative position does not change with time, the object is at relative rest. Both rest and motion are relative concepts, they are dependent on the condition of the object relative to the body that serves as reference. When a train passes the station we say that the train moves relative to the station. But a passenger on the train will say that the station is in motion relative to the train, moving in opposite direction. Another example is shown in Fig. 1. In the figure we indicate two observers O and O' and particle P. The frames of reference XYZ

and *X' Y' Z'* are respectively used by *O* and *O'*. If *O* and *O'* are at rest relative to each other, they will observe the same motion of P. However if O and O' are in relative motion, they will observe the motion of P differently.

The following sections will discuss briefly about laws which govern the motion of particles.

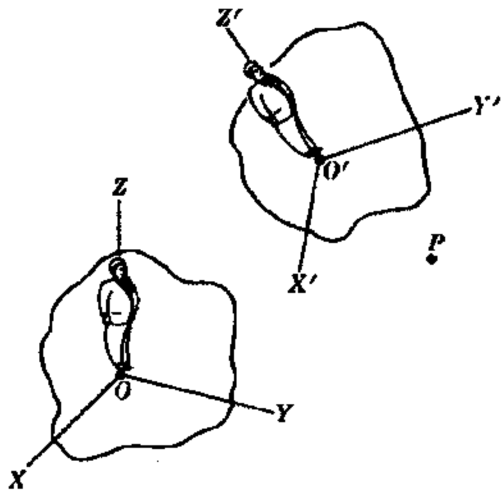


Figure 1. Two different observers study the motion of p.

2.1.2 Newton's laws

Newton's first law:

A body at rest will remain at rest, and a body in uniform motion, i.e., moving with constant velocity, will remain in uniform motion unless a net force acts upon the body.

A body is in equilibrium only if it has constant velocity, with no net force acting on it. This equilibrium includes the state of rest. If the forces are concurrent, applied at the same point, their resultant is their vector sum.

$$\boldsymbol{R} = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3 + \ldots = \sum \boldsymbol{F}_i$$

Thus, a body will be in equilibrium if and only if

$$\sum \boldsymbol{F}_i = 0$$

The second law of Newton gives a quantitative connection between an unbalanced force and the acceleration it produces. Newton's second law states that the acceleration of a body is directly proportional to the net (unbalanced, resultant) force exerted on it by the surroundings. This statement can be written as

$$a \propto \frac{\sum F}{m}$$

where the symbol $\propto$ means is "directly proportional to". Since the resulting force is a vector sum, the direction of $\boldsymbol{a}$ is the same as that of the resultant force, and the magnitude of $\boldsymbol{a}$ is proportional to the net force. m is the mass of a body. The greater the mass of a body, the smaller its acceleration $\boldsymbol{a}$ under the action of a given net force. Newton's second law of motion can be expressed as

$$\sum \boldsymbol{F} = m\boldsymbol{a}$$

which states that the forces produces an acceleration on a mass and in fact is proportional to the mass time acceleration, does not mean that the mass times acceleration defines force. We note that Newton's first law is included as a special case: if $\sum \boldsymbol{F} = 0$, then $\boldsymbol{a} = 0$, and therefore the body is in equilibrium.

Newton's first two laws concern the motion of a single body. The first law of motion tells us whether a force acts on a body, and the second law produces the acceleration of the body if there is an unbalanced force on it. Newton's third law concerns the interactions of pairs of bodies. It states that if a body A exerts a force on body B, then B exerts a force on A of equal magnitude, opposite in direction, and along the same line. It is common to hear the third law expressed as "action equal to reaction". The example can be seen in Fig. 2. A body A exerts an attractive force $\boldsymbol{F_1}$ on body B; by the third law, a body B exerts F_2 on A, such that $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ are equal in magnitude, opposite in direction, and along the line joining A and B.

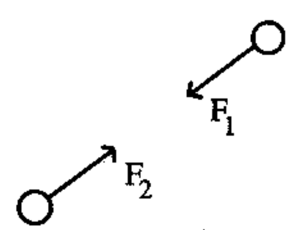


Figure 2. Newton's third law.

2.1.3 Systems of Particles

Consider a body consisting of two particles as shown in Fig 3. Suppose that external force $\boldsymbol{F_1}$ acts on particle 1, particle 2 exerts a force on particle 1, denoted by $\boldsymbol{F_{21}}$. Similarly, particle 1 exerts a force on particle 2, denoted by $\boldsymbol{F_{12}}$. Applying Newton's second law to the motion of particle 1 :

$$F_1 + F_{21} = m_1 a_1 \tag{1}$$

Similarly, for particle 2 :

$$F_2 + F_{21} = m_2 a_2 \tag{2}$$

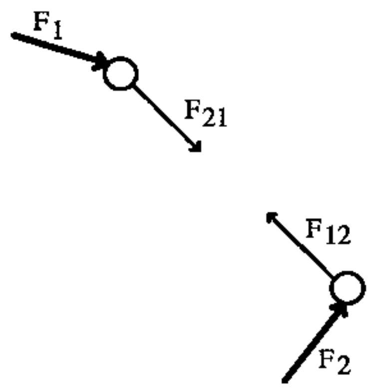


Figure 3. A system composed of two particles.

According to Newton's third law, the force $\boldsymbol{F_{21}}$ exerted on particle 1 by particle 2 and the force $\boldsymbol{F_{12}}$ exerted on particle 2 by particle 1 are equal in magnitude and opposite in direction. Hence, summing these equations, we have

$$\boldsymbol{F}_1 + \boldsymbol{F}_2 = m_1 \boldsymbol{a}_1 + m_2 \boldsymbol{a}_2 \tag{3}$$

The two internal forces have canceled in the equation. Equation (3) shows that external forces on a system are the sum of the external forces on each of its component particles. In general, external forces on a system is expressed as

$$\sum \boldsymbol{F}_i = m_i \boldsymbol{a}_i \tag{4}$$

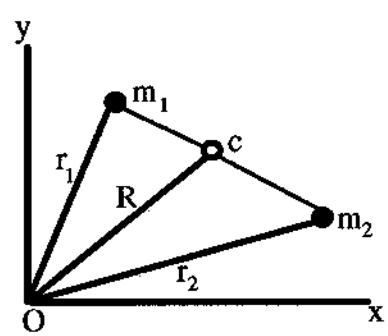


Figure 4. Interaction of two particles.

Consider two particles lying in the xy plane as shown in Fig. 4. The position vectors $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$ show concisely just where m_1 and m_2 are located with reference to origin O of an arbitrary coordinate system. By introducing C as the center of mass of the system of particles, its location is denoted by the position vector

$$\boldsymbol{R_C} = \frac{(m_1\boldsymbol{r_1} + m_2\boldsymbol{r_2})}{M_t} \tag{5}$$

where $M_t = m_1 + m_2$. In general, if a system is composed of any number n of particles, then the position of the center of mass is denoted by

$$R_C = \frac{1}{M_t}\sum_{i=1}^{n} m_i r_i$$

where $M_t = \sum_{i=1}^{n} m_i$. The center of mass is a point at which we may assume all the mass of a body to be concentrated in order to determine the translational (linear) motion of the body. According to Newton's second law, the external force of Eq.(4) should produce an acceleration on the total mass of the system. Therefore the equation becomes

$$\sum \boldsymbol{F} = M_t A_C \tag{6}$$

where $M_t = \sum m_i$ is the total mass of the system, and A_C is the acceleration of the center of mass of the system. When a set of forces whose resultant is $\sum \boldsymbol{F}$ acts on an extended body whose total mass is M_t, the center of mass of the body will undergo a linear acceleration A_C, in the direction of $\sum \boldsymbol{F}$ and with magnitude

$$\frac{\sum \boldsymbol{F}}{M_t}$$

For example, the motion of chain thrown into the air is depicted in Fig. 5. The center of mass of the chain moves as if it were a particle of mass equal to that of the chain and subject to a force equal to the weight of the chain, and therefore the center of mass describes a parabolic path.

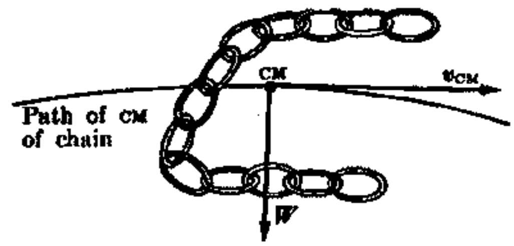


Figure 5. A chain thrown into the air [Alonso77].

2.1.4 Centrifugal, centripetal and corriolis forces

To discuss centrifugal forces, we first have to know about curvilinear motion. Let us consider an example of a body moving in a circular path. For simplicity, assume that the body is a particle of mass *m* moving in a curvilinear path with the radius of curvature of the path as ρ_c,, as shown in Fig. 6. To produce curvilinear motion, the resultant force must be at an angle with respect to the velocity, therefore the acceleration has a component perpendicular to the velocity which will account for the change in the direction of motion. The relation of all the vectors are shown in the figure.

From Newton's second law, we have the component of the force tangent to the path, or tangential force is

$$F_T = ma_T \text{ or } F_T = m\frac{dv}{dt} \qquad (7)$$

and the component of force perpendicular to the path, or the normal or centripetal force is

$$F_N = ma_N \text{ or } F_N = m\omega^2 R \qquad (8)$$

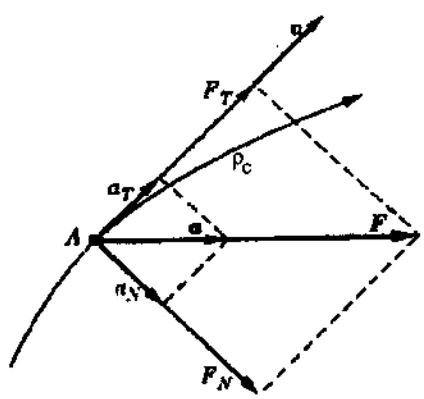


Figure 6. Relationship between the tangential and
normal components of the force and the acceleration in curvilinear motion.

This centripetal force is responsible for the change in direction of the velocity, while the tangential force is responsible to the change in the magnitude. The centripetal force always points toward the center of curvature of the trajectory.

In the case of circular motion with the radius of the circle being R, the particle of mass m moves at a constant angular velocity ω. The velocity v is denoted in the relation [Bornstein68] $v = \omega \ R$, so that the force is also

$$F_N = m\omega^2 R$$

In the case of uniform circular motion the only acceleration is

$$\boldsymbol{a}_N = \frac{d\boldsymbol{v}}{dt} = \boldsymbol{\omega} \times \boldsymbol{v}$$

Therefore $\boldsymbol{F} = m\,\boldsymbol{a} = m\,\boldsymbol{\omega} \times \boldsymbol{v} = \boldsymbol{\omega} \times (m\boldsymbol{v})$. Since $p_m = mv$ is the linear momentum of a particle, we have

$$\boldsymbol{F} = \boldsymbol{\omega} \times \boldsymbol{p_m}$$

The rectangular components of F are

$$F_x = ma_x \text{ and } F_y = ma_y$$

In general, when we include the case in which the mass is variable, thus

$$\boldsymbol{F} = \frac{d(m\boldsymbol{v})}{dt} = \frac{d\boldsymbol{p_m}}{dt}$$

$\boldsymbol{p_m}$ is parallel to the velocity vector, and tangential to the path. Thus $\boldsymbol{p_m}$ can be written $\boldsymbol{p_m} = \boldsymbol{u_T}\, p_m$, where u_T is the unit vector tangent to the curve, therefore we have

$$\boldsymbol{F} = \frac{d\boldsymbol{p_m}}{dt} = \frac{d}{dt}(\boldsymbol{u}_T\, p_m) = \boldsymbol{u_T}\frac{dp_m}{dt} + p_m \frac{d\boldsymbol{u_T}}{dt} \quad (9)$$

If ρ_c is defined as the radius of curvature of the path, as shown in Fig. 7,

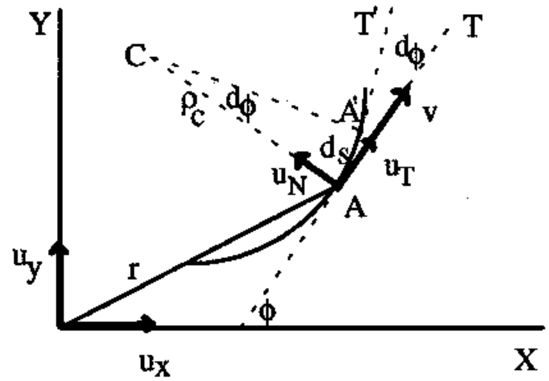


Figure 7. Tangential and normal velocity

then
$$\frac{d\boldsymbol{u}_T}{dt} = \boldsymbol{u}_N \frac{v}{\rho_c}$$

where $\boldsymbol{u}_N$ is the unit vector normal to the curve. Substituting this equation into Eq.(9), we have

$$\boldsymbol{F} = \frac{d\boldsymbol{p}_m}{dt} = \boldsymbol{u}_T \frac{dp_m}{dt} + p_m \frac{d\boldsymbol{u}_T}{dt} = \boldsymbol{u}_T \frac{dp_m}{dt} + \boldsymbol{u}_N \frac{vp_m}{\rho_c}$$

Therefore, instead of Eqs. (7) and (8) we have

$$F_T = \frac{dp_m}{dt} \text{ and } F_N = \frac{p_m v}{\rho_c}$$

In the case of a solid body rotating with constant angular velocity ω about a fixed axis, with the origin lying on the axis of rotation, the velocity of a point of the body at position r is given by simple formula

$$\boldsymbol{v} = \omega \times \boldsymbol{r}$$

This relation can be seen in Fig. 8. The direction of v is perpendicular to the plain containing ω and $\boldsymbol{r}$. The term r is the position vector of a point of the rotating body. The velocity with respect to an inertial observer is expressed as [Kibble66]

$$\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt} = \dot{\boldsymbol{r}} + \omega \times \boldsymbol{r} \tag{10}$$

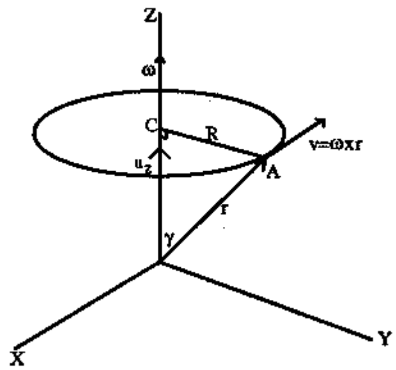


Figure 8. Vector relation between angular velocity ω, linear velocity *v* and position vector *r* of a solid body in circular motion.

It states that the velocity with respect to an inertial observer is the sum of the velocity $\dot{\boldsymbol{r}}$ with respect to the rotating frame and the velocity $\boldsymbol{\omega} \times \boldsymbol{r}$ of a particle at *r* rotating with the body. The rate of change of *v* with respect to an inertial observer is

$$\frac{d\boldsymbol{v}}{dt} = \frac{d^2\boldsymbol{r}}{dt^2} = \dot{\boldsymbol{v}} + \boldsymbol{\omega} \times \boldsymbol{v} \tag{11}$$

From Eq.(10) we have

$$\dot{\boldsymbol{v}} = \ddot{\boldsymbol{r}} + \boldsymbol{\omega} \times \boldsymbol{r} \tag{12}$$

and

$$\boldsymbol{\omega} \times \boldsymbol{v} = \boldsymbol{\omega} \times \dot{\boldsymbol{r}} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) \tag{13}$$

Substituting equations Eq.(11) and Eq.(13) to Eq.(10) results in

$$\boldsymbol{a} = \frac{d^2 r}{dt^2} = \ddot{\boldsymbol{r}} + 2\boldsymbol{\omega} \times \dot{\boldsymbol{r}} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) \tag{14}$$

In the case of a particle moving near the surface of the earth, Eq.(13) is necessary to obtain the equation of motion. For a particle moving under gravity, and under an additional mechanical force ***F***, the equation of motion is

$$m\frac{d^2\boldsymbol{r}}{dt} = mg + \boldsymbol{F}$$

Substituting Eq.(13) into this equation, we have

$$m\ddot{\boldsymbol{r}} = mg + \boldsymbol{F} - 2m\boldsymbol{\omega} \times \dot{\boldsymbol{r}} - m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r})$$

The third and the fourth terms on the right are apparent forces, which arise because of the noninertial nature of the reference frame [Kibble66]. The third term is known as the Coriolis force, while the last term is known as centrifugal force.

2.1.5 Principle of energy conservation and momentum

One of the most important laws in physics is the conservation theorem. The principle of conservation of energy states that the total energy of the system remains unchanged during any process. This principle is valid for both a system of particles and a particle. Let us consider a particle. The total energy of the particle is the sum of its kinetic energy and its potential energy, or

$$E = E_k + E_p = \frac{1}{2}mv^2 + E_p(x,y,z)$$

According to the law of conservation of energy, the energy of the particle is conserved, and we may write

$$E = E_k + E_p = constant$$

For example, in the case of a falling body the potential energy due to gravity is $E_p = mgy$, where m,g and y are the mass of the body, the gravity and the elevation of the body respectively. The conservation of energy gives

$$E = \frac{1}{2}mv^2 + mgy = constant$$

If initially the particle is at height y_0 and its velocity is zero, the total energy is mgy_0, and we have

$$\frac{1}{2}mv^2 + mgy = mgy_0$$

thus $v^2 = 2g(y_0 - y) = 2gh$, where $h = y_0 - y$, that is the height through which it has fallen.

In the case of a system of particles, the proper energy of the system is equal to the sum of the kinetic energies of the particles relative to the inertial observer and their internal potential energy.

$$\begin{aligned} U &= E_k + E_{p,\text{int}} \\ &= \sum_{\text{all particles}} \frac{1}{2}m_i v_i^2 + \sum_{\text{all pairs}} E_{p,i,j} \end{aligned}$$

where E_k is the kinetic energy of the system of particles, and $E_{p,int}$ is internal potential energy of the system. Each pair of particles in the system has the internal potential energy

$$E_p = mg(y_n - y_{n-1}) \tag{15}$$

where y_n is the elevation of the particle *n*. Since each pair of particles has their own internal potential energy of Eq(15), then the total internal energy of the system is

$$E_{p,int} = \sum_{all\ pairs} E_{p,i,j} = E_{p,1,2} + E_{p,1,3} + \ldots + E_{p,2,3} + \ldots + E_{p,n-1,n}$$

The law of conservation of energy for a system of particles states that the change in proper energy of a system of particles is equal to the work done on the system by external forces. This statement is expressed as

$$U - U_0 = W_{ext} \tag{16}$$

where U_0 is the proper energy of the system at time t_0. In an isolated system, there are no external forces acting on the system, so that $W_{ext} = 0$. Then $U - U_0 = 0$. That is, the proper energy of an isolated system of particles remains constant. If the kinetic energy of an isolated system increases, its internal potential energy will decrease by the same amount so their sum remains the same. If there are external forces acting on a system which is also conservative, then $W_{ext} = E_{p,ext,0} - E_{p,ext}$, where $E_{ext,0}$ and $E_{p,ext}$ are the values of potential energy associated with the external forces at initial and final states. Eq.(16) becomes

$$U - U_0 = E_{p,ext,0} - E_{p,ext}$$

$$E = U + E_{p,ext} = E_k + E_{p,int} + E_{p,ext}$$

where E is the total energy of the system. The value of E will be constant during motion of the system under internal and external conservative forces.

The linear momentum of a particle is defined as the product of its mass and its velocity. It is expressed as

$$\boldsymbol{p}_m = m\boldsymbol{v}$$

It is a vector quantity, so that it has the same direction as the velocity. The principle of conservation of momentum says that the total momentum of a system composed of two particles which are subject to their mutual interaction remains constant. Let us consider two particles -1and 2. At particular time *t*, particle 1 is at *A* with velocity $\boldsymbol{v}_1$ and particle 2 is at B with velocity $\boldsymbol{v}_2$. Later, at a time t', particle 1 is at A' and particle 2 is at B'. This situation is shown in Fig. 9. m_1 and m_2 are the masses of particles 1 and 2 respectively. The total momentum at time *t* is

$$\boldsymbol{P}_m = \boldsymbol{p}_{m1} + \boldsymbol{p}_{m2} = m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2 \tag{17}$$

The total momentum at time t' is

$$P_m' = p_{m1}' + p_{m2}' = m_1 v_1' + m_2 v_2' \tag{18}$$

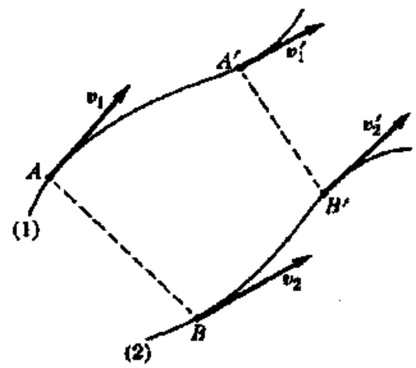


Figure 9. Interaction between two particles.

According to the law of conservation of momentum, Eq.(17) and Eq.(18) must have the same value.

$$p_{m1} + p_{m2} = p_{m1}' + p_{m2}' \quad \text{or}$$

$$p_{m1}' - p_{m1} = p_{m2} - p_{m2}' = -(p_{m2}' - p_{m2})$$

Calling $p' - p = \Delta p$ the change in momentum between t and t', we can write

$$\Delta p_{m1} = -\Delta p_{m2}$$

This indicates that the change in momentum of one particle in a certain time interval is equal and opposite to the change in momentum of the other during the same time interval. If a system is composed many of particles, the conservation of momentum can be expressed as

$$P_m = \sum p_{mi} = p_{m1} + p_{m2} + p_{m3} \ldots = constant$$

2.2 Rigid bodies

A rigid body is a system which is composed of many particles, and the distances between all its component particles remain fixed under the application of a force or torque. During its motion a rigid body conserves its shape. There are two types of motion of a rigid body - translation and rotation. In translation, all particles describe a parallel path so that the lines joining any two points in the body always remain parallel to its initial position. In rotation, all particles describe circular paths around a line.

2.2.1 Angular Momentum

Consider a rigid body rotating around the Z axis with angular velocity ω, which is shown in Fig.10. Each of its particles describe a circular orbit with its center on the Z axis. For example, particle A_i describes a circle of radius $R_i = A_i B_i$ with a velocity $\boldsymbol{v}_i = \omega \times \boldsymbol{r}_i$, where r_i is the position vector relative to the origin O. The magnitude of velocity is $\boldsymbol{v}_i = \omega \times \boldsymbol{r}_i \sin\theta = \omega R_i$. The angular momentum of particle A_i relative to the origin O is

$$\boldsymbol{L}_i = m_i \boldsymbol{r}_i \times \boldsymbol{v}_i$$

where $m_i, \boldsymbol{r}_i$ and $\boldsymbol{v}_i$ represent the mass, position vector relative to the origin and the velocity of particle i, respectively. Its direction is perpendicular to the plane determined by the vector $\boldsymbol{r}_i$ and $\boldsymbol{v}_i$ and lies in the plane determined by $\boldsymbol{r}_i$ and the Z-axis. It therefore makes an angle $\frac{\pi}{2} - \theta_i$ with the axis of rotation Z. The magnitude of $\boldsymbol{L}_i$ is $m_i \boldsymbol{r}_i \boldsymbol{v}_i$ and its component parallel to the Z-axis is

$$\begin{aligned} L_{iz} &= (m_i r_i v_i)\cos(\frac{\pi}{2} - \theta_i) \\ &= m_i r_i \sin\theta\omega R_i = m_i R_i^2 \omega^2 \end{aligned}$$

The component of the total angular momentum of a rotating body along the rotation axis Z is

$$\begin{aligned} L_z &= L_{1z} + L_{2z} + L_{3z} = \sum_i L_{iz} \\ &= (m_1 R_1^2 + m_2 R_2^2 + m_3 R_3^2 + \ldots)\omega \\ &= (\sum_i m_i R_i^2)\omega \end{aligned} \tag{19}$$

The quantity

$$\begin{aligned} I &= m_1 R_1^2 + m_2 R_2^2 + m_3 R_3^2 + \ldots \\ &= \sum_i m_i R_i^2 \end{aligned} \tag{20}$$

is called the moment of inertia of the body relative to the axis of rotation Z. It is obtained by adding, for each particle, the product of its mass times the square of its distance to the axis.

Eq.(19) can be expressed as

$$L_z = I\omega$$

The total angular momentum of the body is

$$L = L_1 + L_2 + L_3 + \ldots = \sum_i L_i \tag{21}$$

2.2.2 Moment of Inertia

As we discussed in the previous paragraph, the moment of inertia is defined as shown in Eq.(20). Since the rigid body is composed of a very large number of particles, that equation must be replaced by integral

$$I = \int R^2 dm$$

If ρ is the density of the body, $dm = \rho dV$. Hence, the above equation can be written as

$$I = \int \rho R^2 dV$$

For a thin plate as indicated in Fig.11, the moment of inertia around the Z-axis is

$$I_Z = \int \rho (x^2 + y^2) dV \tag{22}$$

This moment of inertia has two components - the moments of inertia relative to the X and Y axis, i.e.

$$I_y = \int \rho y^2 dV$$

$$I_x = \int \rho x^2 dV$$

Thus Eq.(22) becomes

$$I_z = I_x + I_y$$

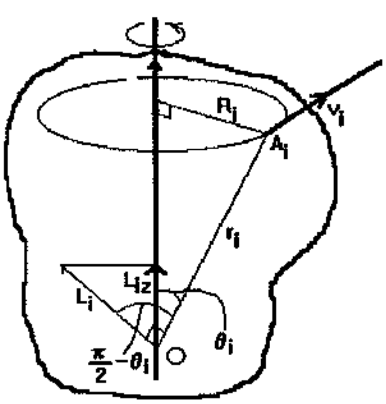


Figure 10. Angular momentum of a rotating rigid body.

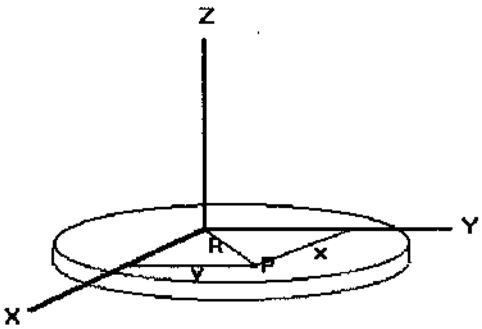


Figure 11. Moment of inertia around Z-axis for thin plate

The moment of inertia relative to parallel axes are related by a very simple formula. Let Z be an arbitrary axis and Z_C a parallel axis passing through the center of mass of the body as shown in Fig.12. To calculate the moment of inertia of the body relative to Z and

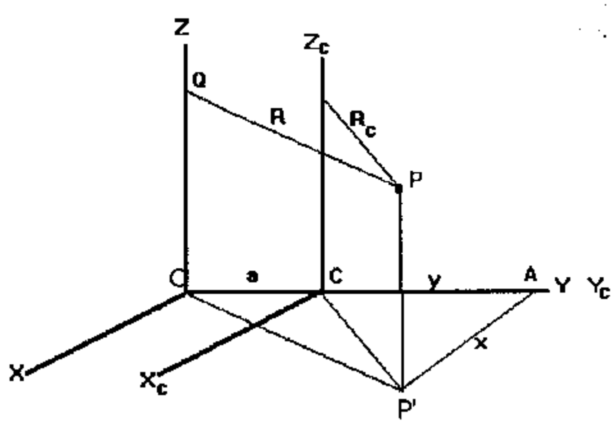


Figure 12. Moment of inertia around Z -axis

Z_C, let us choose the axes $X_C Y_C Z_C$ so that their origin is at the center of mass C, and the Y_C axis coincides with Y. The point P is any arbitrary point in the body M. From the figure, $P'A$ is perpendicular to Y_C and $P'A = x, CA = y$, and $OC = a$, we have

$$R_C^{\,2} = x^2 + y^2$$

$$R = x^2 + (y+a)^2 = R_C^{\,2} + 2ya + a^2$$

The moment of inertia relative to the Z-axis is

$$\begin{aligned} I &= \sum mR^2 = \sum (R_C^{\,2} + 2ya + a^2) \\ &= \sum mR_C^{\,2} + 2a\sum my + a^2 \sum m \end{aligned} \tag{23}$$

The first term is the moment of inertia relative to the Z_C axis which is expressed as I_C, the last term includes the total mass which is denoted as M. Since the center of mass coincides with the origin C of the frame $X_C Y_C Z_C$, then $y = 0$. Hence, the middle term of above equation = 0. Therefore

$$I = I_C + Ma^2$$

Now, we introduce a radius of gyration of a body K_r, which represents the distance from the axis at which all the mass could be concentrated without changing the moment of inertia. The quantity K_r is defined as

$$K_r = \sqrt{\frac{I}{M}}$$

where I is the moment of inertia and M is the mass of the body. Fig.13 shows a list of radii of gyration of some simple bodies.

2.2.3 Equations of motion for rotation of a rigid body

We have already discussed the total angular momentum of a rigid body, which is expressed in Eq.(19). Now, we introduce the relation between the total angular momentum of a rigid body and the total torque τ for a rigid body which is

$$\tau = \frac{dL}{dt}$$

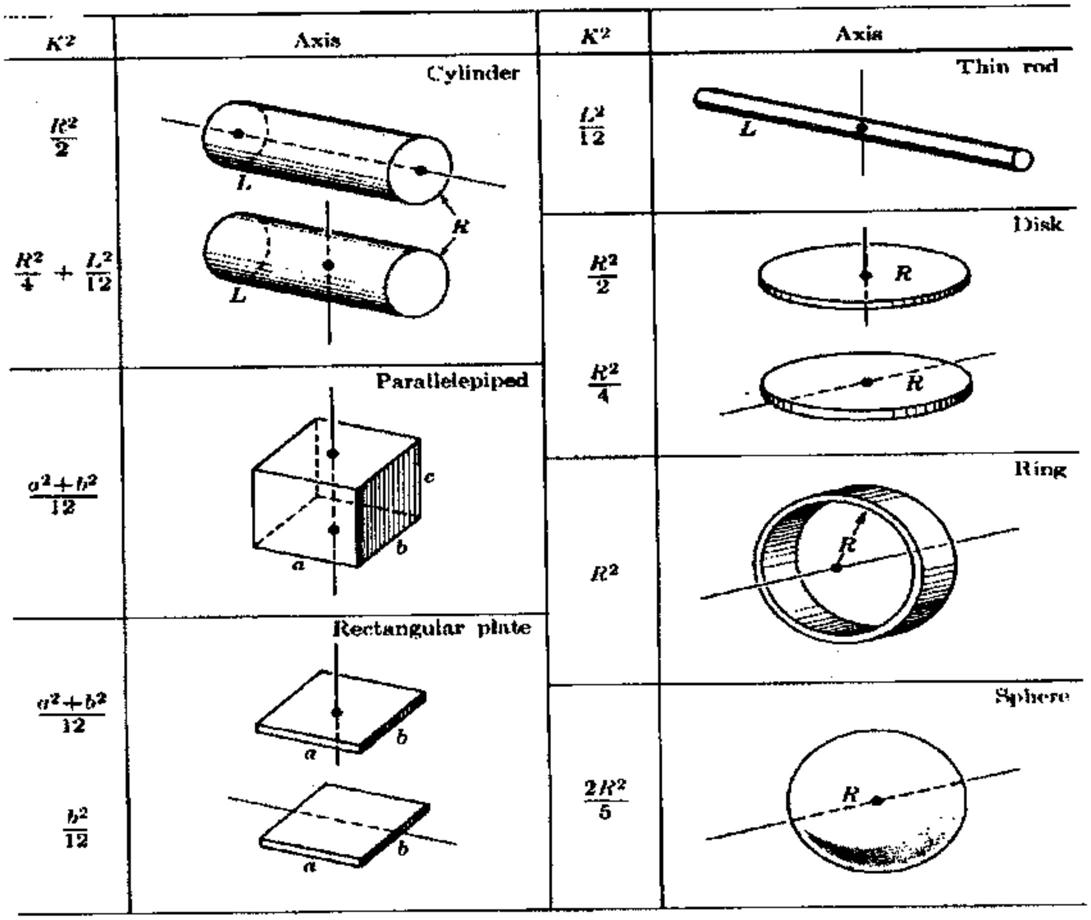


Figure 13. A list of radii of gyration of some simple bodies [Alonso, 1967].

where $\boldsymbol{L} = \sum_i \boldsymbol{L}_i$, and $\tau = \sum_i \tau_i$. This equation is the basic equation for discussing the rotational motion of a rigid body. In the case of a body rotating around a principal axis having a point fixed in an inertial system, the torque is defined as

$$\tau = \frac{dL}{dt} = \frac{d(I\omega)}{dt} \tag{24}$$

where I is the moment of inertia expressed in Eq.(20). If the axis remains fixed relative to the rigid body, the moment of inertia remains constant. Then

$$\tau = \frac{dL}{dt} = I\,\frac{d(\omega)}{dt} \text{ or } \tau = I\alpha_a$$

where $\alpha_a = \frac{d\omega}{dt}$ is the angular acceleration of the rigid body. If $\tau = 0$, Eq.(24) shows that $I\omega = const$, and the moment of inertia is constant. A rigid body rotating around a principal axis moves with constant angular velocity when no external torques are applied. If the body is not rotating around a principal axis, we use Eq.(21) to compute I. If the orientation of the rotation axis is fixed relative to the body, I is constant, hence we have

$$\tau_z = I\frac{d\omega}{dt}$$

where τ_z refers to the component of the total external torque around the rotation axis.

In the case of a body which is rotating around an axis with no point fixed in an inertial system, we have to compute the angular momentum and the torque relative to the center of mass of the body which is

$$\tau_{CM} = \frac{dL_{CM}}{dt}$$

If the rotation is around a principal axis, the equation becomes

$$\tau_{CM} = I\frac{d\omega}{dt}$$

2.2.4 Kinetic energy for a rigid body

A rigid body rotates around an axis with angular velocity ω , the velocity of each particle is $v_i = \omega R_i$ where R_i is the distance of the particle to the axis of rotation. Since a rigid body is composed of a large number of particles, the kinetic energy is

$$\begin{aligned} E_k &= \sum_i \frac{1}{2} m_i v_i^2 \\ &= \sum_i \frac{1}{2} m_i R_i^2 \omega^2 \end{aligned}$$

or, recalling the definition of the moment of inertia which is expressed in Eq.(20).

$$E_k = \frac{1}{2} I\omega^2 \qquad (25)$$

When a body rotates around a principal axis of inertia, the kinetic energy can be expressed

$$E_k = \frac{L^2}{2I}$$

where L is the total angular momentum of the body, which is expressed in Eq.(21).

Consider the general case in which a rigid body rotates about an axis passing through its center of mass, and at the same time has a translational motion relative to the observer. The kinetic energy of a body in an inertial frame is

$$E_k = \frac{1}{2} M v_{CM}{}^2 + E_{k,CM}$$

where M is the total mass. $E_{k,CM}$ is the internal kinetic energy relative to the center of mass. This energy is the rotational kinetic energy relative to the center of mass, which is equal to Eq.(25) , thus we have

$$E_k = \frac{1}{2} M v_{CM}{}^2 + \frac{1}{2} I_C \omega^2 \qquad (26)$$

where I_C is the moment of inertia relative to the axis of rotation passing through the center of mass.

The law of conservation of energy states that the change in proper energy of a system of particles is equal to the work done on the system by the external forces. In the case of a rigid body, it is simply expressed as

$$E_k - E_{k,0} = W_{ext} \qquad (27)$$

where $E_{k,0}$ is the kinetic energy at time t_0, and W_{ext} is the work of the external forces. The external forces acting on a system may also be conservative, so that W_{ext} can be written as

$$W_{ext} = (E_{p,0} - E_p)_{ext} \qquad (28)$$

where $E_{p,0}$ and E_p are the values of the potential energy associated with the external forces at the initial and final state. Substituting Eq.(28) into Eq.(27) and rearranging it, we have

$$E_k + E_p = (E_k + E_p)_0 \qquad (29)$$

where $E_k + E_p$ is the total energy of a rigid body, and denoted as E

$$E = E_k + E_p = constant$$

Substituting Eq.(27) into this equation, we have

$$E = \frac{1}{2} M v_{CM}{}^2 + \frac{1}{2} I_C \omega^2 + E_p = constant$$

For example, if the body is falling under the action of gravity, $E_p = Mgy$, where y refers to the height of the CM (center mass) of the body relative to a horizontal reference plane, and the total energy is

$$E = \frac{1}{2} M v_{CM}^2 + \frac{1}{2} I_C \omega^2 + Mgy = const.$$

If some forces are not conservative, for example when frictional forces in addition to gravitational forces are operating, we must write, instead of Eq.(28)

$$W_{ext} = E_{p,0} - E_p + W'$$

where W' is the work of the external non-conservative force. Equation (29) is now

$$(E_k + E_p) - (E_k + E_p)_0 = W'$$

2.3 Liquid systems

The properties of fluids are unlike that of a solid. The molecules of fluids are farther apart than those of a solid. Thus the attractive forces between the molecules of a fluid tend to change its original shape. Any fluid, no matter how viscous, will yield in time to the slightest stress. A solid needs a certain magnitude of stress to be exerted before it will flow. When external forces act on a fluid, and alter its shape, the tangential stresses between adjacent particles disallow the regaining of the original shape. These tangential stresses depend on the velocity of deformation and vanish as the velocity approaches zero.

A fluid may be a gas or liquid. Compared to a gas, a liquid is relatively incompressible. If all pressure is removed, the liquid does not expand indefinitely, since cohesion between molecules holds them together.

2.3.1 Kinematics of fluid flow

There are two different types of fluid flow. The first type is known as laminar or streamline or viscous flow. The fluid appears to move by the sliding of laminations of infinitesimal thickness relative to adjacent layers. The particles move in definite and observable path or streamlines, as shown in Fig.14.

Figure14. Laminar flow [Daugherty 77].

The second type of fluid flow is known as turbulent flow. The characteristics of this flow are no observable pattern, as in the case of eddies, and no definite frequency, as in wave action. This flow is shown in Fig.15. Turbulence may be found to be a very smoothly flowing stream and one in which there is no apparent source of disturbance.

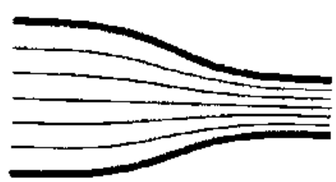


Figure15. Turbulent flow [Daugherty 77].

A steady flow is the flow of a fluid in which all conditions at any point in a stream remain constant with respect to time, but the conditions may be different at different times. A true steady flow is found only in laminar flow. A uniform flow is one in which the velocity is the same in both magnitude and direction at a given instant at every point in the fluid. For a real fluid, where the velocity varies across a section, this type of flow has little meaning. When the shape and the size of the cross section are constant along the length of a channel, the flow can be considered to be uniform.

The flow of particles in fluid can be depicted as a path line in which the trace made by a single particle over a period of time is shown, or streamlines which show the mean direction of a number of particles at the same instant of time. Path lines and streamlines are identical in the steady flow of fluid in which there are no fluctuating velocity components. This is because particles always move along streamlines, since these lines show the direction of every particle. Streamlines are the laminar type of flow, wherein the layers of fluid slide smoothly, one upon another. Path lines and streamlines are not coincident in turbulent flow, the former being very irregular while the latter are everywhere tangent to the local temporal velocity.

Flow rate and mean velocity

One of the most important calculations in fluid systems is the flow rate, which is the quantity of fluid flowing per unit time across any section. Consider Fig.16, which presents a streamline in steady flow lying in the *xz* plane. An element of area *dA* lies in the *yz* plane. The mean velocity at point *P* is u_f. The volume flow rate passing through the element of area *dA* is

$$dQ = u_f . dA = (u_f \cos\theta) dA$$
$$= u_f (\cos\theta dA) = u_f dA'$$

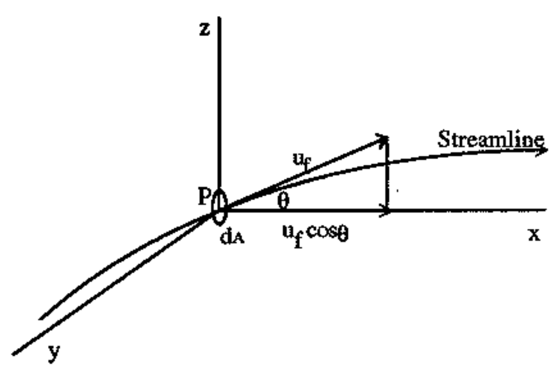


Figure 16. A streamline in a steady flow lying in the xy plane.

In real fluids the velocity u_f will vary across the section in some manner. For example, the velocity adjacent to the wall will be zero, and it will increase rapidly within a short distance from the wall. Hence, the volume flow rate is expressed as

$$Q = \int Au_f dA = Av_f$$

$$G = \gamma_f \int Au_f dA = \gamma_f Av_f$$

or

$$M = \rho \int u_f dA = \rho Au_f$$

where u_f is the temporal mean velocity through an infinitesimal area dA, while v_f is the average velocity over the entire sectional area A. Q is the volume flow rate, G is the weight flow rate, and M is the mass flow rate. γ_f and ρ represent the mass and the weight of fluid per unit volume, respectively. If only average values of v_f are known for the different finite areas into which the total area is divided, then

$$Q = A_1 v_{f_1} + A_2 v_{f_2} + A_3 v_{f_3} + \ldots A_n v_{f_n} = Av_f$$

Thus $v = \frac{Q}{A}$ or $\frac{G}{\gamma_f A} = \frac{M}{\rho A}$.

Equation of continuity

Consider Fig.14, which represents a short length of tube, which may be assumed to be a bundle of streamlines. If m_t is the mass of fluid contained in a volume at time t , and m_{t+dt} is the mass of fluid contained in a volume at time $t + dt$, then we have

$$m_{t+dt} = m_t + (\rho_1 u_{f_1} dA_1)dt - (\rho_2 u_{f_2} dA_2)dt$$

This can also be expressed as

$$m_{t+dt} = m_t + (\frac{d\rho}{dt})\text{vol}$$

where $\frac{d\rho}{dt}$ is the time rate of change of the mean density of the fluid in volume vol.
From the above two equations, we have

$$(\rho_1 u_{f_1} dA_1)dt - (\rho_2 u_{f_2} dA_2)dt = \frac{d\rho}{dt} dt(vol)$$

and

$$\rho_1 \int A_1 u_{f_1} dA \text{-} \rho_2 \int A_2 u_{f_2} dA = \int \frac{d\rho}{d(vol)}$$

The last equation is known as the general equation of continuity for flow through a region with fixed boundaries. It states that the net rate of mass inflow to the control volume is equal to the rate of increase of mass within the control volume. For steady flow, $\frac{d\rho}{dt} = 0$, thus

$$\rho_1 \int A_1 u_{f_1} dA = \rho_2 \int A_2 u_{f_2} dA$$

or

$$\rho_1 A_1 u_{f_1} = \rho_2 A_2 u_{f_2} = M$$

or

$$\gamma_1 A_1 v_{f_1} = \gamma_2 A_2 v_{f_2} = G$$

These continuity equations apply to steady, compressible or incompressible flow within fixed boundaries.

If $\rho = constant$, the fluid is incompressible, and

$$\int A_1 u_{f_1} dA = \int A_2 u_{f_2} dA$$

or

$$A_1 v_{f_1} = A_2 v_{f_2} = Q \qquad (30)$$

This continuity equation applies to incompressible fluids for both steady and unsteady flow within fixed boundaries.

The continuity equation can be expressed in another form, which is often used for the consideration of flow in space, such as in the case of unsteady flow of a liquid in a canal. The principle of conservation of mass indicates that the rate of flow past section 1 minus the rate of flow past section 2 is equal to the time rate of change of storage volume between the two sections; i.e.

$$Q_1 - Q_2 = \frac{d(vol)}{dt}$$

where vol is the volume of liquid contained in the canal between the two sections.

2.3.2 Forces in fluid flow

Newton's second law can be expressed as

$$\sum F = \frac{d(mv_f)}{dt}$$

This statement implies that the external forces on a body are equal to the rate of change of momentum of that body. In fluid systems, the body is defined as the mass of fluid contained in a control volume at time t. Consider a steady flow. The force of the fluid system is

$$\sum F = \frac{d(mv_f)_{out}}{dt} - \frac{d(mv_f)_{in}}{dt} \qquad (31)$$

The force on the fluid mass is equal to the net rate of outflow of momentum across the control surface. Consider the situation shown in Fig.17. This fluid system moves to a new position during time interval dt. It moves a short distance ds_1 at section 1 and a short distance ds_2 at section 2. During the interval dt, the momentum crossing the control surface section 1 and 2 are $(\rho A_1 ds_1)v_{f_1}$ and $(\rho A_2 ds_2)v_{f_2}$ respectively, where ρ is the density of the fluid. Substituting these into Eq.(31), and replacing $\frac{ds}{dt}$ with v_f and Av_f with Q, where Q is the flow rate of the fluid, we have

$$\sum F = \rho Q_2 v_{f_2} - \rho Q_1 v_{f_1}$$

According to Eq.(30) $\rho Q = \rho Q_1 = \rho Q_2$, thus we can write

$$\sum F = \rho Q(v_{f_2} - v_{f_1}) \qquad (32)$$

The direction of $\sum F$ is the same as that of $(v_{f_2} - v_{f_1})$. The $\sum F$ represents the vectorial summation of all forces acting on the fluid mass, such as gravity forces, shear forces and pressure forces including those exerted by fluid surrounding the fluid mass under consideration as well as pressure forces exerted by the solid boundaries in contact with the fluid mass.

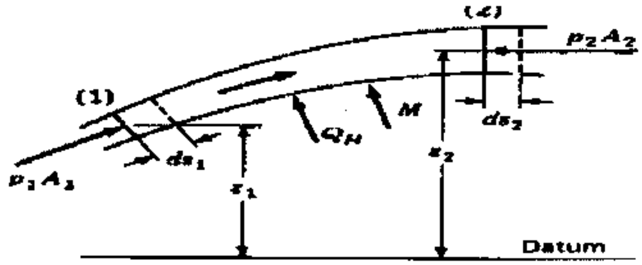


Figure17. Control volume for steady flow [Daugherty 77].

Consider a force exerted on pressure conduits. Fig.18a shows a horizontal flow of fluid to the right through the reducer, while Fig.18b shows a free-body diagram of the forces acting on the fluid mass contained in the reducer. The pressure forces exerted by fluid located just upstream and downstream of the fluid mass under consideration are represented by p_1A_1 and p_2A_2, where p_1 and p_2 are the pressures of fluid at positions 1 and 2, respectively, while A_1 and A_2 represent the cross-sectional areas of surfaces 1 and 2, respectively. The force $(F_{R/F})_x$ represents the force exerted by the reducer on the fluid in the x direction. This force is the integrated effect of the normal pressure forces that are exerted on the fluid by the wall of the reducer.

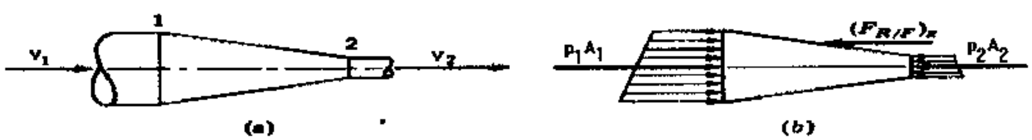


Figure18. A horizontal flow through the reducer [Daugherty 77].

Applying the component of Eq.(32) in the x direction, we have

$$\sum F = p_1A_1 - p_2A_2 - (F_{R/F})_x = \rho Q(v_{f_2} - v_{f_1})$$

$$(F_{R/F})_x = p_1A_1 - p_2A_2 - \rho Q(v_{f_2} - v_{f_1}) \qquad (33)$$

This gives the value of the total force exerted by the reducer on the fluid in the x direction. The force of the fluid on the reducer is, of course, equal and opposite to that of the reducer on the fluid.

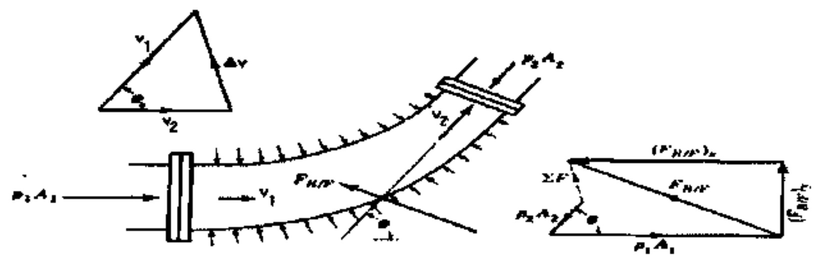


Figure19. Fluid in a reducing pipe bend [Daugherty 77].

If both the direction and velocity are changed, as the fluid undergoes in the reducing pipe bend shown in Fig.19, the computation of the forces of the fluid on the reducing pipe is similar to procedures of the previous case, except that it is convenient to deal with components. In the x direction, we have

$$(F_{B/F})_x = p_1 A_1 - p_2 A_2 \cos\theta - \rho Q(v_{f_2}\cos\theta - v_{f_1})$$

Similarly, in the y direction, we have

$$(F_{B/F})_y = p_2 A_2 \sin\theta + \rho Q v_{f_2} sin\theta$$

Notice that all the forces acting on the fluid which includes the pressure forces on the two ends is $F = \rho Q dv$, while the forces exerted by the bend on the fluid is $(F_{B/F})_x$. The value of $(F_{B/F})_x$ is $\sqrt{(F_{B/F})_x^2 + (F_{B/F})_y^2}$. Thus, the total force exerted by the fluid on the bend is the same value as $(F_{B/F})_x$, but is in the opposite direction.

Consider a force exerted on a stationary vane or blade, such as shown in Fig.20. To obtain the force, we still apply Eq.(32) for the components of the force in the x and y directions. However, since the fluid is in contact with the atmosphere, the *pA* components of the force are neglected. There are two components of the force of the fluid on the blade, $(F_{B/F})_x$ and $(F_{B/F})_y$.

In the case of a moving vane or blade, the force exerted by a stream upon a single moving object can be determined by Eq.(33). Initially, the body has a motion of translation in the same direction as the stream and the fluid flow is steady.

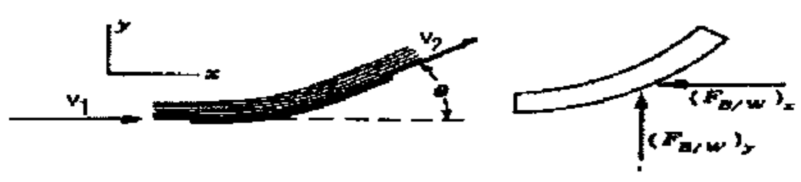


Figure20. Force acting on a stationary vane [Daugherty 77].

In the action upon a moving object, it is necessary to consider relative velocities between the moving object and the stream. The amount of fluid that strikes the single moving object per unit time will be less if the body is moving along the line of action of the stream than if it is stationary. As an extreme case, if the object to be moving in the same direction as the jet and with the same or a higher velocity, none of the fluid will act upon it. If it is moving with less velocity than that of the jet, $v < v_{f_1}$, the amount of fluid striking the body will be proportional to the difference between the two velocities $V_{f_1} = v_{f_1} - v$, where v_{f_1} and v are the velocities of the jet and the object respectively.

Consider an object that is moving away from a nozzle such as depicted in Fig.21. The cross-sectional area of the stream is A_1 and its velocity is v_{f_1}. The rate at which fluid issues from the nozzle is $G = \gamma_f A_1 v_{f_1}$, where γ_f is the specific weight of the fluid. If G' denotes the weight of fluid per second striking a single object moving with velocity v in the same direction as v_{f_1}, then

$$G' = \gamma_f Q' = \gamma A_1 (v_{f_1} - v) = \gamma_f A_1 V_{f_1}$$

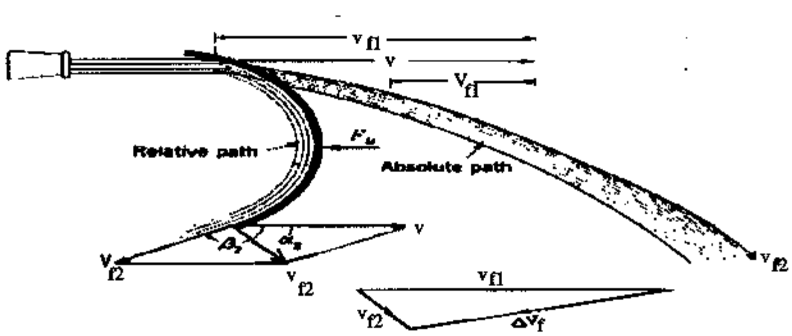


Figure 21. Jet acting on a vane translation [Daugherty 77].

The component of the force in the same direction as v, exerted by the vane on the fluid is

$$F_v = \rho Q' \delta v_v = \frac{G'}{g} \delta v_v$$

where $\delta v_v = v_{f_2} \cos \alpha_2 - v_{f_1}$. The force F_u acts to the left in Fig. 21. Equal and opposite to this force is the force of the jet on the vane acting to the right.

The velocity vector diagram at the entrance and exit to the vane are shown in Fig. 21. The relative velocity at the entrance, just before the fluid strikes the vane, is determined by the relation between v_{f_1} and v. Just after the fluid strikes the vane, its relative velocity must be tangent to the vane surface.

2.3.3 Energy consideration in steady flow

This sections discusses briefly the various forms of energy present in fluid flow. It is known that one of the laws of thermodynamics tells us that energy can be neither created nor destroyed. Kinetic energy of a moving body with its velocity v is equal to $E_k = \frac{1}{2}mv^2$. If a fluid were flowing with all particles moving at velocity v_f, its kinetic energy is also equal to $E_k = \frac{1}{2}mv_f^{\,2}$. The relation of the kinetic energy per unit weight of fluid is expressed as

$$\frac{E_k}{weight} = \frac{\frac{1}{2}mv_f^{\,2}}{\gamma_f(vol)}$$
$$= \frac{\frac{1}{2}[\rho(vol)]v_f^{\,2}}{\gamma_f(vol)} = \frac{v_f^{\,2}}{2g}$$

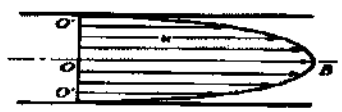


Figure 22. Real fluid [Daugherty 77].

where g is the acceleration due to gravity.

In a real fluid flow as shown in Fig. 22, the velocity of the different particles are not the same. The integration of all portions of the stream is required to obtain the true value of kinetic energy. It is convenient to express the true value in terms of the mean velocity v_f and a factor α_f. Hence

$$\frac{true\ E_k}{weight} = \alpha_f \frac{v_f^{\,2}}{2g} \tag{34}$$

Consider the case where the axial components of the velocity vary across a section, such as shown in Fig. 22. If u_f is the local axial velocity component at a point, the mass flow through an elementary area dA is $\rho dQ = \rho u_f dA$, thus the true flow of kinetic energy per unit time across area dA is $(\rho u_f dA)(\frac{u_f^{\,2}}{2})$. Therefore for the entire section

$$\frac{\frac{true\ E_k}{time}}{\frac{weight}{time}} = \frac{true\ E_k}{weight} =$$

$$\frac{true\ E_k}{weight} = \frac{\frac{\gamma_f}{2g}\int u_f^3 dA}{\gamma u_f dA} = \frac{\int u_f^3 dA}{2g\int u_f dA}$$

Substituting this equation to Eq.(34) for $\frac{true\ E_k}{weight}$, we have an expression for α_f

$$\alpha_f = \frac{\frac{1}{v_f^2}\int u_f^3 dA}{\int u_f dA} = \frac{1}{Av_f^2}\int u_f^3 dA$$

The greater the variation in velocity across the section, the greater the value of α_f will be.

Potential Energy

The potential energy of most of the particles depends on their elevations with respect to an arbitrary datum plane. In fluid flow, the potential energy also depends on where the particle is situated. A particle of weight W located at distance z above the datum possesses a potential energy

$$E_p = Wz$$

Thus, its potential energy per unit weight is

$$\frac{E_p}{W} = z$$

Internal Energy

The internal energy of a fluid is energy due to the motion of molecules and forces of attraction between them. The internal energy is a function of temperature. For an arbitrary interval of time δt, the internal energy per unit of mass is expressed as

$$i = c_v \delta t$$

where c_v is the specific heat at constant volume. The internal energy per unit weight is

$$I_w = \frac{i}{g}$$

General equation for steady flow of any fluid

The first law of thermodynamics states that for steady flow, the change of energy of the system is equal to the sum of the external work done on any system and the thermal energy transferred into or out of the system.

$$\delta E = work + heat$$

Consider Fig. 23a. The fluid system consists of the fluid that was contained between sections 1 and 2 at time t in the control volume. The fluid system moves to the new position during time interval dt, and we assume that the fluid moves a short distance ds_1 at section 1 and ds_2 at section 2 as indicated in the figure. The flow work done by pressure forces $p_1 A_1$ and $p_2 A_2$ on the system is

$$Flow\ work = p_1 A_1\, ds_1 - p_2 A_2\, ds_2 \tag{35}$$

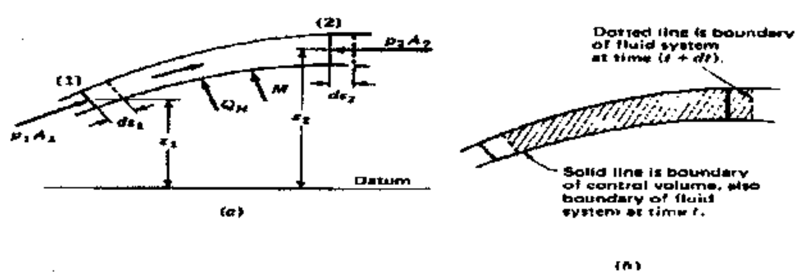


Figure 23. A fluid in a fixed control volume [Daugherty 77].

If h_M is the energy put into the flow by machine per unit weight of flowing fluid, such as a pump or turbine, the shaft work is expressed as

$$\begin{aligned} Shaft\ work &= \frac{weight}{time}\ x\ \frac{energy}{weight}\ x\ time \\ &= (\gamma_{f1} A_1 \frac{ds_1}{dt}) h_M dt \\ &= (\gamma_{f1} A_1 ds_1) h_M \end{aligned} \tag{36}$$

If Q_H is the energy put into the flow by an external heat source per unit weight of flowing fluid, then the heat transfers from an external source into the fluid system is

$$Heat = (\gamma_{f_1} A_1 \frac{ds_1}{dt}) Q_H dt \tag{37}$$

At time $t + dt$ the mass of fluid has moved to a new position as shown in Fig. 23b. The energy of the fluid system is expressed as

$$E_2 = E_1 + \delta E_{out} - \delta E_{in}$$

where E_1 is the energy possessed by the fluid mass at time t, δE_{out} and δE_{in} are the energies that flowed out of and into the control volume, respectively. Hence, the change in energy δE of the fluid system is

$$\delta E = \delta E_2 - \delta E_1 = \delta E_{out} - \delta E_{in}$$

The weight of fluid entering section 1 during time interval dt is $\gamma_{f_1} A_1 ds_1$. A similar expression holds for the weight of fluid leaving ds_2. The energy δE_{in} that enters at section 1 during time dt is

$$\delta E_{in} = \gamma_{f_1} A_1 ds_1 (z_1 + \alpha_{f_1} \frac{v_{f_1}^{2}}{g} + I_{w1})$$

where the first term is its potential energy, the second term is its kinetic energy and the last term is its internal energy.

Similarly, the energy δE_{out} that leaves at section 2 is

$$\delta E_{out} = \gamma_{f_2} A_2 ds_2 (z_2 + \alpha_{f_2} \frac{v_{f_2}^{2}}{g} + I_{w2})$$

Thus,

$$\delta Energy = \delta E = \gamma_{f_1} A_1 ds_1 (z_1 + \alpha_{f_1} \frac{v_{f_1}^{2}}{g} + I_{w1}) - \gamma_{f_2} A_2 ds_2 (z_2 + \alpha_{f_2} \frac{v_{f_2}^{2}}{g} + I_{w2})$$

The work and the heat exchange done by the fluid system are expressed in Eq. (35), Eq. (36) and Eq. (37). Applying the first law of thermodynamics ($work + heat = \delta energy$), and factoring out $\gamma_{f_1} A_1 ds_1 = \gamma_{f_2} A_2 ds_2$, we have

$$\frac{p_1}{\gamma_{f_1}} - \frac{p_2}{\gamma_{f_2}} + h_M + Q_M = (z_2 + \alpha_{f_2}\frac{{v_{f_2}}^2}{g} + I_{w2}) - (z_1 + \alpha_{f_1}\frac{{v_{f_1}}^2}{g} + I_{w1})$$

or

$$(z_1 + \frac{p_1}{\gamma_{f_1}} + \alpha_{f_1}\frac{{v_{f_1}}^2}{g} + I_{w1}) + h_M + Q_M = (z_2 + \frac{p_2}{\gamma_{f_2}} + \alpha_{f_2}\frac{{v_{f_2}}^2}{g} + I_{w2}) \quad (38)$$

This is the general equation for steady flow of any fluid. In many cases this equation may be shortened. Some quantities are equal and cancel each other, or are zero. For example, if two points are at the same elevation, $z_1 - z_2 = 0$. The value of Q_H may be taken as zero, if the temperature of the fluid and that of its surrounding are practically the same. If there is no machine between sections 1 and 2, the term h_M drops out.

For incompressible fluids, the values of γ_{f_1} and γ_{f_2} in Eq.(38) are the same. In turbulent flow, the value of α is a little more than unity, and for simplicity, it will be taken equal to unity. Thus, we have

$$(z_1 + \frac{p_1}{\gamma_{f_1}} + \alpha_{f_1}\frac{{v_{f_1}}^2}{g}) + h_M + Q_M = (z_f + \frac{p_2}{\gamma_{f_2}} + \alpha_{f_2}\frac{{v_{f_2}}^2}{g}) + (I_{w2} - I_{w1}) \quad (39)$$

A change in the internal energy per unit weight of a fluid, δI, is equal to the sum of the external heat added to or taken away from the fluid and the heat generated by fluid friction. Thus

$$\frac{\delta internal\ energy}{unit\ weight} = \delta I_w = I_{w2} - I_{w1}$$

$$= \frac{\delta i}{g} = \frac{c_y}{g}(T_2 - T_1) = Q_H - h_L$$

where c_y is the specific heat of the incompressible fluid, h_L is the fluid-friction energy loss per unit weight of fluid, and δi is a change in the inernal energy per unit mass, respectively. The above equation can be expressed as

$$h_L = I_{w2} - I_{w1} - Q_H = \frac{c_y}{g}(T_2 - T_1) - Q_H \quad (40)$$

$T_2 < T_1$ if the loss of heat ($Q_H < 0$) is greater than h_L and $T_2 > T_1$ if there is any absorption of heat ($Q_H > 0$)

If there is no machine between sections 1 and 2, and if no heat is gained or lost, substituting Eq.(40) into Eq.(38) results in

$$(z_1 + \frac{p_1}{\gamma_{f_1}} \alpha_{f_1} \frac{v_{f_1}^{\;2}}{g}) = (z_2 + \frac{p_2}{\gamma_{f_2}} \alpha_{f_2} \frac{v_{f_2}^{\;2}}{g}) + h_L$$

where h_L represent the energy loss per unit weight of fluid. When it is so small that it may be taken as zero, therefore we have

$$(z + \frac{p}{\gamma_f} + \frac{v_f}{2g}) = constant.$$

This equation is often known as Bernoulli's theorem for frictionless incompressible fluids.

2.3.4 Forces in an immersed body

There are two kinds of forces arising from relative motion between an immersed body and fluid. These are called the drag and lift forces. The former is parallel to the motion, while the latter is at the right angles to the fluid.

The drag forces on a submerged body have two components: a pressure drag F_p and friction or surface drag F_f. The pressure drag is equal to the integration of the components in the direction of motion of all pressure forces exerted on the surface of the body. It is expressed as

$$F_p = C_p \rho \frac{v_f^{\;2}}{2} A$$

where A is the projected area of the body normal to the flow, and C_p is the pressure-drag coefficient, which is dependent on the geometric form of the body, and ρ represents the density of the fluid. The term $\rho \frac{v_f^{\;2}}{2}$ is called the dynamic pressure.

The friction drag is equal to the integration of the components of the shear stress along the boundary of the body in the direction of motion. It is commonly expressed as

$$F_f = C_f \rho \frac{v_f^{\;2}}{2} B_t L_s$$

where C_f is the friction-drag coefficient, dependent on viscosity. L_s. is the length of surface parallel to the flow, and B is the transverse width, conveniently approximated for irregular shapes by dividing the total surface area by L_s.

The total drag on a body is equal to the sum of the friction drag and pressure drag, i.e.

$$F_D = F_f + F_p$$

In the case of a well-streamlined body, such as the hull of a submarine, the friction drag is the major part of total drag. Usually when the wake resistance becomes significant, one is interested in total drag only by employing a single equation

$$F_D = C_d \rho \frac{{v_f}^2}{2} A \qquad (41)$$

The Reynolds number, N_R , is defined as the ratio of inertia to viscous forces

$$N_R = \frac{\rho {v_f}^2 D^2}{D v_f \mu} = \frac{D v_f}{\vartheta}$$

where μ is the coefficient of viscosity, and ϑ is the kinematic viscosity. D is a length that is significant in the flow pattern. For example, for a pipe completely filled, D is the diameter of the pipe.

In the case of the flow around a sphere, with very low Reynolds number $D\frac{v_f}{\vartheta} < 1$, the flow about the sphere is completely viscous and the friction drag is given by Stoke's law

$$F_D = 3\pi\mu D$$

Equating this equation to Eq.(41) results in $C_d = \frac{24}{N_R}$, where A is defined as $\pi\frac{D^2}{4}$, the frontal area of the projected sphere.

2.4 Nuclear reactions

A nuclear reaction is a process of interacting an external nuclear particle with an atomic nucleus. A compound nucleus is temporarily formed by the merging of the incident particle with the target nucleus. The compound particle lasts a very short time before splitting into a product nucleus. The properties of the individual atomic nucleus is changed because of the actions of external influences. If this nucleus is left to itself, it retains its physical properties over a long period of time. This kind of nucleus is called a *stable nucleus*. However, not all nuclei are stable. Some of them have the capability of transforming spontaneously into other nuclei with different masses and charges. This process is called *radioactive decay*. The initial unstable radioactive nuclei are often called *parent* nuclei, while the resulting nuclei are called *daughter* nuclei. A decay constant λ is a rate at which a particular nucleus transforms into a different type of nucleus. Suppose there are N radioactive nuclei with a decay constant λ . The rate of the number of radioactive nuclei changing is

$$\frac{dN}{dt} = -\lambda N \qquad (42)$$

The minus sign indicates that the number of radioactive nuclei is decreasing. If N_0 is the initial number of radioactive nuclei present at time 0, then after a time t, the number N_t that remain can be calculated by integrating the decay equation (42).

$$\int \frac{N_0}{N_t} \frac{dN}{N} = -\int_0^t \lambda N dt$$

The solution is

$$N_t = N_0 e^{-\lambda t} \qquad (43)$$

The most common method for expressing the rate decay of radioactive nuclei is that of expressing the half-life of radioactive nuclei. The half-life time $t_{\frac{1}{2}}$ is required to reduce by one-half the number of radioactive nuclei in a given sample. Referring to equation (43), $t_{\frac{1}{2}}$ is given by

$$\frac{N_t}{N_0} = \frac{1}{2} = e^{-\lambda t_{\frac{1}{2}}}$$

$$t_{\frac{1}{2}} = \frac{0.693}{\lambda}$$

Thus, after one half-life, the number of nuclei is reduced to $\frac{1}{2}$. Consequently after *n* half-lives the number of radioactive nuclei is reduced to $(\frac{1}{2})^n$.

2.4.1 Radioactivity (alpha, beta and gamma decays)

Alpha decay.

Decay of a radioactive nucleus which causes alpha particle emission is called alpha-decay. The α particles are identical with the nuclei of helium atoms. In this kind of radioactivity, a product nucleus has an atomic number which is reduced by 2 and the mass number by 4.

A typical example of natural alpha decay is that of Uranium 238, the most abundant isotope of Uranium [Borowotitz and Bornstein, 1968].

$${}_{92}U^{238} \rightarrow {}_{90}Th^{234} + {}_{2}\alpha^{4}$$

Beta decay.

A radioactive process in which an unstable nucleus spontaneously ejects a fast electron is called beta decay. In this kind of decay, the daughter nucleus has an electric charge different from that of the parent nucleus. In this process, one of the neutrons in the nucleus breaks up, forming a photon and an electron, and the electron is cast off. For example, Pb 214 emits a β particle which converts it into a different element, bismuth.

$${}_{82}Pb^{214} \rightarrow {}_{63}Bi^{214} + {}_{-1}\beta^{0}$$

Gamma decay.

The third type of radioactivity emits a gamma ray which is a photon. The photon carries away neither charge nor mass. In the gamma decay process, the daughter nucleus is almost identical with the parent. This process usually follows an alpha or beta decay, which leaves the resultant nucleus in an excited state. An example of gamma emission is

$${}_{88}Ra^{226} \rightarrow {}_{86}Rd^{222^*} + {}_{2}\alpha^{4}$$

$${}_{86}Rd^{222*} \rightarrow {}_{86}Rd^{222} + {}_{0}\gamma^{0}$$

The asterisk sign indicates that the Radon is initially in an excited state.

Energy in nuclear decay.

Energy released in nuclear decay comes from the conversion of mass. According to Einstein's relation, the kinetic energy is equal to $E = mc^2$. This energy is carried off by the α or β particle whenever nuclear decay occurs. The total energy released in nuclear decay is calculated from the difference Δm between the mass of the original nucleus, and the mass of products of the reaction. For example, in this particular beta-decay

$$_{82}Pb^{214} \rightarrow {}_{83}Bi^{214} + {}_{-}\beta^{0}$$

we have the following masses

mass of original nucleus, $_{82}Pb^{214}$	:	213.9998 amu
mass of products, $_{83}Bi^{214}$ and $_{-}\beta^{0}$	:	(213.99792 + 0.00055) amu

$$\Delta m = 0.00137 \ amu$$

where 1 $amu = 913\ MeV$
Thus, the energy released in this beta decay is = $\Delta E = 1.3\ MeV$. The energy released in alpha decay can be similarly calculated.

The process of alpha decay and beta decay usually is accompanied by gamma emission. The emission of gamma rays shows that the nucleus is settling down to a state of lower excitation. The electromagnetic energy is released at the time an atom undergoes transition from a higher to lower energy state.

2.4.2 Induced nuclear reaction

In general, a nuclear reaction may be expressed as

$$a + X \rightarrow Y + b \quad \text{or} \quad X(a,b)Y \tag{44}$$

where *X* and *Y* are the target and product nuclei, respectively; *a* and *b* are the incident and product particles, respectively. When a target nucleus is bombarded by energetic charged particles it may undergo transformation into a new nucleus accompanied by the ejection of particles different from the incident one. It requires sufficient energy from the incident particle to overcome the repulsive force between the particle and target nucleus. Based on the product and the target nucleus, there are some types of induced nuclear reactions, such as [Bush62]

Nuclear transmutation - The incident particle may be an α particle, proton or neutron. The product particle may be a charged particle or neutron.

Radioactive capture - A particle may combine with a nucleus to produce a new nucleus which is in an excited state. The excess energy is emitted in the form of a γ-ray photon. This reaction is very common with neutrons; e.g.

$$H^1(n,\gamma)H^2$$

Photodisintegration - If the incident particles are γ-rays, nuclear disintegration may occur according to Eq.(44) with *a* representing a γ-ray photon. A typical reaction is

$$Be^9(\gamma,n)Be^8$$

Fission - For a heavy target nucleus *X*, the products of the reaction may be two nuclei Y and Z of comparable mass according to

$$a + X \rightarrow Y + Z$$

The incident particle may be a neutron, an α-particle or a γ-ray photon.

Elastic scattering - It is sometimes observed that an incident particle *a* approaches a nucleus *X* and then a similar particle leaves the vicinity of *X* traveling in a different direction to the incident particle. The only energy exchange which is observed is in the kinetic energy of the particle and of the nucleus which would be expected for the purely dynamic collision between perfectly elastic point masses.

Inelastic scattering - The scattered particle may lose kinetic energy in excess of that required for an elastic collision with a nucleus, there being a corresponding increase in the internal energy of the nucleus. Both elastic and inelastic scattering are represented as

$$a + X \rightarrow X + a$$

The kinetic energies of incident particles and products of a nuclear reaction can be obtained directly.

2.4.3 Fission reaction

As mentioned above, a nuclear reaction can be expressed by *X(a,b)Y*. For a heavy target nucleus *X*, the product of the reaction *Y* and *b* may be two nuclei of comparable mass. Such a reaction is called a fission reaction. This reaction can occur spontaneously, but it is usually induced by neutrons. It may also be induced by alpha particles, protons, deuterons and gamma rays. This type of reaction is exoergic, in which energy is released in the reaction, and the kinetic energy of the products is greater than that of the incident particle.

In a fission reaction, a self-sustaining reaction is possible. The energy is released per fission together with neutrons which may be utilized to produce further fission. In almost all fission processes a few neutrons as well as gamma radiations are released at the instant of fission. Thorium and Uranium are examples of nuclei which can be induced by neutrons of moderate energy. This reaction occurs naturally in large quantities. For example, plutonium (Pu^{239}) may be considered as a possible fuel in a chain reactor, since the chain reactor produces considerable amounts of plutonium. This kind of reaction is shown below

$$_{92}U^{238} + _0n^1 \rightarrow _{92}U^{239} \rightarrow _{93}Np^{239} \rightarrow _{94}Pu^{239}$$

Energy released by fission

The energy released by fission also comes from the conversion of masses. Since a chain reaction possibly occurs, a considerable amount of energy must be released during the fission process. Some of the energy released is delayed, though, and appears during the radioactive decay of fission products. For example, a neutron induced fission of U^{235} produces the light and heavy fragments Sr^{94} and Xe^{140}. The reaction is shown below

$$n + U^{235} \rightarrow U^{236} \rightarrow Xe^{140} + Sr^{94} + 2n$$

A successive beta-ray emission occurs in both Xe^{140} and Sr^{94} decays. These types of decays are shown below

$$Xe^{140} \rightarrow Cs^{140} \rightarrow Ba^{140} \rightarrow La^{140} \rightarrow Cs^{140}$$

$$Xr^{94} \rightarrow Y^{94} \rightarrow Zr^{94}$$

The total energy released by fission can be determined from the difference between the masses of the original nuclei (*n* and U^{238}) and the stable product nuclei (Ze^{94}, Cs^{140} and *2n*). In our example,

the mass of U^{235} is	235.116600 *amu*
the mass of a neutron is	1.008982 *amu*
the total mass of the original nuclei is	236.125582 *amu*
the total mass of 2 neutrons is	2.017964 *amu*
the mass of Cs^{140} is	139.947600 *amu*
the mass of Zr^{94} is	93.935800 *amu*
the total mass of product nuclei is	235.903524 *amu*

Thus, the difference between the original and the product nuclei is 0.21958 *amu* which is equivalent to 206.8 *MeV*.

The energy released by a fission process is much larger than that of previously discussed radioactive decays. The large amount of energy release in a fission process has been developed into a practical source of energy in generating power plants.

2.4.4 Fusion reaction

A fusion reaction is a nuclear reaction wherein two light nuclei are combined to form a single one. This type of reaction is exoergic, and it will release the greatest energy for the lightest elements. The following are examples of fusion reactions. The first two reactions are referred to as $D + D$ reactions, and the third as a $D + T$ reaction.

	Energy release (MeV)
$H_2 + H_2 \rightarrow H_3 + n$	3.25
$H_2 + H_2 \rightarrow H_3 + H_1$	4
$H_3 + H_2 \rightarrow H_4 + n$	17.6

In fusion reactions, if target nuclei are bombarded at rest, most of the kinetic energy of the bombarding particles leads to ionization, and energy transfers to the atomic electrons of the target nuclei. The ionization mentioned above is the process of removing an electron from an atom.

Chapter 3

Qualitative Simulation

Qualitative Simulation (QSIM) was developed by Kuipers[1986]. QSIM basic objective is to derive a qualitative description of dynamic behavior of physical system. A behavior of a physical system is described by values of a set of variables as a function of time. His approach starts with a set of constraints abstracted from differential equations. QSIM guarantees to produce a qualitative behavior corresponding to any solution to the original equations.

The central inference within his approach is qualitative simulation: derivation of a description of the behavior of a mechanism from qualitative constraint equations. A differential equation describes a physical system in terms of a set of state variables and constraints. The solution to the equation may be a function representing the behavior of the system over time.

Kuipers introduces the concept of qualitative function and expresses all constraints on the behavior of the functions. At any time, a function takes on one of two types of values, a landmark, or an interval between two landmarks. Time is described by means of distinguished time-points, which occur whenever a function reaches or leaves a landmark. Time is either equal to a distinguished time-point, or it equals to an interval between two such points of time. He also points out that both analytical and qualitative solutions are abstractions of the real behavior of a physical system (see Fig. 24).

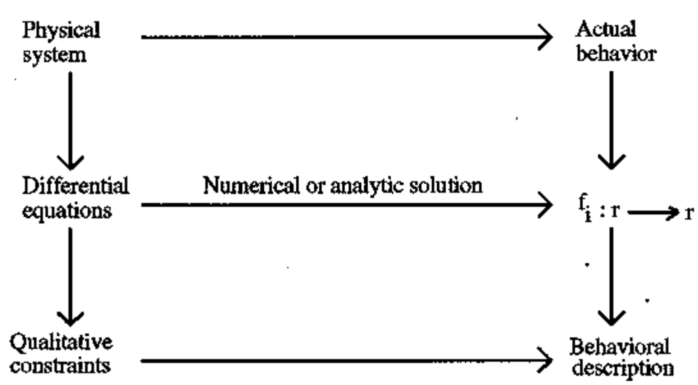


Figure 24. Qualitative simulation and differential equations are both abstractions of actual behavior.

Kuipers introduces five types of constraints which are shown in Fig.25.

The first two constraints shown in Fig. 25 are arithmetic constraints. The third merely describes a change of sign. The fourth expresses that one function is the derivative of another. This is mainly used the other way around, to perform a qualitative integration computing the next value of function *g(t)*. The last two constraints express functional relationships. The last constraint states that whenever *f(t)* increases, *g(t)* decreases.

constraints	hold if and only if
ADD(f,g,h)	$f(t) \oplus g(t) = h(t)$ for every $t \in [a,b]$
MULT(f,g,h)	$f(t) \otimes g(t) = h(t)$
MINUS(f,g)	$f(t) = - g(t)$ for every $t \in [a,b]$
DERIV(f,g)	$\frac{df(t)}{dt} = g(t)$ for every $t \in [a,b]$
$M^+(f,g)$	$f(t)=h(g(t))$, h differentiable $h'(x)>0$
$M^-(f,g)$	$f(t)=g(h(t))$, h differentiable $h'(x)<0$

Figure 25. Constraints used in QSIM.

3.1 Qualitative differential and constraints

The approaches of mapping a set of Ordinary Differential Equations (ODEs) into a set of constraints are similar to the following example. Consider an ODE of Eq. 45.

$$\frac{d^2u}{dt^2} - \frac{du}{dt} + arctan(k.u) = 0 \qquad (45)$$

This equation is transformed into a set of simultaneous equations. The simultaneous equations are transformed to a set of constraints. Both the equations and the constraints are shown in Fig. 26.

simultaneous equations [a]	constraints [b]
$f_1 = \frac{du}{dt}$	$DERIV(u, f_1)$
$f_2 = \frac{df_1}{dt}$	$DERIV(f_1, f_2)$
$f_3 = k.u$	$MULT(k, u, f_3)$
$f_4 = arctan\, f_3$	$M^+(f_3, f_4)$
$f_2 - f_1 + f_4 = 0$	$ADD(f_2, f_4, f_1)$

Figure 26. A set of simultaneous equations and its constraints.

Any solution of Eq. 45 will also satisfy the set of functions f_1 to f_4, since these are equivalent to Eq. 45. A set of simultaneous equations [a] derives a set of constraints [b], therefore any solution for Eq. 45, will automatically be a solution for the constraints. For each set of constraints like [b], there are several different set of ODEs which map onto that particular set of constraints. Consider Eq. 46.

$$\frac{d^2u}{dt^2} - \frac{du}{dt} + e^{k.u} = 0 \qquad (46)$$

This equation would result in a different equation defining f_4 in [a], but it still produces the same set of constraints.

QSIM makes an assumption that every physical system is described by a specific set of ODEs. Each set of ODEs can be mapped onto a set of constraints. This implies that the qualitative analysis derives the behavior of a class of systems, not the behavior of a single system.

3.2 Derivation of behavior

The notion of a qualitative state is one of the important concepts of qualitative reasoning. Qualitative state is defined [Forbus84] as the following. The qualitative state of a variable is the qualitative value of the variable. The qualitative state of a physical system is the combination of the states of all variables describing the system. Kuipers made an extension of the notion of a state by including derivatives on an equal basis with the values of functions. Kuipers thus defines the state and behavior of a single function as follows.

State of QSIM function

Let $L_1 < \ldots < L_n$ be the landmark values of $f:[a,b] \rightarrow R$. For any $t \in [a,b]$, the qualitative state of *f* at *t*, *QS(f,t)* is a pair <*qval,qdir*> defined as follows:

$$qval = \begin{cases} L_i & \text{if } f(t) = L_i, \text{ a landmark value} \\ (L_i, L_{i+1}) & \text{if } f(t) \in (L_i, L_{i+1}) \end{cases}$$

$$qdir = \begin{cases} inc, & \text{if } f'(t) > 0 \\ dec, & \text{if } f'(t) < 0 \\ std, & \text{if } f'(t) = 0 \end{cases}$$

Qualitative behavior

The qualitative behavior of *f* on $[t_a, t_b]$ is the sequence of the qualitative states of *f*:

$$QS(f,t_a), QS(f,t_a,t_{a+1}), \ldots, QS(f,t_{b-1},t_b), QS(f,t_b)$$

alternating between qualitative states at distinguished time points, and qualitative states on intervals between distinguished time points.

A physical system is characterized by a set of variables. QSIM performs an analysis of a system by examining the set of qualitative functions $\{f_1,..,f_m\}$ corresponding to the set of characteristic variables. There are two types of qualitative states for this kind of system, either at landmark or between landmarks. These are specified as indicated below.

$$QS(F,t_i) = [QS(f_1,t_i), QS(f_2,t_i), \ldots, QS(f_m,t_i)]$$
$$QS(F,t_i,t_{i+1}) = [QS(f_1,t_i,t_{i+1}), \ldots, QS(f_m,t_i,t_{i+1})]$$

The qualitative behavior of the system *F* is specified as the sequence of the qualitative states of *F*, alternating between landmark and interval values.

$$QS(F,t_0), QS(F,t_0,t_1), QS(F,t_1), \ldots, QS(F,t_n)$$

To derive formally the sequence of states constituting the qualitative description, it is required to identify the possible transitions between states. There are two types of qualitative state transitions [Kuipers86]: *P-transitions*, moving from a landmark to an interval, and *I-transitions*, moving from interval to a landmark. The definitions of these transitions are given below.

An *I-transitions* of *f* is a pair of adjacent qualitative states of *f*,

$$QS(f,t_i) \Rightarrow QS(f,t_i,t_{i+1})$$

whose first state is the qualitative state on the interval between distinguished time-points. A *P-transitions* of *f* is a pair of adjacent qualitative states of *f*, whose first state is the qualitative state at distinguished time-point.

$$QS(f,t_{i-1},t_i) \Rightarrow QS(f,t_i)$$

Fig.27 shows the possible state transitions in QSIM.

The qualitative simulation algorithm is given the following description of a mechanism [Kuipers86] is its input.

1. Set $\{f_1,\ldots,f_m\}$ of symbols representing the functions in the system.
2. Set of constraints applied to the function symbols. Each constraint may have associated corresponding values for its functions.
3. Each function is associated with a totally ordered set of symbols representing landmark values; each function has at least the basic set of landmark $\{-\infty, 0, \infty\}$.
4. Each function may have upper and lower range limits, which are landmark values beyond which the current set of constraints no longer apply.
5. An initial-point symbol, t_0, and qualitative values for each of the f_1 at t_0 are given

	P-transitions	
	$QS(f,t_i)$	$\Rightarrow$ $QS(f,t_i,t_{i+1})$
P1	$< L_i, std >$	$< (L_i), std >$
P2	$< L_i, std >$	$< (L_i, L_{i+1}), inc >$
P3	$< L_i, std >$	$< (L_{i-1}, L_i), dec >$
P4	$< L_i, inc >$	$< (L_i, L_{i+1}), inc >$
P5	$< (L_i, L_{i+1}), inc >$	$< (L_i, L_{i+1}), inc >$
P6	$< L_i, dec >$	$< (L_{i-1}, L_i), dec >$
P7	$< (L_i, L_{i+1}), dec >$	$< (L_i, L_{i+1}), dec >$

	I-transitions	
	$QS(f,t_i,t_{i+1})$	$\Rightarrow QS(f,t_{i+1})$
I1	$< L_i, std >$	$< L_i, std >$
I2	$< (L_i, L_{i+1}), inc > c$	$< L_{i+1}, std >$
I3	$< (L_i, L_{i+1}), inc >$	$< L_{i+1}, inc >$
I4	$< (L_i, L_{i+1}), inc >$	$< (L_i, L_{i+1}), dec >$
I5	$< (L_i, L_{i+1}), dec >$	$< L_i, std >$
I6	$< (L_i, L_{i+1}), dec >$	$< L_i, dec >$
I7	$< (L_i, L_{i+1}), dec >$	$< (L_i, L_{i+1}), dec >$
I8	$< (L_i, L_{i+1}), dec >$	$< L^*, std >$
I9	$< (L_i, L_{i+1}), inc >$	$< L^*, std >$

Figure 27. Possible state transitions in QSIM.

The result of qualitative simulation is one or more qualitative behavior descriptions for the function symbols given. Each qualitative behavior description consists of the following [Kuipers86]:

1. A sequence $\{t_0, \ldots, t_n\}$ of symbols representing the distinguished time points of the system's behavior.
2. Each function *f*, has a totally ordered set of landmark values, possibly extending the original set.
3. Each function has at each distinguished time-point or interval between adjacent time-points, a qualitative state description expressed in terms of the landmark values of that function.

To illustrate the mechanism of QSIM, consider a simple system consisting of a ball thrown upward in a constant gravitational field. The set of constraints is

$$DERIV(y,v), \quad DERIV(v,a), \quad a(t) = g < 0$$

where *y, v, a* represent symbols of the height where the ball reaches, the velocity and the acceleration of the ball, respectively. We start with an active state, $t = t_0, t_1$, whose description is:

$$QS(a,t_0,t_1) = <g, std>$$
$$QS(v,t_0,t_1) = <(0,\infty), dec>$$
$$QS(y,t_0,t_1) = <(0,\infty), inc>$$

The set of possible qualitative state transitions for a, v and y from the current state is given by

a	*I1*	$<g,std>$	$\Rightarrow$	$<g,std>$
v	*I5*	$<(0,\infty),dec>$	$\Rightarrow$	$<0,std>$
	I6	$<(0,\infty),dec>$	$\Rightarrow$	$<0,dec>$
	I7	$<(0,\infty),dec>$	$\Rightarrow$	$<(0,\infty),dec>$
	I9	$<(0,\infty),dec>$	$\Rightarrow$	$<L^*,std>$
y	*I4*	$<(0,\infty),inc>$	$\Rightarrow$	$<(0,\infty),inc>$
	I8	$<(0,\infty),inc>$	$\Rightarrow$	$<L^*,std>$

Figure. 28. Possible transitions used in QSIM of ball throwing example.

Since the current state represents the time-interval ($t = t_0, t_1$) only I-transitions of Fig. 27 are applicable. *I2* and *I3* transitions for *y* are excluded since y is assumed to be finite ($y(t_1) \neq \infty$).

Next, for every constraint, generate all possible combinations of tuples (pair or triples) of transitions generated above. This will result in

DERIV(y,v)	*mark*	*DERIV(v,a)*	*mark*
(I4,I5)	*ic*	*(I5,I1)*	*ic*
(I4,I6)	*ic*	*(I6,I1)*	
(I4,I7)		*(I7,I1)*	
(I4,I9)	*iw*	*(I9,I1)*	*ic*
(I8,I5)	*iw*		
(I8,I6)			
(I8,I7)	*ic*		
(I8,I9)	*ic*		

For each constraint, eliminate all tuples which are inconsistent with that constraint. For example, tuple*(I4,I5)* would require *y* to continue increase while *v=0*, an obvious inconsistency, therefore this tuple must be eliminated. Similarly, those tuples marked with *ic*, and *iw* are eliminated by constraint consistency filtering and pairwise consistency filtering respectively. The constraint consistency filtering states that the directions-of-change tuple must be consistent with the constraint in the state resulting from the transition. The pairwise consistency filtering states that tuples in adjacent constraints must assign the same transition to the function they share. For example, since tuples *(I4,I9)* from constraint *DERIV(y,v)* and *(I9,I1)* from constraint *DERIV(v,a)* are adjacent, they must assign the same transitions to *v, I9.* However the transition *I9* to the tuple *(I4,I9)* from constraint *DERIV(v,a)* has already been deleted because of constraint inconsistent filtering, then the tuple *(I4,I9)* from constraint *DERIV(y,v)* is marked by *iw* an must be deleted.

Next, generate all possible global interpretations from the remaining tuples. A global interpretation is equivalent with a permitted state for the system. The two global interpretations formed from the remaining tuples are

	y	*v*	*a*
1.	*I4*	*I7*	*I1*
2.	*I8*	*I6*	*I1*

From the first interpretation, it can be inferred that *y* continues to increase while *v* continues to decrease, and *a* remains constant. From the second, we can reason that *y* becomes steady at *L**(new landmark value) while *v=0* and *a* remains constant.

Chapter 4
Envision

This approach was developed by Kleer, J.D and Brown, J.S.[1984]. They presented framework for modeling the generic behavior of individual components of a device which is based on the notions of qualitative differential equations (confluences) and qualitative state. Their approach was implemented in a program called the "Envision" Envision first establishes a qualitative model of the system, and then simulates the model in an attempt to derive the behavior of the system. One of the main features of this approach is that it considers a physical system to consists of a simpler components. Envision builds a qualitative model of a system from the following inputs: a list of components constituting the system, a topological description of how the components are interconnected, and a library of generic component-models.

4.1 Device structure and relations

There are three basic entities in the Envision approach: components, connections and materials. Physical behavior is accomplished by operating on and transporting materials such as water, air, and electrons. Components are constituents that can change the form and characteristic of material. Connections are simple constituents which transport material from one components to another and cannot change any aspect of the material within them. Some example of conduits are pipes, wires and cables. Variables are associated with the material flow. Components act on the materials causing the associated variables to change their values.

The components are building blocks. Each components is described by a generic qualitative model. The behavior of a system is derivable from the generic component models and the topological description of how components are interconnected.

Kleer and Brown always employ the {+,0,-} syntax for both variables and derivatives. They adopted a notation '$[]_Q$' to indicate the qualitative value of expression within brackets with respect to quantity space. More general, the simple quantity space of x<a, x=a, and x>a are denoted by $[x]_a$. Thus $[x]_0$=+ iff x>0, $[x]_0$=0 iff x=0, and $[x]_0$=- iff x<0. The addition and multiplication used to specify how different variables relates to each other are shown on Fig.29. Kleer and Brown denotes these constraints as confluences.

[X]+[Y]

[X] / [Y]	-	0	+
-	-	-	
0	-	0	+
+		+	+

[X]×[Y]

[X] / [Y]	-	0	+
-	+	0	-
0	0	0	0
+	-	0	+

Figure 29. Addition and multiplication operations.

A set of values satisfies a confluence if [Kleer84]: (1) Qualitative equality strictly holds using arithmetic table shown in Fig. 29 or (2) The left hand side of the confluence cannot be evaluated.

A set of values contradicts a confluence if (1) Every variable has a value, (2) The left hand side of the confluence can be evaluated (3) The confluences are not satisfied.

4.2 Structure to function

A device consists of physically disjoint parts which are connected together. The structure of the device is described in terms of its components and interconnections. Each component has type, whose generic model is available in the model library. The approach is to infer the behavior of a physical device from a description of its physical structure. The task is to determine the behavior of a device given its structure and access to generic models in the model library.

The requirement that models in the library components should be applicable for analysis of different systems places heavy demands for modularity in the component models. This attempt is taken care of through "No Function In Structure (NFIS)" principle. This states that the laws of the parts of device may not presume the functioning of the whole. Consider an electrical switch as an example. A model of a switch which states that the current is flowing when the switch is closed violates the NFIS principle because there are many closed switches through which current does not necessarily flow (such as, two switches in series). Current flows in the switch only if the switch is closed and there is a potential difference for current to flow.

A model for a component may be derived from a mathematical model of the component. The Envision procedure is somewhat simpler than constraints based approach since component are primitive units, which would rarely be modeled by anything more complicated than first differential equation.

The distinction between a component and a connection is important. In a reality, a connection is nothing but an abstraction of component which is modeled in an extremely simple manner. For example, consider a pipe connecting two containers. If it is desirable to take account of any drop in pressure in the pipe, it must be modeled as a component. Modeling the pipe as a connection merely makes Envision equate the pressures at both ends of the pipe. A connection has no capacity for storing materials and does not influence the variables of the materials passing through it.

Fig. 30 shows a simplified model of a valve. The relationships between area, A, the drop in pressure over the restriction, p, and the flow, q are expressed in Eq. 47 and Eq. 48, in which ∂p, ∂q and ∂A represent the qualitative derivatives of p,q and A respectively. Assuming that ∂A equals zero, ∂p and ∂q must always attain identical values. If the pressure increase, the flow must increase as well. This figure shows that the direction of flow is the same as the direction of the drop in pressure. This model presumes only one specific direction for the flow through the valve.

$$p + A = q \qquad (47)$$

$$\partial p + \partial A = \partial q \qquad (48)$$

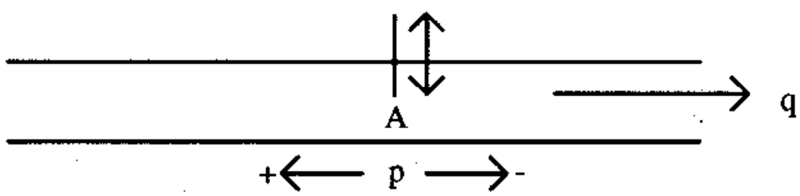


Figure 30. A simple valve mechanism.

The Eq. 47 and Eq. 48, are called pure confluences, since they do not relate variables and derivatives in the same expression. A more complicated model accounting for the possibilities of flow in both direction is given by

$$\partial p + (p \times \partial A) - \partial q = 0 \qquad (49)$$

The basic behavioral of the valve is that an increase in the area available for flow always reduces the absolute value of the pressure drop across the valve. Eq. 49, an example of a mixed confluence, relates variables and derivatives in the same expression. A mixed confluence can always be converted to a set of pure confluences by introducing more qualitative states on the component model. For example, the state for which $\partial p + (p \times \partial a) - \partial q = 0$ holds can be split into three states, thereby producing a pure model: one which $p>0$ so that $\partial p+\partial a-\partial q=0$, one which $p=0$, so that $\partial p-\partial q=0$, and one in which $p<0$ so that $\partial p-\partial a-\partial q=0$. By rewriting the model to avoid mixed confluences, the model in Fig. 31 is obtained.

4.3 Deriving behavior

In addition to the confluences describing the components of a physical system, the topology of the system constraints its behavior. Two principles, continuity and compatibility, are used to create additional confluences relating to topology. The continuity principle is applicable for stream like processes such as electrical current and flows of liquids It states that the sum of all material flows entering a connection is zero. Since connection are not allowed to accumulate material, this is a sound physical principle.

The compatibility principle is applicable for pressure-like variables. It is states that whichever path is taken between two points in the topology, the sum of the pressure drops along different path must be equal. For electrical systems, this analogous to Krichoff's voltage law. .

Mode	*State*		*Confluences*
OPEN	$A=[A_{max}]$	$p=0$	$\partial p=0$
WORKING+	$0<A<A_{max},,p>0$	$p=q$	$\partial p+\partial A-\partial q=0$
WORKING0	$0<A<A_{max},,p=0$	$p=q$	$\partial p-\partial q=0$
WORKING-	$0<A<A_{max},,p<0$	$p=q$	$\partial p-\partial A-\partial q=0$
CLOSED	$A=0$	$q=0$	$\partial q=0$

Figure 31. A valve model for Envision.

Applying both continuity and compatibility principle to the fullest extent possible produces a redundant set of confluences. This may provoke difficulties when deriving behavior. Therefore, Envision restricts the number of confluences generated from the topology. It includes only one continuity confluence for every component, and a compatibility confluence for every three conduits [Kleer84].

To understands the behavior of a device which is derived from its structures, Envision decomposes a device's behavior into two dimensions, one being interstate behavior and the other being intrastate behavior.

The intrastate behavior concerns the behavior within a qualitative state, the change of values of derivatives while the system remains in a specific state. Interstate behavior concerns the possible transitions between states. Consider a pressure regulator shown in Fig. 32 whose device topology is shown in Fig. 33.

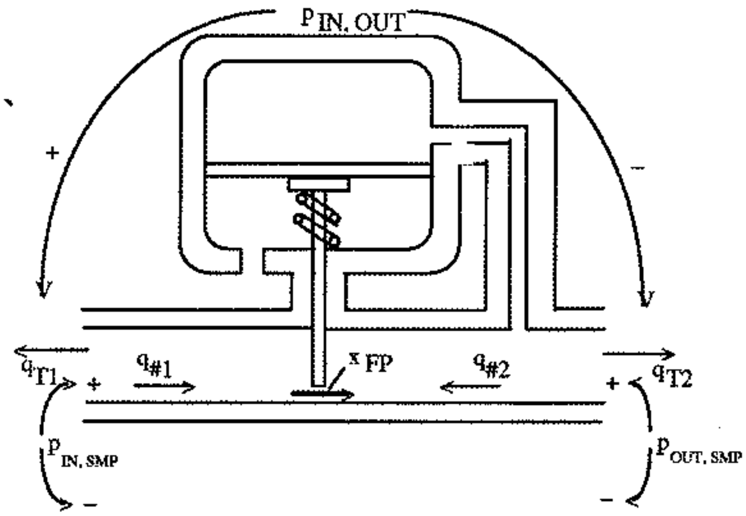


Figure 32. Pressure regulator [Kleer84].

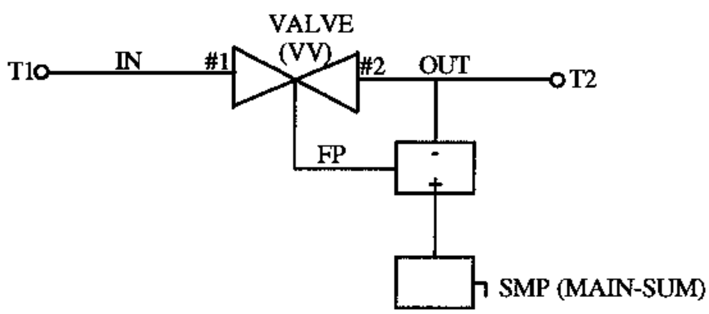


Figure 33. Device topology of the pressure regulator.

The specific confluences governing the behavior of the device can be constructed from the device topology, the library of component models and the composite device state. The pressure-regulator has two components, each of which is modeled by one confluence. The remaining confluences describe the behavior of the material. The confluence for the valve is

$$\partial p_{IN,OUT} - \partial q_{*1(VV)} + \partial x_{FP} = 0$$

where $\partial p_{IN,OUT}$ is the pressure drop from input to output, $q_{*1(VV)}$ is the flow from terminal #1 into the valve, and ∂x_{FP} is the position of the valve control and is qualitatively equal to the area available for flow. The confluence for the pressure-sensor is

$$\partial x_{FP} + \partial p_{OUT,SMP} = 0$$

where $P_{OUT,SMP}$ is the pressure at the output of the pressure-regulator. The position of the valve varies inversely with the output pressure. The compatibility of the pressure-regulator is

$$\partial p_{IN,OUT} + \partial_{OUT,SMP} - \partial p_{IN,SMP} = 0$$

There are three conduits: IN,OUT and SMP, and one component VV in the pressure-regulator that processes fluid. The continuity confluences for all conduits, except SMP, are

$$\partial q_{T2} + \partial q_{*2(VV)} = 0$$
$$\partial q_{T1} + \partial q_{*1(VV)} = 0$$
$$\partial q_{*1(VV)} + \partial q_{*2(VV)} = 0$$

A load is connected to pressure-regulator

$$\partial q_{T2} + \partial q_{\#2(VV)} = 0$$

Envision is also possible to model the qualitative values of the device variables. These values help to determine the state of devices and hence which derivative confluences apply. For example the qualitative value of the position of the valve varies with the qualitative value of input pressure. The model of their qualitative values is

$$[p_{IN,OUT}] - [q_{\#1(VV}] = 0$$

Since area is, by definition, always positive

$$[x_{FP}] = +$$

In state WORKING p>0

$$[p_{IN,OUT}] = +$$

Given input condition $[p_{IN,SMP}] = +$, $[p_{IN,OUT}] = +$ necessarily follows given the remaining confluences. The confluence for the pressure sensor is

$$[x_{FP}] + [p_{OUT,SMP}] = +$$

The continuity confluences are :

$$[q_{T2}] + [q_{\#2(VV)}] = 0,$$
$$[q_{T1}] + [q_{\#1(VV)}] = 0,$$
$$[q_{\#1(VV)}] + [q_{\#2(VV)}] = 0$$

The pressure-regulator is connected to a positive load:

$$[q_{T2}]-[p_{OUT,SMP}]=0$$

All confluences grouped by components and conduits are

$\partial p_{IN,OUT} - \partial q_{\#1(VV)} + \partial x_{FP} = 0$	$[x_{FP}]=+, \quad [p_{IN,OUT}]=+,$
	$[p_{IN,OUT}]+[q_{\#1(VV)}]=0$
$\partial x_{FP} + \partial p_{IN,SMP} = 0$	$[x_{FP}]+[p_{IN,OUT}]=+$
$\partial p_{IN,OUT} + \partial p_{OUT,SMP} - \partial p_{IN,SMP} = 0$	$[p_{IN,OUT}]+[p_{OUT,SMP}]-[p_{IN,SMP}]=0$
$\partial q_{T2} + \partial q_{\#2(VV)} = 0$	$[q_{T2}]+[q_{\#2(VV)}]=0$
$\partial q_{T1} + \partial q_{\#1(VV)} = 0$	$[q_{T1}]+[q_{\#1(VV)}]=0$
$\partial q_{\#1(VV)} + \partial q_{\#2(VV)} = 0$	$[q_{\#1(VV)}]+[q_{\#2(VV)}]=0$
$\partial q_{T2} - \partial p_{OUT,SMP} = 0$	$[q_{T2}]+[p_{OUT,SMP}]=0$

4.4 State diagrams

Constructing the state diagram is analogous to solving a set of simultaneous differential equations characterizing the behavior of a physical system. The process of constructing a particular state transition is equivalent to performing integration involving first-order derivatives operating in the original state [Kleer84]. Consider the operation of the pressure-regulator whose valve is modeled by the following confluences

OPEN	$A=[A_{max}]$	$p=0$	$\partial p=0$
WORKING	$0<A<A_{max}, p>0$	$p=q$	$\partial p+\partial A-\partial q=0$
CLOSED	$A=0$	$q=0$	$\partial q=0$

There are three states in this model: OPEN, WORKING, and CLOSED. As the valve is the only component of the pressure-regulator that has state, the composite device, like wise, has only three states [OPEN],[WORKING], and [CLOSED]. Suppose the input pressure is decreasing and the pressure regulator is in state [WORKING] then $\partial x_{FP}=+$, which increases A, the cross sectional area available for flow. This causes the possibility that $A<A_{max}$ may no longer hold. If this occurs, the state ends and the device transitions into a new one with valve pinned in state OPEN. The pressure-regulator does not provide regulation at all, in this state, because the input pressure is less than the regulator's target output pressure. The resulting state diagram is shown in Fig. 34.

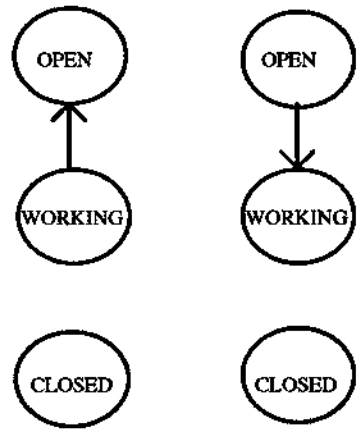


Figure 34. State diagram for the pressure regulator with decreasing input (a) and increasing input (b).

Consider, a model for the pressure-sensor which relates pressure directly to force $\partial p = \partial q$. The effect of a force on an object with mass is based on Newton's law *F=ma.* As mass is always positive unchanging, the confluence for this is

$$[F] = \partial v$$

where ∂v is the qualitative derivative of velocity. This mixed confluence equation can be modeled as three states

$$F > 0 \quad [F > 0], \quad \partial v = +,$$
$$F = 0 \quad [F = 0], \quad \partial v = 0,$$
$$F < 0 \quad [F < 0], \quad \partial v = -.$$

The model for a spring of an object with mass is derived from Hooke's law *F=kx* or *dF/dt = kv:*

$$\partial F = [v]$$

This is also a mixed confluence equation which can be modeled as three states

$$v > 0; \quad [\mathrm{v} > 0], \quad \partial \mathrm{F} = +$$
$$v = 0; \quad [\mathrm{v} = 0], \quad \partial \mathrm{F} = 0$$
$$v < 0; \quad [\mathrm{v} < 0], \quad \partial \mathrm{F} = -$$

The model for the valve is the same as before, except for the fact that ∂A is the time derivative of distance assuming area is proportional to the distance the valve is open) and hence it is a velocity *v*. The confluence is still mixed (*q* is flow rate)

$$\partial p - [v] - \partial q = 0$$

This mixed confluence is modeled as three states of a pure model:

$$v>0; \quad [0<A<A_{max}, \quad p<0, \quad v>0], \quad \partial p - \partial q = +$$
$$v=0; \quad [0<A<A_{max}, \quad p<0, \quad v=0], \quad \partial p - \partial q = 0$$
$$v<0; \quad [0<A<A_{max}, \quad p<0, \quad v=0], \quad \partial p - \partial q = -$$

The new device topology is shown in Fig.35. The device begin in state *v=0, F=0.* The confluences describing the behavior of valve's mass, valve and spring are obtained from the models just presented.

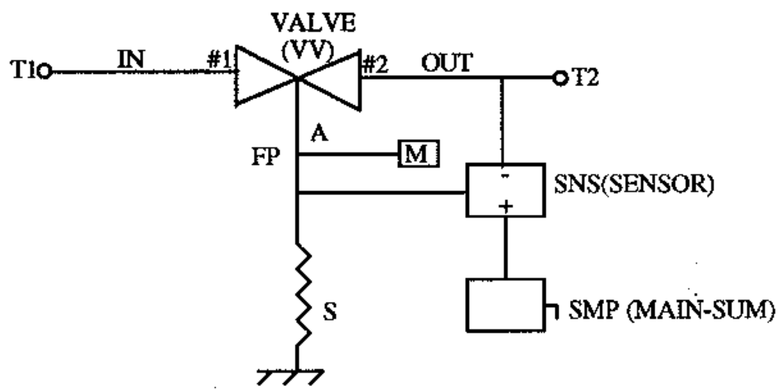


Figure 35. Device topology of the pressure -regulator with mass and spring.

As v= 0 the confluence describing the pressure regulator is

$$\partial p_{IN,OUT} - \partial q_{\#1(VV)} = 0$$

Since $F=0$, the applicable confluence is

$$\partial F_{\#1(S)} = 0$$

The confluence for the pressure-sensor is

$$\partial p_{OUT,SMP} + \partial F_{\#1(S)} + \partial F_{A(M)} = 0$$

The remaining confluences are similar to that of pressure-regulator without spring.

$$\partial F_{\#1(S)} + \partial F_{\#2(S)} = 0 \qquad \partial p_{IN,OUT} + \partial p_{OUT,SMP} - \partial p_{IN,SMP} = 0$$
$$\partial q_{T1} + \partial q_{\#1(VV)} = 0, \qquad \partial q_{T1} + \partial q_{\#2(VV)} = 0,$$
$$\partial q_{\#1(VV)} + \partial q_{\#2(VV)} = 0, \quad \partial q_{T2} - \partial p_{OUT,SMP} = 0, \quad \partial p_{IN,SMP} = +$$

Fig.36 shows the state diagram for the behavior of the-pressure regulator based on these models. Circles indicate momentary states, and squares indicate states that may exists for an interval of time.

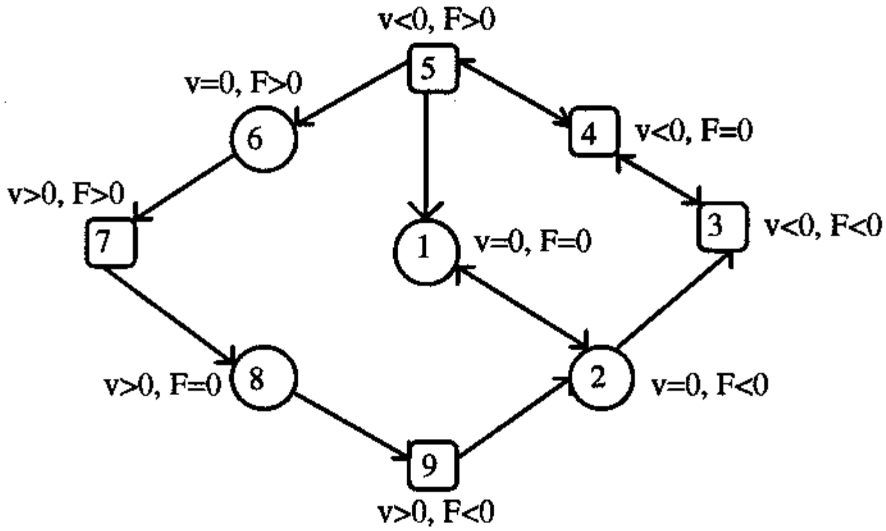


Figure 36. State diagram for pressure-regulator with continuing input signal, mass, spring and no friction.

The analysis given by Envision are:

In state 1, *v=0, F=0.*

Because $\partial F_{AM} = -$, M immediately changes state to *F<0.*

The device immediately changes state to 2: *v=0, F<0.*

In state 2, *v=0, F<0.*

Because $\partial v_{FP} = -$, S and VV immediately changes state to *v<0.*

In state 3, *v<0, F<0.*

The value of ∂F_{AM} is ambiguous.

If ∂F_{AM}=-, M immediately changes state to *F=0.*

The device may change state to 4: *v<0, F=0.*

In state 4, *v<0, F=0.*

The value of ∂F_{AM} is ambiguous.

If ∂F_{AM}=-, M immediately changes state to *F<0.*

If ∂F_{AM}=+, M immediately changes state to *F>0.*

Therefore, the device may change to state to one of states 5: *v<0, F>0*; 1: *v<0, F<0.*

In state 5, *v<0, F>0.*

The value of ∂F_{AM} is ambiguous.

If ∂F_{AM}=-, M may change state to *F=0.*

Because $\partial v_{FP} = +$, S and VV may change state to v=0.

Therefore, the device may change state to one of

6: *v=0, F>0;* 1: *v=0, F=0;* 4: *v<0, F=0.*

In state 6, *v<0, F>0.*

Because $\partial v_{FP} = +$, S and VV immediately changes state to *v>0.*

Because ∂F_{AM}=-, M may change state to *F=0.*

Therefore, the device must immediately changes state to one of :

8: $v>0, F=0$; 7: $v>0, F>0$.

In state 7, $v>0, F>0$.

Because ∂F_{AM}=-, M may change state to $F<0$.

The device may change state to 8: $v>0, F=0$.

In state 8, $v>0, F=0$.

Because ∂F_{AM}=-, M immediately changes state to $F<0$.

The device immediately changes state to 9: $v>0, F<0$.

In state 9, $v>0, F<0$.

Because $\partial F_{FP} = -$, S and VV may change state to $v=0$

The device may change state to 9: $v=0, F<0$.

Chapter 5

Dimensional Analysis

5.1 Dimensional and non-dimensional quantities

Every phenomenon in physics is determined by a series of variables, such as mass, energy, velocity, or stress, which take definite quantities in given cases. The quantities in an experiment result from a measurement on some system under controlled conditions, or can be derived from mathematical formulations. For example, the volume of a cube is a derived quantity since its determination can be reduced to the measurement of length. Quantities are called non-dimensional or abstract, when their values are independent of the system of measurement units. If the values of quantities depend on the systems of measurements units; such as the scale used, these quantities are called dimensional. Examples of dimensional quantities include length, time, energy, and moments. Charges, angles, the ratio of the square of length to an area, the ratio of energy to moment are the examples of non-dimensional quantities.

The subdivision of quantities into dimensional and non-dimensional is, to a certain extent, a matter of convention. If angle is defined as the ratio of subtended arc of a circle to its radius, the angular unit of measurement will be defined uniquely. If angle is measured only in radians in all systems, then it can be considered as non dimensional quantity. The same argument applies to acceleration. Acceleration is usually considered as a dimensional quantity with unit of length divided by time squared. The acceleration due to gravity g is equal to $9.81 \text{ m} / \text{sec}^2$. This constant can be selected as a fixed unit of measurement for the acceleration in all systems. Weight is the ratio of the magnitude of a measured acceleration to the magnitude of acceleration due to gravity. Any acceleration can be measured by a weight factor. It is a numerical value which will not vary when a transformation is made from one unit of measurement to another. The weight factor is a non-dimensional quantity. Non-dimensional quantities can be expressed in various numerical forms. In fact, the ratio of two lengths can be expressed as an arithmetic ratio, as a %.

The basic dimensional quantities in physics are length, mass, time and charge. The dimension of derived quantities can be expressed using the basic dimensional quantities. Length and time are expressed in terms of many different units besides the meter, foot, yard or second. Among these are the rod, furlong, mile and light-year for length and minute, month and century for time. For many purposes in physics the notion of the quantity under discussion may be defined in terms of some combination of fundamental quantities of length, time, mass, and electric charge. The dimension of the quantity is written as a symbol between brackets. For example, the dimension of length is indicated as $[L]$ and that of time as $[T]$. The dimension of a derived quantity such as velocity, which is distance divided by a time, is $\left[LT^{-1}\right]$. The unit of velocity may be meters per second or furlongs per century or whatever, but when we are concerned only with the fact that a velocity means a certain combination of a length and a time, we specify its dimension.

5.2 Applications of dimensional analysis

Dimensional analysis has been used to derive formulas in college physics, for purposes in engineering, and it has been applied to a fairly diverse set problems in engineering [Sedov59; Langhaar51].

Another application of dimensional analysis has been proposed by Bhaskar and Nigam[90]. Their work is to reason about physical systems or devices without explicit knowledge of the physical laws that govern the operation of such devices. Their method requires knowledge of the relevant physical variables and their dimensional representation. The dimensional representations of physical variables encode a significant amount of the physical processes, and they can be obtained without explicit knowledge of the underlying laws of physics. A variety of partial derivatives are computed to characterize the behavior of the system. These partials are used to reason qualitatively about the behavior of devices and systems.

To carry out the scope of dimensional analysis as a method for qualitative reasoning about physical systems, Baskar and Nigam developed "regimes". Regimes are a conceptual machinery for reasoning with dimensionless numbers, using elementary notions about partial differentiation. Their method is useful for tackling qualitative reasoning problems, such as the following [Bhaskar90]:

(a) to resolve, under certain circumstances, some ambiguities inherent in reasoning with a {+,0,-} qualitative calculus;

(b) to provide a comparative qualitative representation for a physical process;

(c) to derive the causal structure of a device's behavior, given the inputs and the outputs of a device.

Dimensional Analysis

The principle of dimensional homogeneity in all physics can be stated as follows. If

$$y_i = \sum_i a_i x_i$$

is a physical law or equation, then $a_i\ x_i$ must have the same dimensions as y_i. If the a_i are dimensionless constants, then each of x_i must have the same dimensions as y_i.

A dimensionless product π has the following form :

$$\pi_i = y_i \times (x_i\alpha_{di1} \ldots x_r\alpha_{dir})$$

where $\{x_i \ldots x_r\}$ are the repeating variables, $\{y_1 \ldots y_{n-r}\}$ are the performance variables and $\{\alpha_{dij} | 1 \le n-r, 1 \le j \le r\}$ are the exponents. The basis is the set of variables x_j that repeat in each π. A term π_i is a regime, which refers to a particular physical aspect of the system. An ensemble is a collection of regimes. If a system has n variables and a dimensional matrix of rank r, then the ensemble contains $n-r$ regimes. A pivot or a contact variable is a variable, x_k, that occurs in both π_i and π_j.

The regime π_i offers us a dimensionally homogeneous equation connecting the variable y_i with the basis variables $x_1 \ldots x_r$ as the product form

$$y_i = \pi_i \times x_1^{\alpha_{di1}} \ldots x_r^{\alpha_{dir}}$$

where $1 \le i \le n-r$. There are three kind of regimes that are used for reasoning about the behavior of a device or a physical system.

1. Intra-regime partials.
   They are used for examining how the variables within a regime are related to one another. From the expression obtained from a regime, π_i the change in y with respect to a basis variable x_j can be obtained by

$$\frac{\partial y_i}{\partial x_j} = -\frac{\alpha_{dij} y_i}{x_j}$$

2 Inter-regime partials.
   They are used to relate performance variables y_i and y_j that occur in the regimes π_i and π_j respectively. The inter-regime partial models the changes in y_i and y_j in response to a change in contact variable x_p. The notation for inter-regime partial is

$$[\frac{\partial y_i}{\partial x_j}]^{x_p} = \frac{\frac{\partial y_i}{\partial x_p}}{\frac{\partial y_j}{\partial x_p}}$$

   From the regime $\pi_i, \frac{\partial y_i}{\partial x_p} = -\frac{\alpha_{dip}}{x_p}$ and from the regime $\pi_j, \frac{\partial y_j}{\partial x_p} = -\frac{\alpha_{djp}}{x_p}$ thus

$$[\frac{\partial y_i}{\partial x_j}]^{x_p} = \frac{\alpha_{ip}}{\alpha_{jp}}$$

3. Inter-ensemble partials.
   Inter-ensemble partials are used to reason about the behavior of a device or systems consisting of coupled components or subsystems.

When dealing with a device with several components, the ensemble of each component or subsystem should be obtained. In order to reason about the behavior of the entire device, we need to reason about coupling, which manifests itself in terms of coupling quantities which are used to obtain a coupling regime.

Consider two ensembles A and B, regimes π_{Ai} and π_{Bj} belonging to these ensembles and variables y_{Aj} and y_{Bj} that are described by the regimes. The notation for interensemble partials is

$$[\frac{\partial y_{Ai}}{\partial y_{Bj}}]^{\pi_c}$$

where π_C is the contact regime.

When reasoning about the behavior of a system, the objective is to compute the direction of change of a performance variable in response to a change in the basis variable(s). To reason about change in a performance variable y_i as a result of a change in some variables x_j, the following can be used:

If x_j is in the basis and occurs in π_i, then use intra-regime partials.

If x_j is in the basis but not in π_i, then reason using chains of inter-regime partials.

If x_j is not in the basis, then use the appropriate inter-regime partial linking π_i and π_j.

Reasoning about physical systems with dimensional analysis proceeds along the following lines:

1 List the n variables that characterize the problem, and write their dimensional representations; let r be the number of distinct dimensions that are used.

2 By Buckingham's theorem [Langhaar51] *(n-r)* dimensionless products, π , can be calculated as follows:

   a. Select r variables to be the basis, $x_i \dots x_r$ and let δ_i be the dimensional of x_i

   b. Each π_i is represented in the form

$$y_i \times x_i^{\alpha_{di1}} \dots x_r^{\alpha_{dir}}$$

   Replacing each x and y by its corresponding dimensional representation δ leads to the expression $\partial_i \, \partial_i^{\alpha_{di1}} \dots \partial_r^{\alpha_{dir}}$. As π_i is dimensionless, the exponents of each dimension should add up to zero. The values of the exponents α_i can be determined, and hence the expressions for π_i.

3. Use π to reason about system and component behavior by computing partial derivatives of the form $\frac{\partial y_i}{\partial x_i}$ and $[\frac{\partial y_i}{\partial y_j}]^{x_k}$.

The following examples of analyzing the behavior of physical systems through regime analyses are discussed by Bhaskar and Nigam [Bhaskar90].

First, consider a horizontal oscillation of a block and spring system. Once the block is displaced from its rest position, it oscillates about this position (see Fig.37). How do we reason about the behavior of this system in terms of the time period of oscillations?

Figure 37. Oscillation of a frictionless block [Bhaskar90]

The quantities that characterize the system and their associated dimensions are

time period	$t\,[T]$
mass	$m[M]$
spring constant	$K\,[MT^{-2}]$

There are three quantities, $n = 3$, and two dimensions *(M and T)*, $r = 2$; thus there is $(n-r)$ π , i.e.a single π .

$$\pi = tm^{\alpha_{d1}} K^{\alpha_{d2}}$$
$$\pi = t[M]^{\alpha_{d1}} [MT^{-2}]^{\alpha_{d2}} \qquad (50)$$

π is dimensionless, and the exponents of the M and T dimensions should each add up to zero. Thus we have equations:

$$\text{M-homogeneity} : \alpha_{d1} + \alpha_{d2} = 0$$
$$\text{T-homogeneity} : 1 - 2\alpha_{d2} = 0$$

Solving those equations, we have $\alpha_{d1} = -\frac{1}{2}$ and $\alpha_{d2} = \frac{1}{2}$
Substituting these values into equation (50) results in

$$\pi = tm^{-\frac{1}{2}} K^{\frac{1}{2}} \qquad (51)$$

Using equation (50), we can reason about the behavior of the system in terms of the time period of oscillation by computing the value of intra-regime partials; $\frac{\partial t}{\partial m}$ and $\frac{\partial t}{\partial K}$. The solutions are $\frac{\partial t}{\partial m} > 0$ and $\frac{\partial t}{\partial K} < 0$. Hence, we can reason that a heavier mass will oscillate with a larger time period, while a stiffer spring will cause the mass to oscillate with a smaller time period. To know the effect of a change in K and m on the system, variables K and m are grouped together as $\frac{K}{m}$ or $\frac{m}{K}$ and use the partials $\frac{\partial t}{\partial(\frac{K}{m})}$ or $\frac{\partial t}{\partial(\frac{m}{K})}$. The result of $\frac{\partial t}{\partial(\frac{K}{m})}$ is negative, and that of $\frac{\partial t}{\partial(\frac{m}{K})}$ is positive. We can reason that if m increases and K decreases, t (the time period of oscillation) will be larger.

The second example of using regime analysis is the motion of a projectile. Consider a projectile which is shot vertically with initial velocity v. It rises a certain height and then falls back to the earth. In order to be more realistic, we include two more variables; a surface area and an air resistance which is force per unit area per unit mass. The objective is to reason about the height attained and the times of rise and fall. The physical quantities characterizing the system along with its dimensions are

time of rise	t_1 $[T]$
time of fall	t_2 $[T]$
acceleration due to gravity	g $[LT^{-2}]$
maximum height	h $[L]$
initial velocity	v $[LT^{-1}]$
air resistance	r_a $[L^{1}T^{-2}]$
surface area	S $[L^2]$

There are seven physical quantities, and two dimensions (L and T); thus we have five πs, $\pi_1 ... \pi_5$. The performance variables are t_1, t_2, h, r_a and S, while the basis variables are g and v. We have equations

$$\pi_1 = t_1 g^{\alpha_{d11}} v^{\alpha_{d12}} \tag{52}$$
$$\pi_2 = t_2 g^{\alpha_{d21}} v^{\alpha_{d22}}$$
$$\pi_3 = h g^{\alpha_{d31}} v^{\alpha_{d32}}$$
$$\pi_4 = r_a g^{\alpha_{d41}} v^{\alpha_{d42}}$$
$$\pi_5 = t_5 g^{\alpha_{d51}} v^{\alpha_{d52}}$$

Substituting the dimension mentioned above into each equation, and using the principle of dimensional homogeneity, the resulting πs can be determined. For example, after substituting the dimensions into Eq. (51), we have

$$\pi = [T][LT^{-2}]^{\alpha_{d11}}[LT^{-1}]^{\alpha_{d12}} \tag{53}$$

As π is dimensionless, the exponent of the L and the T dimensions should each add up to zero. Thus we have equations

$$\text{T-homogeneity} : 1 - 2\alpha_{d11} - \alpha_{d12} = 0$$

$$\text{L-homogeneity} : \alpha_{d11} + \alpha_{d12} = 0$$

Solving these equations for α_{11} and α_{12} results in

$$\pi = t_1 \frac{g}{v} \tag{54}$$

Similarly, the values of other πs can be determined which result in

$$\pi_2 = t_2 \frac{g}{v}$$
$$\pi_3 = h \frac{g}{v^3}$$

$$\pi_4 = r_a \frac{v}{g^3} \tag{55}$$

$$\pi_5 = S \frac{g^2}{v^4} \tag{56}$$

The two dimensionless products Eq.(54) and (56) can be combined into a single product :

$$\pi_4 = r_a \frac{S}{g} \tag{57}$$

To reason about a performance variable t_1, as a result of a change in $r_a S$, we obtain the inter-regime partial using Eq.(54) and Eq.(57), i.e.

$$[\frac{\partial t_1}{\partial (rS)}]^g = \frac{[\frac{\partial t_1}{\partial g}]}{[\frac{\partial (rS)}{\partial g}]}$$

Calculating intra-regime partials from Eq.(54) and Eq.(57) results in

$$\frac{\partial t_1}{\partial g} < 0$$

$$\frac{\partial (rS)}{\partial g} > 0$$

Hence, the inter-regime partial

$$[\frac{\partial t_1}{\partial (rS)}]^g < 0$$

From this inter-regime partial we can reason that $\delta(r_a S) > 0$ leads to $\delta t < 0$. Thus a projectile with a larger surface area will have a shorter rise time. Also a projectile traveling in a medium with greater air resistance will have a shorter rise time. The projectile ensemble is shown in Fig.38.

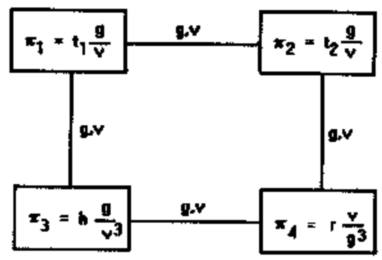


Figure 38. The projectile ensemble.

Consider a pressure regulator as shown in Fig. 39 as the third example of using dimensional analysis. The device is analyzed in terms of two components -a pipe with an orifice and a spring valve assembly. The function of the device is to maintain a constant pressure at the output.

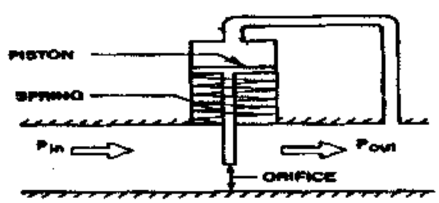


Figure 39. The pressure regulator [Bhaskar90].

Pipe orifice ensemble

The variables characterizing a pipe orifice ensemble are as follows

outlet pressure p_{out} $[ML^{-1}T^{-2}]$
orifice flow rate Q $[L^3T^{-1}]$
inlet pressure p_{in} $[ML^{-1}T^{-2}]$
orifice opening A_{open} $[L^2]$

fluid density ρ $[ML^{-3}]$ ρ

We have five quantities and three dimensions; choosing p_{in}, A_{open} , and ρ as the basis variables leads to the followings πs

$$\pi_{A1} = Qp_{in}^{\alpha_{d11}} A_{open}^{\alpha_{d12}} \rho^{\alpha_{d13}} \tag{58}$$

$$\pi_{A2} = Qp_{out} p_{in} A_{open}^{\alpha_{d12}} \rho^{\alpha_{d13}}$$

Applying the principle of homogeneity to Eq.(58), the exponents of the L, the M and the T dimensions should each add up to zero

L-homogeneity : $\alpha_{d11} + \alpha_{d13} = 0$
M-homogeneity : $3 - \alpha_{d11} + 2\alpha_{d12} - 3\alpha_{d13} = 0$

T-homogeneity : $-1 - 2\alpha_{d11} = 0$

Solving these equations for $\alpha_{d11}, \alpha_{d12}$ and α_{d13} results in

$$\pi_{A1} = \frac{Q\rho^{\frac{1}{2}}}{A_{open} p_{in}^{\frac{1}{2}}}$$

Similarly, π_{A2} can be determined

$$\pi_{A2} = \frac{p_{out}}{p_{in}}$$

The resulting intra-regime partials $\frac{\partial Q}{\partial p_{in}}, \frac{\partial Q}{\partial A_{open}}$, and $\frac{\partial p_{out}}{\partial p_{in}}$ are all positive. Hence, the inter-regime partial $[\frac{\partial p_{out}}{\partial Q}]^{p_{in}}$ is also positive. We can reason that if the input pressure p_{in} increases, the flow rate Q and the outlet pressure p_{out} will increase. Similarly, if the orifice opening A_{open} decreases then Q will also increase. Since the inter-regime partial $[\frac{\partial p_{out}}{\partial Q}]^{p_{in}}$ is positive, an increase in Q will lead to an increase in p_{out}.

Spring valve ensemble

Consider the spring valve assembly, which has pressure applied to a piston that is connected to a spring. The quantities that characterize the system are

spring displacement $x\ [L]$
pressure $P\ [ML^{-2}\ T^{-2}]$
spring constant $K\ [MT^{-2}]$

There are three quantities, and three dimensions. In order to obtain π_{B1}, the dimensions of $[MT^{-2}]$ appear in both *P* and *K* can be seen as a single dimension. Choosing *P* and *K* as the basis leads to the following

$$\pi_{B1} = xP^{\alpha_{11}}K^{\alpha_{12}}$$

Substituting the dimensions and applying the principle to this equation, leads to the resulting π_{B1}

$$\pi_{B1} = x\frac{P}{K}$$

From this ensemble the intra-regime partial is $\frac{\partial x}{\partial P}$ is negative. Hence we can reason that if the pressure *P* applied on the piston increases, *x* decreases.

There are two coupling regimes. The one, π_{C1}, comes from the connection that transmits the outlet pressure in the pipe to the piston in the spring valve assembly, the other one, π_{C2} encodes the geometric constraint that motion of the piston affects the orifice opening. The two regimes are

$$\pi_{C1} = \frac{P}{p_{out}}$$

$$\pi_{C2} = \frac{x}{A_{open}^{\frac{1}{2}}}$$

From these regimes, both intra-regime partials $\frac{\partial P}{\partial p_{out}}$ and $\frac{\partial x}{\partial A_{open}}$ are positive. Hence, if p_{out} increases, *P* will increase; also if the spring is compressed, the orifice will be reduced. Based on the ensembles developed above, we can reason that the pressure regulator exhibits the following behavior :

From π_{A2}, an increase in p_{in} leads to an increase in p_{out}. This increase in p_{out} leads to an increase in P in the spring valve ensemble (from π_{C1}). From π_{B1}, an increase in *P* leads to a decrease in *x*. The decrease in *x* causes A_{open} to decrease (from π_{C2}). In the pipe orifice ensemble, the decrease in A_{open} causes *Q* to decrease. Finally the decrease in *Q* leads to a decrease in p_{out} (from inter-regime partial $[\frac{\partial p_{out}}{\partial Q}]^{p_{in}}$). From these regimes, both intra-regime partials $\frac{\partial P}{\partial p_{out}}$ and $\frac{\partial x}{\partial A_{open}}$ are positive. Hence, if p_{out} increases, *P* will increase, also if spring is compressed, the orifice size will reduce. The inter-ensemble analysis of the pressure regulator is depicted in Fig. 40.

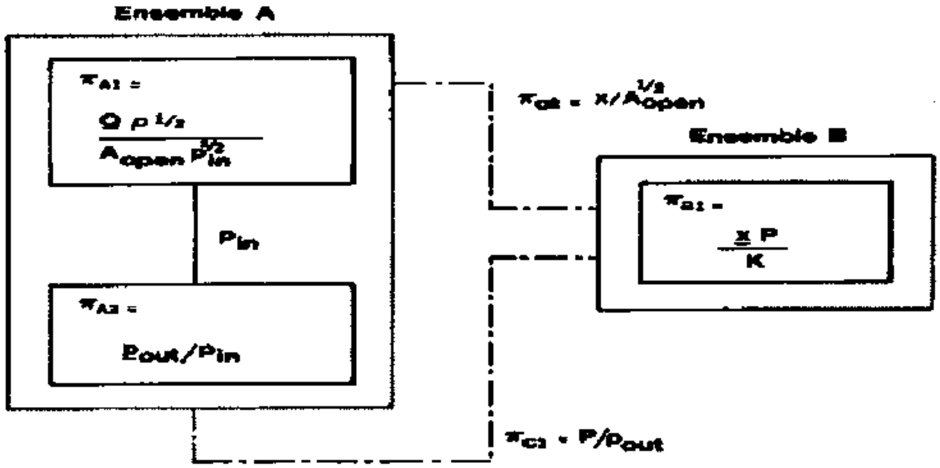


Figure 40. Inter-ensemble of the pressure regulator [Bhaskar90].

Chapter 6

Qualitative Process Theory

Research in qualitative reasoning was originally motivated by the fact that human beings function well in their physical surrounding, without resorting to quantitative computations. Researchers in qualitative reasoning, therefore, argued that it should be possible to formalize knowledge providing humans with this capability by means of pure symbolic methods. Qualitative reasoning provides valuable insight and methods for analyzing physical systems. A pure qualitative approach developed for analyzing physical systems is Qualitative Process Theory (QPT) [Forbus84]. Forbus presented a QPT which defines a simple notion of physical process which is useful as a language to write dynamic theories. It provides a way of specifying processes and their effects in a way that allows both deduction of what process occurs and how they might change. Processes represent the activities that occur in physical situations. The physical situation is described by a collection of objects, their properties, the relation between them and the processes that are occurring.

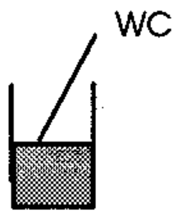
6.1 Quantity

The state of a physical process system is characterized by values of a set of variables. In the QPT, variables are denoted by quantities. The objects which have continuous parameters such as mass, temperature, and pressure are represented by quantities.

Processes affect objects in various ways. Many of these can be modeled by changing parameters of an object, properties whose values are drawn from a continuous range [Forbus84]. Examples of parameters that can be represented by quantities include the pressure of a gas inside a container, one-dimensional position, the temperature of some fluid, and the magnitude of the net force on an object.

The predicate *Quantity-Type* indicates that a symbol is used as a function that maps objects into quantities. To express that an object has a quantity of a particular type, QPT uses the relationship *Has-Quantity.* For example, some quantities that are used in representing the liquid in a cup are shown in Figure 41.

The quantities consists of two parts, an amount and a derivative, each of which are numbers. The derivative of a quantity can in turn be the amount of another quantity. For example, the derivative of one-dimensional position is the amount of one-dimensional velocity. Functions A and D map from quantities to amount and derivative respectively.

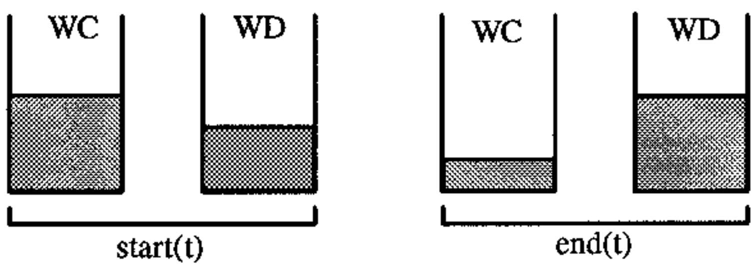


Quantity-Type(amount-of)	Has-Quantity(WC,amount-of)
Quantity-Type(level)	Has-Quantity(WC,level)
Quantity-Type(pressure)	Has-Quantity(WC,pressure)
Quantity-Type(volume)	Has-Quantity(WC,volume)

Figure 41. Some quantities representing the liquid in a cup.

Every number has both sign and magnitude. The functions *s* and *m* map from numbers to sign and magnitude, respectively. The combinations of these functions that select parts of quantities are denoted by A_s, A_m, D_s, D_m where

A_m is magnitude of the amount
A_s is sign of the amount
D_m is magnitude of the derivative, or rate
D_s is sign of the derivative



(M Am[amount-of (WC)] start(t)) > (M Am[amount-of(WD)] start(t))
(M Am[amount-of (WC)] end(t)) < (M Am[amount-of(WD)] end(t))
(M Ds[amount-of (WC)] t) = -1
(M Ds[amount-of (WD)] t) =1

Figure 42. M describes values at different times.

Numbers, magnitude and signs take on values at a particular time. The function *M* is used to refer to the value of a quantity or some part of it at a particular time t. For example, to indicate the value of level W at time *t*, QPT uses an expression

(M level(W) t)

while to indicate the increment of the level of fluid in the tank W at time *t*, it uses expression

$$(M\ D_s\ [level(W)\ ]\ t) = 1$$

This statement is read as "the value of the sign of the derivative of level W at time *t* is 1". An example of using the above notations is shown in Fig. 42. Some facts about the two containers is expressed as a relationship between their quantities.

6.2 Individual View

Objects can exist or not, and their properties can change dramatically. For example water can be poured into a cup and then drunk and springs can be stretched and broken. To model these kinds of changes which depend on values of quantities, QPT introduces *Individual Views* which describe objects and their states. The *Individual Views* consist of (1) *Individuals,* the objects which must exist before it is applicable, (2) *Quantity Conditions* which are statements about inequalities between quantities involving the individual and statements about whether or not certain other *Individual Views* hold, (3) *Preconditions* which are still further conditions that must be true for a view to hold, and (4) a collection of *Relations* that are imposed by that view. Fig. 43 illustrates a simple description of the fluid in a cup.

For every collection of objects that satisfies the description of the individual for a particular type of individual view, there is a view instance (VI), that relates them. The status of VI is active whenever the preconditions and quantity conditions are satisfied, otherwise the status of VI is inactive. If a VI is active the specified relations hold between its individuals.

```
Individual View Contained-Liquid(p)
  Individuals:
      con a container
      sub a liquid
  Preconditions
      Can-Contain-substance(con,sub)
  Quantity conditions:
      Am[amount-of-in(sub,con)]>ZERO
  Relations:
      There is p ∈ piece-of-stuff
      amount-of(p)=amount-of-in(sub,con)
      made-of(p)=sub
      container(p)=con
```

Figure 43. Individual view describe objects and states of objects [Forbus84]

This figure shows a simple description of fluid contained in a cup. It says that whenever there is a container that contains some liquid substance then there is a piece of stuff in that container.

6.3 The quantity space

In QPT, variables are denoted by quantities. The permitted values for quantities are defined in its quantity space. A quantity space is defined as a set of symbols defining the values a variable may attain, and a set of partial orderings between those symbols.

QPT allows quantity spaces with an arbitrary number of partialy ordered symbols. The order of the symbols may change during analysis. Consider a system as shown in Fig. 45. Assume that the initial situation is as shown in the figure.

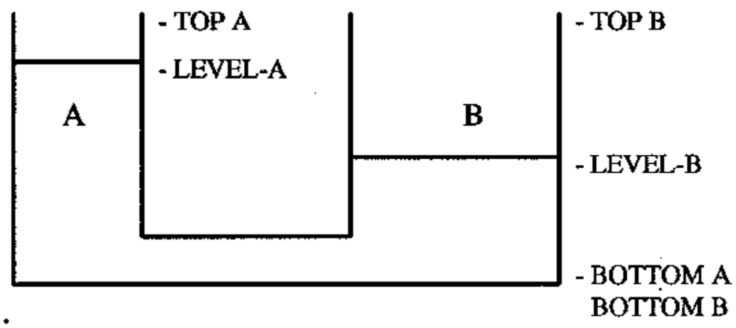


Figure 44. Connected liquid containers.

The initial quantity space may look like the one shown in Fig. 45. Two elements that are ordered and with no elements in the ordering known to be between them are called neighbors.

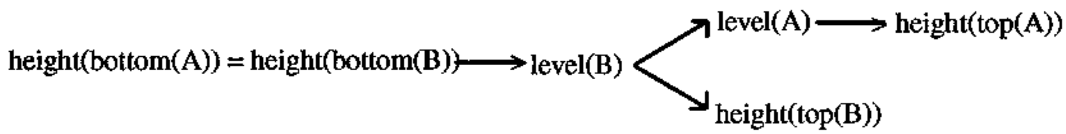


Figure 45. Graphical notation for quantity space.

In Fig. 45, level(B) has neighbors height(top B) and level(A) but not height(top A). This partial ordering expresses that it is known that level(B) is less than both level(A) and top (A), but it is initially not known whether level(A) is less than top(B)

6.4 Functional relationships

QPT introduced the concept of direct and indirect influences to describe interactions between quantities. Direct influences are used to describe dynamic interactions whereas indirect influences are used to describe functional relationships.

There are two kinds of direct influences, positive and negative. These are denoted as I+(q,n) and I-(q,m), pronounced *n influenced by q positively* and *m influenced by q negatively,* respectively.

The direct influences describe a relation between the influencing numbers, *m* and *n*, and the influenced quantity, q. The value of the derivatives of q are to be computed as the qualitative sum of all numbers influencing it. For *q* above, *n* would contribute positively, while *m* would be added with a negative sign. The influencing numbers, *m* and *n,* are usually specified as the value of some quantity.

Indirect influences are written $q \propto_{Q+} m_1$ and $q \propto_{Q-} m_2$. This is pronounced *q* is *qualitatively proportional to* m_1 and *q* is *inversely qualitatively proportional to* m_2, respectively. This means that there exist a functional relationship $q = f(\ldots, m_1, m_2, \ldots)$ such that the qualitative derivative of *q* equals that of m_1 when everything else is held constant. Similarly, the direction of change for *q* opposes that of m_2 when m_2 is changing while everything else is held.

First note that indirect influences are unidirectional. This implies that the influences above may be used to infer something about how *q* changes when m_1 or m_2 is known to be changing. Indirect influences can never be used to directly derive the value of a quantity, for example the value of *q* cannot be derived directly from the known values of m_1 and m_2 . Only the derivative may be propagated. Finally, if m_1 and m_2 were increasing simultaneously, they would influence *q* in opposing directions. In general nothing can be inferred about the direction of the change in *q* in this case. However, if m_1 was increasing while m_2 was decreasing, their influences would work in the same direction which would allow us to infer that *q* was decreasing.

The notion of correspondence allows information about inequalities to be transferred across qualitative proportionalities ($\propto_Q$'s). Correspondences are the means of mapping value information (inequalities) between quantity spaces via $\propto_Q$ [Forbus84]. For example, a correspondence of the force exerted by an elastic band is zero when it is rest is

Correspondence(A[internal-force(band)],ZERO],
(A[length(band)], A[rest-length(band)]))

If the length of the band described above is greater than its rest length the internal force is greater than zero (see Fig. 46). The rough shape of the graph below is determined by $\propto_Q$, and the equality between the two points is determined by the correspondence.

Any quantity may simultaneously influence several other quantities, some directly, others indirectly. A quantity may simultaneously be directly influenced by any number of other quantities. Similarly, a quantity may be influenced indirectly by any number of other quantities. No quantity may simultaneously be both directly and indirectly influenced.

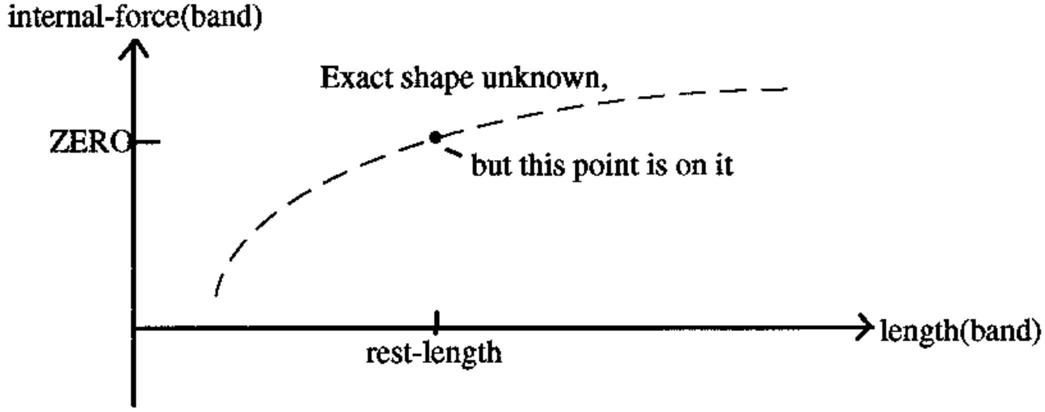


Figure 46. Graph of an example of correspondence.

6.5 Landmark, time, episodes and histories

A value of a set of variables typically characterize the state of a physical system. In the qualitative approach, the values assigned to the variables should reflect some kind of qualitative difference. The set of values which a given variable may attain is specified as a set of symbols, and the meaning of these symbols are defined in a quantity space. For example, take the temperature of an amount of water, T. A viable quantity space is shown in Fig. 47.

quantity value	corresponding region in real line
F_{qv}	$T < 0$
M_{qv}	$T = 0$
N_{qv}	$0 < T < 100$
B_{qv}	$T = 100$
V_{qv}	$100 < T$

Figure 47. A quantity space for the temperature of water.

The set of symbols tabularized in the above figure captures some of the characteristic properties of water as a function of temperature. Only the symbols M_{qv} and B_{qv} are strictly required. Every other value can be expressed in terms of M_{qv} and B_{qv}, i.e. N_{qv} is equal to the open interval (M_{qv}, B_{qv}). The symbol M_{qv} and B_{qv} are denoted as landmark values, i.e. values where something important happens. Landmark values are points on the real line, while the other qualitative values are intervals between landmarks.

A distinguished time point is a point in time where "something important happens". Typically, the value of a variable passes a landmark. Thus, each variable may be associated with a sequence of a distinguished time points. A system described by several variables is associated with the sequence of distinguished time points where one of its variables changes its value, i.e. the union of distinguished time points for the variables of the system.

Forbus represents how things change through time using the notion of history. A history is made up of *episodes* and *events*. They are different in temporal aspects. An episode is equivalent to the interval between two distinguished time points.

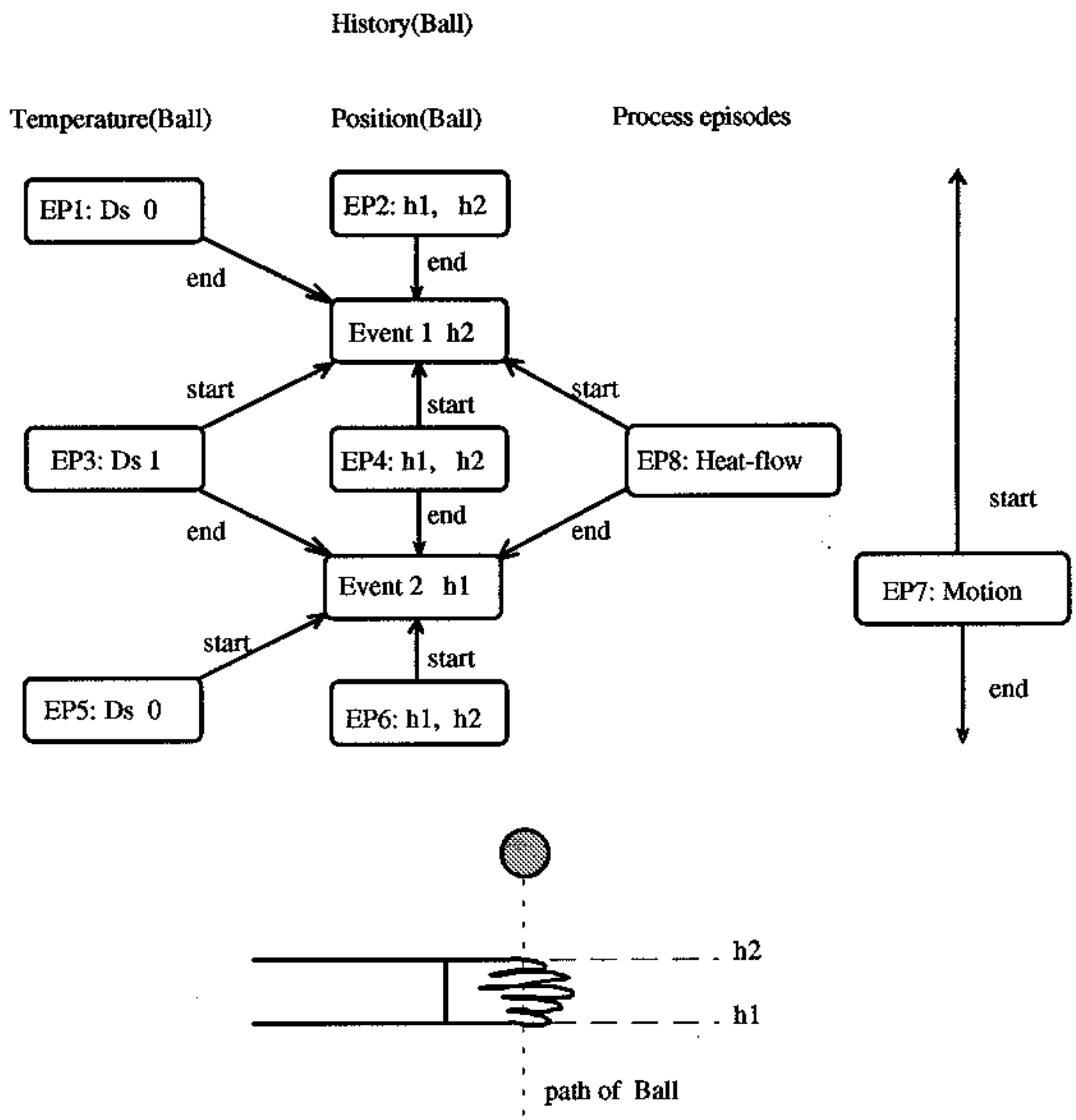


Figure 48. History for a ball dropping through a flame.

Episodes have a *start* and *end* which are events that serve as its boundary. Episodes occur over an interval of time; events usually last for an instant. The start of some piece of history is directly after the end of the previous without any time interval. This enables us to say, for example, the episode of heating water on a stove is ended by the event of the water reaching its boiling temperature; during the episode the temperature was below the boiling point. Fig. 48 illustrates the history for a ball dropping through a flame. A history is a sequence of episodes.

6.6 Process

A physical process is a thing that acts through time to cause changes. The concept of a process is used to capture the effects of dynamic interaction in a system. Take a pan on a hot plate as an example. The mere fact that a container is placed on a hot-plate does not entail that a flow of heat will occur. But if the temperature in the plate is greater than temperature in the container, a flow of heat will arise. This will cause the temperature in the container to rise and the temperature in the plate to decrease. The notion of a process allows us to formalize conditions for when a process will exist as well as the consequences of its activity. A process is specified by five parts :

Individuals : description of the objects which the process acts on.

Quantity Conditions : Inequality statements and status assignments which must be true for the process to be active.

Preconditions : Statements other than Quantity Conditions that must be true for the process to be active.

Relations: The relationships between the individuals which hold when the process is active.

Influences : Descriptions of the direct effects of the process.

Figure 49 depicts the process of heat flow.

process heat flow(src,dst,path)

Individuals:
  src an object, Has-Quantity(src,heat)
  dst an object, Has-Quantity(dst,heat)
Precondition
  Heat-Aligned(path)

Quantity-Condition:
  A[temperature(src)]>A[temperature(dst)]

Relations:
  Let flow rate be quantity
  As[flow-rate] > ZERO
  Flow-rate α_{Q+} (temperature(src)–temperature(dst))
Influences:
  I-(Heat(src),A[flow-rate])
  I+(Heat(dst),A[flow-rate])

Figure 49. Physical process definition for heat flow.

The difference between quantity conditions and preconditions is that the quantity conditions are those statements that can be expressed solely in QPT. They are inside the theory, in the sense that the truth values of these conditions may change when QPT assigns new values to the quantities during the qualitative analysis, such as requiring the temperature of two bodies to be different for heat flow to occur as a prerequisite to boiling. Preconditions are those factors that are outside the theory, in the sense that the truth values for preconditions may change independently of the analysis in the QPT. For example, consider a pipe with an operating valve in it. A fluid will only flow in the pipe when the valve is open; this should be stated as a precondition for the fluid flow.

Relations and influences formalize how an active instance affects the system. The relations usually describe indirect effects via functional relationships between quantities, such as the flow rate in heat flow being qualitatively proportional to the difference in the temperature of the source and destination. Indirect influences and logical statements, which hold when the instance is active are also specified here.

The influence field, found in the definition of process only, is used to specify dynamic interaction by means of direct influences between quantities. These quantities are either associated with one of the objects specified in the individuals field or with the instance. A quantity must be either directly or indirectly influenced at a particular time. There is no quantity that is both directly and indirectly influenced at once.

Like an individual view, a process is a time dependent thing, except that it has influences. For every collection of objects that satisfy the individual specifications for a particular process, there is a process instance (PI) that relates them. A process instance is active whenever both the preconditions and quantity conditions are true. This active process instance represents the process acting between these individuals. A process instance has a status Active or Inactive according to whether or not the particular process it represents is acting between its individuals.

6.7 Basic deductions

QPT representation allows several deductions as categorized below [Forbus84].

6.7.1 Finding possible processes

All changes in physical systems are caused directly or indirectly by processes. This is a central assumption of QPT[Forbus84]. As a consequence, a vocabulary of processes that occur in a domain must be incorporated in the physics for the domain. This process vocabulary can be viewed as specifying the dynamics theory for the domain.

This process vocabulary determines the types of processes that can occur. Given a collection of individuals and a process vocabulary, the individual specifications from the elements in the process vocabulary must be used to find collections of individuals that can participate in each kind of process. These process instances (PI) represent the potential processes that can occur between set of individuals[Forbus84]. A similar deduction is used for finding view instances.

6.7.2 Determining activity

A status of process instance, whether active or inactive according to whether or not the particular process it represents acting between its individuals. A status instance can be assigned to each process instance for a system by determining whether or not the preconditions and the quantity conditions are true. The process and the view structures are defined as the collection of the active PI's and VI's of a system, respectively. This process structure represents what is happening to the individuals in a particular system.

6.7.3 Determining changes

Most of changes in an individual are represented by the D_s values for its quantities. A D_s value of -1 indicates the quantity is decreasing, a value of 1 indicates the quantity is increasing, and a value of 0 indicates that it remains constant. There are two ways for a quantity to change; caused directly by a process or influenced indirectly by $\propto_Q$. Determining the value of D_s for a quantity is called resolving its influences [Forbus84].

If a quantity is directly influences, resolving that quantity requires adding up the influences. If all influences have the same signs, then the D_s value is simply that sign. Since there is no numerical information, ambiguitis can arise. Sometimes an answer can be found by sorting the influences into positive and negative sets, and using inequality information to prove that one set of influences must, taken together, be a larger set then the other set. However there is not always enough information to do this, so direct influences are not always resolvable[Forbus84].

Resolving an indirectly influenced quantity involves gathering the D_s statements that specify it as a function of other quantities [Forbus84]. In many cases indirect influences cannot be resolved within QPT because of the lack of detail information about the form of the function. For example, suppose there is a quantity q_o such that a particular process structure:

$$q_0 \propto_{Q+} q_1 \wedge q_0 \propto_{Q-} q_2$$

where $\wedge$ represents a boolean operation AND. If we also know that $D_s[q_1] = 1$ and $D_s[q_2] = 1$, then $D_s[q_0]$ cannot be determined, since there is not enough information to determine which indirect influence dominates. However, if we had $D_s[q_1] = 1$ and $D_s[q_2] = 0$, then we can conclude that $D_s[q_0] = 1$.

The collection of qualitative proportionalities which hold at any particular time is loop-free. If A is qualitatively proportional to B, then it cannot also be the case that B is proportional to A. Physical systems, such as feedback system, always contain a derivative relationship, which is modeled by a direct influence, not a qualitative proportionality. For example, in considering about fluid flow, one might observe that a change in amount of liquid causes a change in flow rate, which in turn affects the amount of liquid. But flow rate is qualitatively proportional to the amount of liquid (via its dependence on pressure, which depend on the level, which in turn depends on the amount of liquid), the flow rate is a direct influence on the amount of liquid.

6.7.4 Describing behavior

In qualitative reasoning, behavior is described as a sequence of qualitative states, or equivalently, as a history. The behavior is derived by a qualitative form of simulation. The basic strategy is to start with a known qualitative state. By applying the constraints which describe the relations between values and derivatives, the possible values of derivatives are identified. By extrapolating the current state in the direction given by derivatives, the potential successor states for the system are derived. This procedure is iterated until either a state where all derivatives are zero is reached, or a previously explored state is revisited. Consider a physical system which is shown in Fig. 50.

According to Fig. 50, the only existing block is B, and it is connected to the spring S, which in turn is connected to a fixed object, the wall. B and S thus satisfy the individuals requirements for *b* and *s*. The view SBS(B,S), which is referred to as SB in the following, may thus be instantiated. This is the only possible instantiation of a view in this example.

Consider the view and process definition shown in Fig. 51. Assume that B can only move in a straight line perpendicular to the wall. This yield two potential directions of motion to be denoted as INBO (inbound) and OUTB(outbound). Given that the view SB has been instantiated, the four processes: MOT(SB,OUTB), MOT(SB,INBO), ACC(SB,OUTB) and ACC(SB,INBO) may be instantiated.

It has been established that SB is active. The activity of the process depends on the values of *force* and *veloc*. Assume B in Fig. 50 has been pushed toward the wall, released and just started to move from the wall. Assuming a simple quantity spaces of {+,0,-} type, the following are the initial values specified for the quantities; *force* is [+], *pos* is[-], *veloc* is [+], *kin-ene* is[+], and *pot-ene* is [+].

The validity of the quantity conditions may now be checked. Since *veloc* and *acc* are positive, corresponding to the direction OUTB, the two instances with *dir* bound to INBO, MOT(SB,INBO) and ACC(SB,INBO), are inactive, because the conditions on the direction are not satisfied. On the other hand, A[*veloc*(SB)]>0 in MOT(SB,OUTB) hold true, similarly A[*force*(SB)]>0 holds for ACC(SB,OUTB). For these two instances, the conditions on directions hold as well. The process MOT(SB,OUTB) and ACC(SB,OUTB) are therefore active.

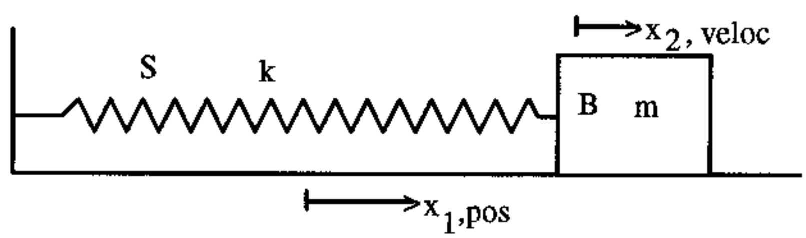


Figure 50. A sliding block attached to a spring.

INDIVIDUAL VIEW SBS(b,s) : sliding blcok spring

Individuals:	b an object, Mobile(b), Has-quantity(b,mass)
	s a spring, Connected(b,s) Connected(s,fixed-object).
Relations:	Let *pos* be a quantity
	Let *veloc* be a quantity
	Let *force* be a quantity
	$force \propto_{Q-} pos$
	correspondece(A[*force*]=0, A[*pos*] =0)
	Let *acc* be quantity
	Let $acc \propto_{Q+} force$
	Let $acc \propto_{Q-} mass(b)$
	correspondece(A[*acc*]=0, A[*force*] =0)
	Let *kin-ene* be quantity.
	Let $kin-ene \propto_{Q+} veloc$
	Let *pot-ene* be quantity.
	Let $pot-ene \propto_{Q+} veloc$

PROCES MOT(sb,dir)	;(motion)
Individuals:	sb an instance-of SBS, dir a direction,
Preconditions:	Free-direction(sb,dir)
Quantity Conditions:	A[*veloc*(sb)]>Zero Direction-off(dir,A[*veloc*(sb)])
Influences:	I+(*pos*(sb),A[*veloc*(sb)])

PROCES ACC(sb,dir)	;(acceleration)
Individuals:	sb an instance-of SBS, dir a direction,
Preconditions:	Free-direction(sb,dir)
Quantity Conditions:	A[*force*(sb)]>Zero Direction-off(dir,A[*force*(sb)])
Influences:	I+(*veloc*(sb),A[*acc*(sb)])

Figure 51. View and process definition of a sliding block attached to a spring.

Process structure is a set of all active instances of views and processes in a given system. In our example, the process structure consists of SB, MOT(SB,OUTB) and ACC(SB,OUTB). The next step in the QPT analysis is to extract the direct and indirect influences specified by these instances. The extracted influences form a set of constraints which constitute the qualitative model of the physical system in the present qualitative state.

The qualitative model is now applied to predict the behavior of the system. The values of the derivative of the directly influenced quantities are first computed as the qualitative sums of the numbers influencing each variable. The direct influence currently in effect are I+(*veloc*(SB),A[*acc*(SB)] and I+(*pos*(SB),A[*veloc*(SB)]). In this case, there are only two directly influenced variables, each of them is being influenced by one number only. Since both *veloc* and *acc* are positive, the derivatives of *pos* and *veloc* are positive.

The derivatives of indirectly influenced variables are computed by propagating the known values. Since *force* is inversely qualitative proportional to *pos* which is increasing, the derivative of *force* is negative. Since *acc* is qualitative proportional to *force*, and mass of B is assumed to be constant, the derivative of *acc* is negative. The values of the derivatives of the rest of the indirectly influenced quantities are computed in a similar way.

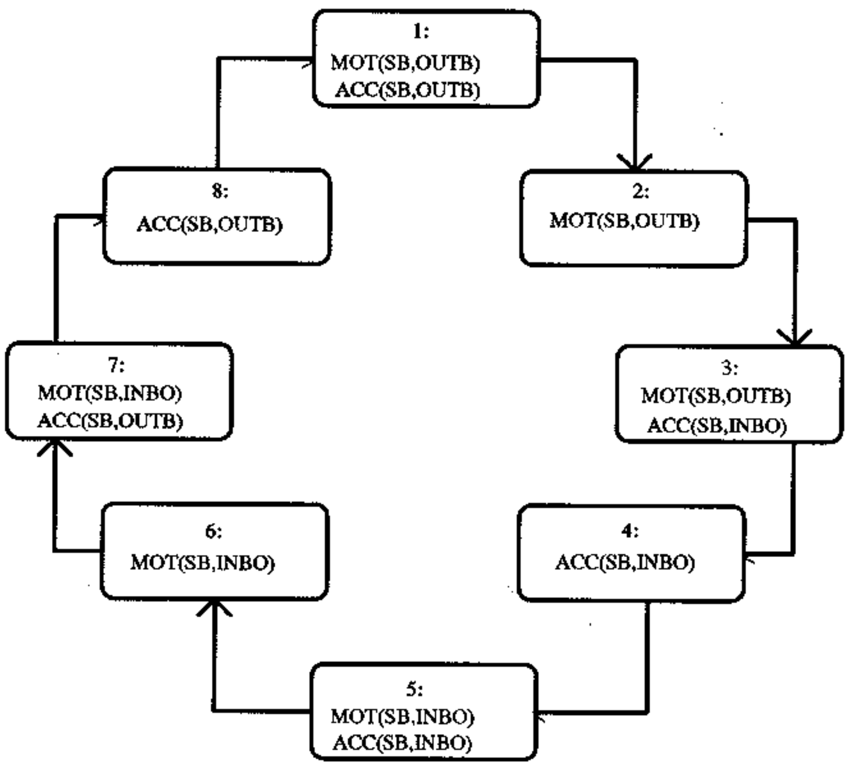


Figure 52. Sequence of process structure of Fig. 50.

Next, the QPT proceeds to what is known as limit analysis. Changes in quantities can result in the process and view structures themselves changing. Determining these changes and changes in D_s is called limit analysis [Forbus84]. Obtaining limit analysis requires the extrapolation from current values in the direction given by the values of derivatives, followed by an attempt to infer which quantity first reaches a landmark. In the present situation, *veloc* is positive and increasing. Since there is no element in its quantity space which is greater than positive, it cannot change its value in the present situation.

The other quantities are more interesting, both *pos, force* and *acc* are moving towards the value 0. Normally, the QPT would now have difficulties in establishing which quantity would reach its new value first. In this case, the two correspondences in the view definition saves the situation. They specify that all three quantities reach their new values simultaneously, this happens when B reaches its resting position.

The *force* has become 0, thus the quantity condition specifying a positive *force* is violated. This cause the ACC[SB,OUTB] process to become inactive. In order to identify a new process structure which will provide an updated set of constraints, the analysis must return to the step described above. The program iterates the task of identifying a set of constraints, computing the values of derivatives, and performing limit analysis. Fig. 52 shows the complete solution for the system in terms of the sequences of process structures. The permitted transitions between structures are drawn as lines between the boxes.

Chapter 7

Hybrid Phenomena Theory

Woods [1991] presented a Hybrid Phenomena Theory (HPT) which inherited basic concepts like views, phenomena and influences from QPT. The HPT defines those concepts in a more precise manner in order to represent physics knowledge with the accuracy needed to develop full parametric models. Parameters have quantitative interpretations which are required by the majority of application areas in science and engineering. Since a qualitative reasoning fails to produce quantitative models which physicists and engineers normally use, they understandably see little merit in the enterprise.

7.1 Components of HPT

HPT comprised of three models of components; topological, phenomenological and state space models. Each of which will be discussed in the following.

The term topological model is used to refer to a set of objects, and a set of logical statements. The objects will provide descriptions of entities such as pipes, valves and control volumes of physical substances. The logical statement will describe properties of these objects and the topological relationships which exist between them. Any given object, as well as any logical statement, may in principle be valid or invalid at a particular point of time. The state of topological model is thus defined by those objects and relationships which are valid at that particular point of time. This model is also stated as the knowledge level of HPT[Woods91b].

The term phenomenological model is used to refer a set of objects describing instances of physical phenomena which may potentially occur in a given system. All objects describing instances of phenomena will not necessarily be active at a given time. The state of the phenomenological model is thus defined by the set of active phenomena instances. This model is also stated as the symbolic level of HPT[Woods91b].

The term state space model is used to derive behavioral predictions in the quantity part of the framework. It is also known as the numeric level of HPT[Woods91b]. The fact that an instance of a view or phenomena exist does not entail that it will influence the analysis of the system. Only the active instances influence the analysis. To be active, all pre- and quantity conditions for an instance must be satisfied. In addition, all objects bound to individuals in the instance have to be active.

To derive the behavioral prediction in the quantity part of a framework, HPT will span a superset of a state space model. This is accomplished by treating the terms of the equations as distinct entities which, at a given point of time, are included in the state space model at that point of time. After having active instances at a given point of time, by combining all influences defined by these instances, the HPT will produce a set of equations describing the behavior of the system.

HPT model refers to the combined model comprising the topological model, the phenomenological model, the state space model and a set of relationships describing how the state and validity in the respective component models affect the state and validity in the other component models.

The HPT generalizes and extends the QPT. The QPT employs qualitative constraints to describe the relation between variables, while the HPT utilize parametric state space models. Provided that the values of the parameters of the model are known or can be estimated, this will allow us to avoid problems like ambiguous solutions which are inherent in the qualitative simulation techniques [Woods91a]. Fig. 53 shows the model components in HPT. The qualitative model includes both the topological and phenomenological components. In addition, the qualitative model incorporates certain dependencies between topological and phenomenological components which are not attributed to either component. Similarly, in addition to the qualitative and quantitative components, the HPT model incorporates a set of dependencies among these components which are not considered to belong to either component.

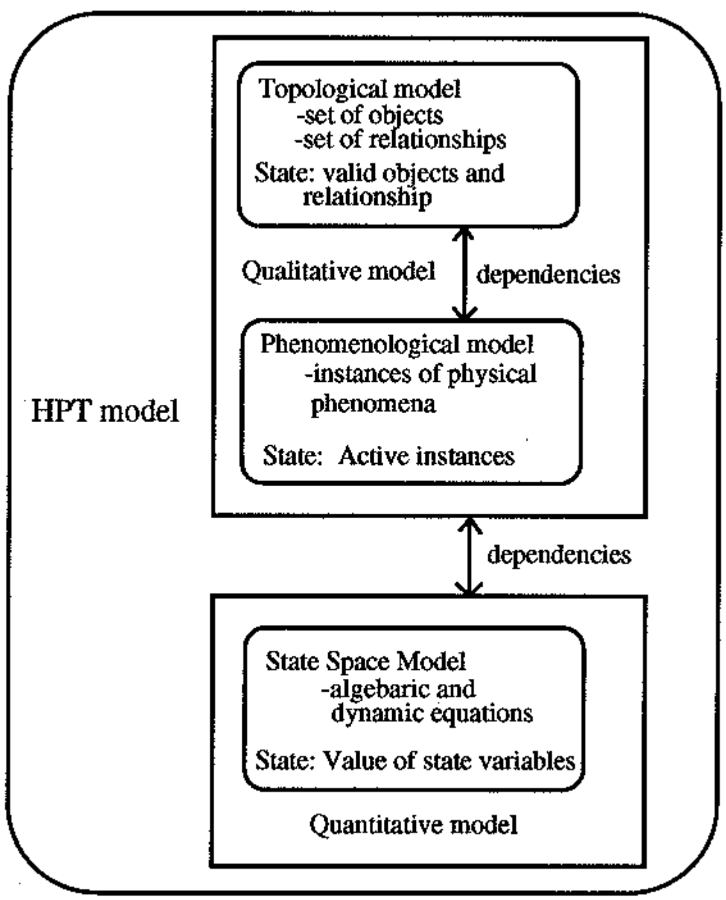


Figure 53. Model components in HPT

The most significant difference between HPT and QPT lies in the fact that the HPT employs quantitative values to characterize the properties of the objects, whereas the QPT employs qualitative values. Apart from this, the actual objects used for all practical purposes are identical in both approaches. In addition, the HPT logical model introduces a type of dependency between the entities in the phenomenological model which is called subsumption and has no counterpart in QPT.

7.2 Mathematical terminology

HPT uses term state space model to refer a set of equations comprising two different subsets, a set of dynamic equations and a set of algebraic equations. The former corresponds to the direct influence of QPT, the latter corresponds to indirect influence. Equations (59) and (60) gives a general formulation of a state space model. The first constitutes the dynamic subset, the second constitutes the algebraic subset.

$$\dot{x} = f(k,u,s,\theta_1,\theta_2) \quad (59)$$

$$s = g(k,u,\theta_1,\theta_2) \quad (60)$$

where

k is a vector of state variables
u is a vector of independent or control variables
s is a vector of dependent variables
θ_1 is a vector time varying parameters
θ_2 is a vector of constants

Both the dynamic and the algebraic equations are functional relationships. The dynamic equations define the value of the derivatives of the state variables with respect to time. The algebraic equations define the value of the dependent variables. The state space models are heavily used in the domain of dynamic simulation. In this case, the functional relationships prescribe how to compute future values for the variables.

HPT uses the term quantity as a common descriptor for any element in one of the vectors k,u,s,θ_1, or θ_2. It uses variable to refer an element in any of k,u or s. It uses parameter as a common name for an element in either θ_1 or θ_2.

HPT derives parameteric state space models, the parameters may have a quantitative interpretation. The derived mathematical model provides a basis for quantitative simulation, filtering and estimation schemes developed within fields such as control and chemical engineering [Woods91a]. The concept of an equation is important in HPT, since a large class of problems involving the physical world depends on quantitative knowledge. Influences specify what can cause a quantity to change. A direct influence describes how the value of one quantity influences the derivative of another. A change in the influencing quantity only directly affects the value of influenced quantity.

In HPT, influences express how the value of a given variable is affected by one or a set of other variables. There are two types of influences; dynamic influences and algebraic influences. The former corresponds to the direct influence of QPT and the latter corresponds to indirect influence. Their syntax is

```
(dyn-inf <influenced variable>
      <list of influencing variables>
      <numeric function>)
```

and

```
(alg-inf <influenced variable>
      <list of influencing variables>
      <numeric function>)
```

respectively. An example where a variable x_1 is dynamically influenced by two variables x_2 and x_3 is shown in Equation (61). Here, the amount of influence is computed as a nonlinear function of x_2 and x_3. If this is the only influence affecting x_1, the derivative of x_1 will be computed from Equation (62).

$$(\mathrm{dyn-inf}\, x_1 (x_2 x_3)(\mathrm{sqrt}(x_2 x_3))) \quad (61)$$

$$\dot{x}_1 = \sqrt{x_2 x_3} \quad (62)$$

The complete semantic interpretation for dynamic influences is that ; the derivative of a dynamically influenced variable equals to the sum of the numeric functions specified in the dynamic influences affecting the variable. Each dynamic influence is interpreted as the specification of a term in the equation defining the derivative of the dynamically influenced variable.

For algebraic influences mentioned above, the semantic interpretation is that: an algebraically influenced variable equals to the sum of the numeric functions specified in the algebraic influences affecting that variable. Each dynamic influence is interpreted as the specification of a term in the equation defining the value of the algebraically influenced variable.

To illustrate how influences are combined, consider a closed container which is partially filled with liquid and gas taking up the remaining volume. The liquid has level l, the gas a pressure p_1. The pressure at the bottom of the container, p_2, is affected by two different influences as shown bellow.

$$(\mathrm{alg-inf}\, p_2 (p_1) p_1)$$
$$(\mathrm{alg-inf}\, p_2 (l)(rgl))$$

If no other influence specifies p_2 as influenced variable, this implies that p_2 is given by expression

$$p_2 = p_1 + \rho g l$$

There are several differences between the algebraic and dynamic influences of the HPT and the indirect and direct influences of the QPT. The first is that the HPT-influences are no longer restricted to relate just two quantities. The second is that the HPT-influences define complete non-linear functions of any number of variables. The third is that the HPT will combine algebraic influences in the same manner as it combines dynamic influences. The QPT is modular with respect to dynamic variables, it computes the value of the derivative of all directly influenced variables as the sum of all influences affecting it. However QPT is not fully modular with respect to dependent variables. Although several views and processes may specify indirect influences affecting a given variable, the QPT fails to specify a general mechanism for deriving the value of an indirectly influenced variable whenever two or more influences are pushing it in different directions. The HPT is fully modular in both kinds of variables.

7.3 Subsumption

HPT incorporates a mechanism called subsumption, which enables us to make correct simplifying assumptions. Obtaining a suitable model requires that the correct assumptions be made. The effect of a specific assumption propagates through the modeling procedure and will typically influence a number of later decisions.

Consider the physical system shown in Figure 54; i.e. a pan containing an amount of water placed on a hot plate. We want to model the effect of heat flow from the hot plate to the water.

In this situation, the heat flow is from hotplate to pan, and from the pan to the water. The common simplifying assumption would be to consider the pan and water as an object with respect to heat flow by summing up their heat capacities. So the heat flow is from the hotplate to the combined object. An instance of this combined object will satisfy the individual condition in the definitions describing heat flow from hotplate. The water and pan do not exist as objects anymore. This will cause problems, because the water and pan are bound to individuals in some instance of a view or phenomenon.

To avoid problems with assumptions involving more than one object, HPT incorporates a mechanism called *subsumption* which is defined as follows. For any two instances INS_1 and INS_2 of a given view or phenomenon definition with individual D_1 $D_2..D_m$, INS_2 is subsumed by INS_1 if and only if, the object bound to D_i of INS_2 is an individual of an instance INS where INS is the object bound to D_i of INS_1.

This mechanism marks one of instances as subsumed without removing any objects. Subsumed objects may still be bound to individuals in other instances. For example, an instance *heat-flow-1(heat-bridge-1 pan water)* describes the heat flow from water to pan, while an instance *heat-flow-2(hotplate container-with-liquid-1 heat-bridge-2)* describes the heat flow from hotplate to the combined object *container-with-liquid.* The instances *heat-bridge-1* and *heat-bridge-2* describe the heat connections between pan and water, and between hotplate and *container-with-liquid* respectively, while the instance *container-with-liquid-1* describes a view of a liquid inside the container. Since pan and water are the objects of the instance *heat-flow-1* and they are the individuals of the instance *container-with-liquid-1* (see Fig. 57), then the instance *heat-flow-1* is subsumed by *heat-flow-2*.

7.4 Deriving an HPT model

So far, we have discussed the components of HPT as well as the influences of HPT. This section will demonstrate how the mechanism of HPT works. Consider a physical system which is shown in Fig. 54. Its initial description of the topological model is shown in Fig. 55.

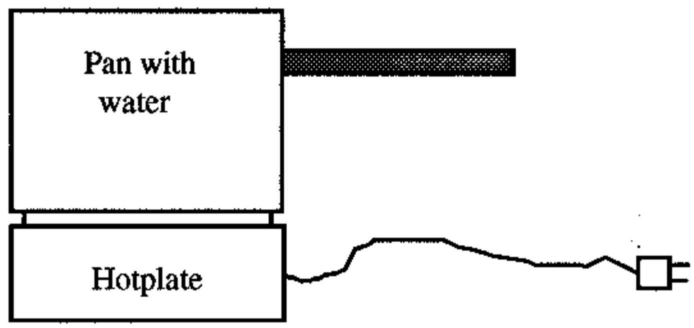


Figure 54. A pan with water on a Hotplate

The topological model consists of two sets of objects. The first set represents the physical entities in the system to be modeled. In the example, this is the HOTPLATE, the PAN and the WATER. The second set represents properties of the individual physical entities and relationships among two or more of them. These objects describe the topological organization of the physical entities. In the input description, two properties are specified; that the HOTPLATE has-heat-leading-top and the PAN has-heat-leading-bottom. In addition, there are two relationships; place-in (WATER PAN) and rest-on (PAN HOTPLATE).

No additional basic physical entities are produced by program, since none of the phenomena definition employed in this example specifies that additional basic physical entities shall be created upon instantiation of the phenomena definition. However, several additional properties and relationships will be created, thus extending the topological model from what was specified by user. The complete set of logical relationships created for the example is shown in Fig.56.

The next step is determining the phenomenological model. The phenomenological model includes all instances of phenomena created by the program. Assuming all relevant phenomena are included in the knowledge base, the instantiated phenomena constitute all phenomena instances which can conceivably affect the system being modeled given the specified input description. This model also includes a number of objects representing how the activity level of instances depend on logical facts and quantitative state in the system. The combined truth values of these relationships determine which phenomena instances are active. The set of active instances is denoted as qualitative state.

```
(defobject pan 'container-with open-top
    (construct-material steel)
    (bottom-area 0.018)
    (capasity 0.003)
    (heat-capacity 3000)
    (has-heat-leading-bottom)
    )

(defobject hotplate 'electric-hotplate
    (area 0.018)
    (heat-capacity 2000)
    (power-consumption 1500)
    (has-heat-leading-top)
    )

(defobject water 'water-liquid
    (boiling-temperature 100)
    (vaporization-hat 2400000)
    (specific-heat-capacity 4200)
    (density 1000)
    )

(declaration placed-in (water pan))

(declaration rests-on (pan hotplate)
```

Figure 55. Input description of topological model of Fig. 54.

```
has-heat-leading-bottom(PAN)
has-heat-leading-top(HOTPLATE)
can-contain-liquid(PAN)
placed-in(WATER PAN)
rest-on(PAN HOTPLATE)
has-heat-leading-bottom(CONTAINER-WITH-LIQUID-1)
rest-on(CONTAINER-WITH-LIQUID-1 HOTPLATE)
heat-connected(PAN WATER)
heat-connected(WATER PAN)
heat-connected(HOTPLATE PAN)
heat-connected(PAN HOTPLATE)
heat-connected(CONTAINER-WITH-LIQUID-1 HOTPLATE)
heat-connected(HOTPLATE CONTAINER-WITH-LIQUID-1)
```

Figure 56. The complete set of logical relationships of the system (Fig. 54).

```
(defview container-with-liquid (c l)
 (inst-name view-inst-with-heat-capasity)
  (c  (is-a container-with-open-top)
      (can-contain-liquid)
  (l  (is-a liquid))
  (placed-in l c))
 (precondition
  (subsuming
   (assume-one-object-liquid-container (c l))))
 (quantitiyconditions
   (subsuming (> mass(l)   0)))
 (relations
   (define-variable h
     (:value 0
      :what? "liquid level in container"
      :unit "meter"))
   (define-variable p
     (:value 0
      :what? "bottompressure in container"
      :unit "Pa"))
   (define-variable temp
     (:value 0
      :what? "temperature"
      :unit "K"))
   (define-variable m
      :what? "temperature"
      :unit "K"))
   (define-variable c
      :what? "heat capacity"
   (connect-quantity area area (c))
   (if (rest-on(c x))
   (if (has-heat-leading-bottom(c))
      then (has-heat-leading-bottom(self)))
   (alg-inf m (mass(l))
      (+ mass(l) mass(c)))
   (alg-inf h (mass(l))
      (/ mass(l) (* density(l) area(c))))
   (alg-inf p (h) (*  h density(l) g(global))))

(defphenomenon electric-heat-hotplate(elh)
 (individuals
  (elh  (is-a electric-hotplate)))
 (preconditions
  (power-on(elh)))
  (placed-in l c))
 (dynamics
  (dyn-infl temp(elh) (power(elh))
      (/ power(elh) c(elh))))
)
```

```
(defview heat-bridge (ob1 ob2)
(individuals
 (ob1 (is- object-with-heat-capacity))
 (ob2 (is- object-with-heat-capacity))
(quantityconditions (> mass(obj1) 0)
                    (> mass(obj2) 0))
 (heat-connected ob1 ob2))
 (relations
  (define-parameter alpha
    (:value 1200 : what? "heat transfer coefficient"
     : unit "J s-1 K-1 m-2"))
  (define-parameter kappa
    (:what? "heat transfer per degree Kelvin"
     : unit "J s-1 K-1"
     : compfunc
       (* alpha (MIN area(ob1) area(ob2)))))))


(defphenomenon heat-flow (src dst hbr)
 (individuals
   (hbr (instance-of heat-bridge(src dst)))
   (src (is-a object-with-heat-capacity))
   (dst (is-a object-with-heat-capacity)))
 (quantityconditions
   (> temp(src) temp(dst)))
 (relations
    (define-variable hstr
       (:what? "heatflow from src to dst"
        :unit "J s-1"))
    (alg-infl hstr (temp(src) temp(dst))
       (* kappa(hbr)  (-temp(src) temp(dst)))))
    (dynamics
       (dyn-infl temp(dst)
          (hstr) (/ hstr hcp(dst)))
       (dyn-infl temp(src)
          (hstr) (- (/ hstr hcp(src)))
 )

(defphenomenon boiling (lc hf)
(individuals
  (lc (instance-of container-with-liquid(nil nil)))
  (hf (instance-of heat-flow(nil lc nil))))
(quantityconditions (> temp(lc) btem(lc(l))))
(dynamics
  (dyn-infl mass(lc(l)) (hstr(hf))
      (- (/ hstr(hf) h(lc)))))
  (dyn-infl temp(lc(l)) (hstr(hf))
      (- (/ hstr(hf) c(lc)))))
)
```

Figure 57. Definition of views and phenomena [from Woods, 1991].

```
(dephenomenon boil-v2 (lc hf)
(individuals
   (lc (instance-of container-with-liquid(nil nil)))
   (hf (instance-of heat-flow(nil lc nil))))
(quantityconditions
   (on (> temp(lc)  (+ (btemp(lc)   .1)))
   (off (> temp(lc)  (- (btemp(lc)   .1)))
(dynamics
    (dyn-infl mass(lc(l)) (hstr(hf))
        (- (/ hstr(hf) h(lc)))))
    (dyn-infl temp(lc(l)) (hstr(hf))
        (- (/ hstr(hf) c(lc)))))
)
```

Figure 58. Continuation of definition of views and phenomena.

The view and phenomena definitions for this example is shown in Fig. 57. The HPT-objects created for the example are shown in Fig. 59. This includes references to global variables which is never bound to any individual. The three objects, HOT-PLATE, PAN and WATER are the basic objects specified in the input description. Then follows all instances of views and phenomena created for this example.

The first four instances of HEAT-BRIDGE characterize the heat paths between the three basic objects. There are two instances for each pair of objects as the description carries a notion of direction. For each of these instances of HEAT-BRIDGE, there is an instance of HEAT-FLOW which binds the corresponding HEAT-BRIDGE instance.

```
GLOBAL
HOT-PLATE
PAN
WATER
HEAT-BRIDGE-1(PAN WATER)
HEAT-BRIDGE-2(WATER PAN)
HEAT-BRIDGE-3(HOT-PLATE PAN)
HEAT-BRIDGE-4(PAN HOT-PLATE)
ELECTRIC-HEAT-HOT-PLATE-1(HOTPLATE)
CONTAINER-WITH-LIQUID-1(PAN WATER)
HEAT-FLOW-1(PAN HOTPLATE HEAT-BRIDGE-4)
HEAT-FLOW-2(HOTPLATE PAN HEAT-BRIDGE-3)
HEAT-FLOW-3(WATER PAN HEAT-BRIDGE-2)
HEAT-FLOW-4(PAN WATER HEAT-BRIDGE-1)
HEAT-BRIDGE-5(HOT-PLATE CONTAINER-WITH-LIQUID-1)
HEAT-BRIDGE-6(CONTAINER-WITH-LIQUID-1 HOT-PLATE)
HEAT-FLOW-5(CONTAINER-WITH-LIQUID-1 HOT-PLATE HEAT-BRIDGE-6)
HEAT-FLOW-6(HOT-PLATE CONTAINER-WITH-LIQUID-1 HEAT-BRIDGE-5)
BOILING-1(CONTAINER-WITH-LIQUID-1 HEAT-FLOW-6)
BOIL-V2-1(WATER HEAT-FLOW-4)
```

Figure 59. Objects, view and phenomena instances.

The next instance describes the heat generation in the HOTPLATE. Then follows the object implementing the assumption that the PAN and the WATER objects may be considered as one object with respect to any heat flow. The latter instance breeds two additional instances of HEAT-BRIDGE which subsequently give rise to two additional instances of HEAT-FLOW. Finally, there are two instances describing the boiling phenomenon, but created from different definitions.

In order to determine which phenomena instances are active, the numerical values for variables and parameters involved in the relevant activity conditions must be specified. The initialization of any variables or parameters which are involved in quantity conditions must be provided by a user. For example, all variables describing the mass of an object is initialized to 2, all temperatures are set to 0, and the power consumption of the hotplate is set to 1000.

Once the initial values for the relevant variables have been specified, the quantity conditions are tested. A list of preconditions is presented to the user. Each precondition must be set either true or false. In this example, it is specified that the hot plate is not turned on, and the assumption of WATER and PAN be considered as one object is set to true.

In this condition, the first four instances of HEAT-BRIDGE and the corresponding instances of HEAT-FLOW (see Fig. 59.) are all inhibited by the subsumption mechanism. The only active instances are CONTAINER-WITH-LIQUID-1, HEAT-BRIDGE-5, and HEAT-BRIDGE-6. Since the power is still off, the phenomenon instance describing the heat generation in the HOTPLATE is not active. The two heat-flows corresponding to the active instances of HEAT-BRIDGE remain inactive due to the fact that the temperature difference between two objects are zero.

By combining all active instances, the resulting state space model is only consits of some algebraic relationships

$$u_1 = 2$$
$$z_1 = z_2 \rho_1 g$$
$$z_2 = \frac{u_1}{\rho_1 a_1}$$
$$z_3 = u_1 + m_2$$

where u_1 and ρ_1 are mass and density associated with WATER; z_1, z_2, and z_3 are bottom pressure, liquid level, and mass associated with CONTAINER-WITH-LIQUID-1, respectively; m_2, a_1 are mass and bottom area associated with PAN, respectively.

Next, the power in the HOT-PLATE is turned on. The phenomenon instance ELECTRIC-HEAT-HOT-PLATE-1(HOTPLATE) now is active, but no other phenomenona instances are affected. The dynamic influence of this instance now gives rise to a dynamic equation describing how the temperature in the HOT-PLATE is affected by the heat generated. The corresponding state space model is

$$\dot{x}_1 = \frac{u_2}{c_3}$$
$$u_1 = 2$$
$$u_2 = 1000$$
$$z_1 = z_2 \rho_1 g$$
$$z_2 = \frac{u_1}{\rho_1 a_1}$$
$$z_3 = u_1 + m_2$$

where x_1, c_3 and u_2 are temperature, heat capacity, and power consumption associated with HOT-PLATE, respectively.

Now, the value of the temperature in HOTPLATE is modified to 0.1 to anticipate the consequences of the now active heat generation in HOTPLATE. There is temperature difference between two objects, HOTPLATE and CONTAINER-WITH-LIQUID-1. Consequently, the HEAT-FLOW-6(HOT-PLATE CONTAINER-WITH-LIQUID-1 HEAT-BRIDGE-5) changes to be active. The resulting state space model is

$$\dot{x}_1 = \frac{u_2}{c_3} - \frac{z_4}{c_3} \quad (63)$$
$$\dot{x}_2 = \frac{z_4}{c_4} \quad (64)$$
$$u_1 = 2$$
$$u_2 = 1000$$
$$z_1 = z_2 \rho_1 g$$
$$z_2 = \frac{u_1}{\rho_1 a_1} \quad (65)$$
$$z_3 = u_1 + m_2$$
$$z_4 = k_5 (x_1 - x_2)$$

where z_4 is the heat flow from source to destination associated with HEAT-FLOW-6(from HOT-PLATE to CONTAINER-WITH-LIQUID-1), x_1, c_3 represent temperature and heat capacity associated with HOTPLATE respectively, x_2, c_4 are temperature and heat capacity associated with CONTAINER-WITH-LIQUID-1, and k_5 represents heat transfer per degree Kelvin associated with HEAT-BRIDGE-5.

The heat flow specifies two dynamic influences. The first (Eq. 63) describes the negative impact on the temperature of the object giving up the heat, HOT-PLATE. The change in temperature of HOT-PLATE is influenced by the heat come from power consumption and the heat lost to CONTAINER-WITH-LIQUID-1. The second (Eq. 64) describes the effect on the temperature of the object receiving the heat.

The temperature in both HOT-PLATE and CONTAINER-WITH-LIQUID-1 now gradually increases. However, there is no change in the qualitative state until the temperature of the liquid reaches the point where boiling occurs. The boiling temperature of the liquid was specified to 100, changing the temperature of the WATER to 100.1 thus causes BOILING-1 to change to active. The resulting state space model is

$$\dot{x}_3 = -\frac{z_4}{h_2} \qquad (66)$$

$$\dot{x}_2 = \frac{z_4}{c_4} - \frac{z_4}{c_4} \qquad (67)$$

$$\dot{x}_1 = \frac{u_2}{c_3} - \frac{z_4}{c_3} \qquad (68)$$

$$u_2 = 1000$$

$$z_1 = z_2 \rho_1 g$$

$$z_2 = \frac{u_1}{\rho_1 a_1} \qquad (69)$$

$$z_3 = x_1 + m_2$$

$$z_4 = k_5 (x_1 - x_2)$$

where x_3, h_2 are mass and vaporization-heat associated with WATER respectively.

The first dynamic influence of the boiling instance (Eq. 66) specifies how the boiling phenomenon will cause the liquid to evaporate. The second describes how the boiling phenomenon consume the heat. The heat, which come into CONTAINER-WITH-LIQUID-1, is totally consumed.

If this situation persists long enough, the water will eventually evaporate away. The value of the mass of WATER may be gradually reduced until it reaches zero. At this point, the quantity condition in CONTAINER-WITH-LIQUID-1 shift to false.

Now, the WATER has ceased to exist. All objects binding to the WATER, including the CONTAINER-WITH-LIQUID-1 must thus be inhibited. All of previous active instances except for ELECTRIC-HEAT-HOT-PLATE-1 are now deactivated. The only phenomenon instance containing any influences which switches to active in the new situation is HEAT-FLOW-2. This instance describes the heat flowing from the HOTPLATE to the PAN. The state space model corresponding to the qualitative state is

$$\dot{x}_1 = (-\frac{z_5}{c_3}) + \frac{u_2}{c_3}$$

$$\dot{x}_4 = \frac{z_5}{c_6}$$

$$u_2 = 100$$

$$z_5 = k_7 (x_1 - x_4)$$

where z_5 is the heat flow from source to destination associated with HEAT-FLOW-2 (the heat flow from the HOTPLATE to the PAN), c_3, c_6 represent the heat capacities associated with HOTPLATE and PAN respectively, and x_4 is the temperature associated with PAN.

Now, the PAN and the WATER can be treated as two distinct objects with respect to heat flows. This is accomplished by setting the corresponding subsuming pre-condition to false. The WATER object now can be reestablished by setting its mass to 2 again, the temperature of the PAN is simultaneously changed to 100 and temperature of the PAN is set to 200. The following phenomena instances become active in the new situation: HEAT-BRIDGE-1, HEAT-BRIDGE-2, HEAT-BRIDGE-3, HEAT-BRIDGE-4, ELECTRIC-HEAT-HOT-PLATE-1, HEAT-FLOW-2, and HEAT-FLOW-4. Therefore, by combining all influences of active instances, the resulting state space model can be determined in a similar manner.

Chapter 8

Discussion and Conclusions

8.1 A Commercial Tool G2

Before summarizing our discussion, a brief introduction on G2 is given. G2 is a commercial tool that can be used for reasoning about physical systems.

G2 is a tool for developing and running real-time expert systems for complex applications that require continuous and intelligence monitoring, diagnosis and control. G2 has several capabilities as follows:

1. It can reason about and control events in continuously changing environment.
2. It can respond to events when they occur (without continually having to poll sensors).
3. It can apply both procedural knowledge and rule-based heuristics.
4. It can simulate real-time conditions for testing purposes.
5. It can develop models and schematics for system development
6. It can express relationships between objects.
7. It can interact with users, both locally and at remote computers.
8. It can communicate with other Gs applications, such as external simulation programs.

Developing an application with G2 proceeds along the following steps:

1. Define each class of object in the application; what it looks like and what its attributes are.
2. Having defined the classes of objects that are found in the application, user then create a model of the application by placing objects on workspace and connecting them to show the relationships. This will result in a schematic diagram of the application. Associated with each object in this diagram is table describes that object. This table is created from definition of the object's class.
3. After creating the schematic, the user must indicate the source of values for each variable. Possible data sources include the G2 real-time inference engine, the G2 simulator, and other data sources such as real sensors.

Inferring Variables.

In G2 variables may receive values from several sources. Possible data sources include the G2 real-time inference engine, the G2 simulator, and other data sources such as real sensors.

For variables to receive values from G2 real-time inference engine, the user must write rules that tell G2 how to conclude those values. The user can also create other rules that indicate how to respond and what to conclude from changing conditions within the application--what to conclude from trends, how to determine a failure, what actions to minimize loss and maximize safety and productivity.

For variables to receive values from the G2 simulator, the user must create simulation formulas that tell the G2 simulator how to find values for those variables. These formulas may be algebraic, difference, or first order differential equations. The user can also use simulation formulas in expressing complex, high order models as a set if first order state variables models which may be linear or non-linear.

User can create either rules or formulas which are either generic and applicable to classes of objects or specific to particular objects.

8.2 Summary

From the discussions presented in the previous chapters it can be summarized that:

1. Both QPT and HPT provide a framework, or vocabulary, to formalize knowledge on physical interactions. This is the idea of views and phenomena definitions comprising prescription for relations and influences between individual quantities which will hold for instances of the definition. In addition, QPT includes a set of view and process definitions which may be instantiated. Envision also provides such framework, this is the idea of generic component models, each consisting of a number of confluences. Each confluence describes a component in a specified qualitative state. In addition, Envision includes a library of generic component models.

2. Both QPT and HPT have a mechanism which produces a set of constraints describing the system in the current state. The mechanism consists of two steps. First instantiating the view and process definitions for each set of objects satisfying the individual conditions for each definition. Next establishing a process structure consisting of instances whose pre- and quantity conditions are satisfied in the current situation, and extracting the influences from these instances. Envision also has a similar mechanism which consists of two steps. First, generating the compatibility and continuity constraints. Next, selecting which confluences describe each component in the system's current qualitative state.

3. QSIM provides neither a framework to formalize knowledge on physical interactions nor a mechanism such as mentioned in 2. However QSIM formulates

the problem in terms of constraints to be used. QSIM does not provide assistance in modeling the system except for those instances when a model in terms of ODEs exits, in which case a procedure for converting ODEs to constraints may be applied. Envision provides a library of components. The user needs only to specify what kind of components is his system built from and how they are interconnected for Envision to select the right confluences modeling system. QPT and HPT are similar in that the user only specifies which objects take apart and relations between these objects.

4. QPT, HPT and Envision use the set of generated constraints to derive possible successor states. A new set of constraints is produced whenever a new qualitative state is reached. This approach is iterated until quiescence occurs or until the successor state equals a previously identified state. Only HPT results in a mathematical model in each state.

5. Envision and QSIM describe behavior in terms of qualitative state while QPT and HPT describe behavior in terms of process structures. QPT and HPT make use of notions of phenomena explicitly and relate behavior to the occurrence of physical phenomena. Neither QSIM nor Envision embodies a counterpart to the notion of phenomena. Therefore, QPT and HPT provide us with a stronger conceptual framework for formalizing our knowledge about physical systems. Through the introduction of pre and quantity conditions, QPT allows us to formalize conditions for when these phenomena occur. QSIM approach only addresses the aspects related to simulation, while Envision, QPT and HPT address an additional issue of deriving a model, expressed as constraints, from topological description of a process.

6. HPT generalizes and extends QPT. QPT employs qualitative constraints to describes relation between variables. HPT utilizes a parametric state space model. Among the approaches reviewed, only HPT has a representation which allows us to modify mathematical models in accordance with the actual situation.

7. None of those approaches have direct differential equation representations. High order differential equations must be represented in terms of first order differential equations.

8. None of those approaches can represent nuclear physical processes, since none of them has a mechanism capable of reasoning about probabilistic events such as occur in nuclear physical processes.

A comparison of physics reasoning systems reviewed in this report; Envision, QSIM, Dimensional Analysis, QPT, HPT and the commercial tool G2, is tabularized as shown in Fig. 60.

Comparison of physics reasoning systems

Property	QSIM	Envision	Dimensional analysis	QPT	HPT	G2
input	constraints	components and topology	variables and their dimensions	objects and relations	objects and relations	objects and relations
model is derived from	differential equations	qualitative differential equations of its components and their interconnections	quantities' dimensions	topological description of process	topological description of process	rule conditions
library	n	y	n	y	y	y
constraints	M^+, M^-, Add,Mult,Deriv	confluneces	regimes	direct and indirect influences	algebraic and dynamic influences	variable ranges
mechanism to produce constraints	derived from differential equations	- generate continuity and compatibility -generate their confuences	-derived from physical quantities' dimensions	-instantiate view and process definitions -establish process structure -extract influences	-instantiate view and process definitions -establish process structure -extract influences	rule conditions
quantity value	dec, 0, inc	-, 0, +	dec, 0, inc	-1, 0, 1	R	R
framework	n	generic component models	n	view and phenomena definitions	view and phenomena definitions	rule library
behavioral description	qualitative states	qualitative states	change in variables of interest	in term of process structure	state space model and quantitative states	quantitative states
can represent state at discrete times	y	y	n	y	y	y
can represent state variables	n	n	n	n	y	y

Figure 60. Table of comparison of physics reasoning systems.

Property	QSIM	Envision	Dimensional analysis	QPT	HPT	G2
can represent a feedback system	y	y	y	y	y	y
qualitative results	y	y	y	y	y	y
quantitative results	n	n	n	n	y	y
modularity	n	y	n	y	y	y
computer program	QSim	Envision	n	GIZMO	HPT	G2

Figure 60. Table of comparison of physics reasoning systems (continuation).

8.3 Open problems

8.3.1 Probabilistic events

None of the physics reasoning systems discussed above were designed to reason about non-deterministic physics processes, such as nuclear physics processes which occur in the core of a reactor. Consequently, they have no mechanism for describing non-deterministic events. An example of such a process is shown below. A nuclear physics processes resulting from neutron absorption in ${}^{235}_{92}U$ may be written in general form as

$$ {}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{A_1}_{Z_1}F_1 + {}^{A_2}_{Z_2}F_2 + \nu {}^{1}_{0}n + energy $$

where ${}^{A_1}_{Z_1}F_1$ and ${}^{A_2}_{Z_2}F_2$ indicate possible products produced in a fission reaction, and $\nu {}^{1}_{0}n$ represents the release of neutrons. One example in which the ${}^{A_1}_{Z_1}F_1$ and the ${}^{A_2}_{Z_2}F_2$ are isotopes of krypton and barium, respectively is

$$ {}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{90}_{36}Kr + {}^{144}_{56}Ba + 2\,{}^{1}_{0}n + energy $$

The fission fragments ${}^{A_1}_{Z_1}F_1$ and ${}^{A_1}_{Z_1}F_1$ are isotopes with their mass numbers ranging from 75 to 160, with the most probable being around 92 and 144 [Murray, 1988]. Although fission is the dominant process, a certain fraction of the absorption of neutrons in Uranium result in radioactive capture, according to

$$ {}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{236}_{92}U + \gamma $$

The probability of fission is about 86%, while the probability of radioactive capture is 14% [Murray, 1988].

8.3.2 Multiphase systems

The physics reasoning systems discussed above are not capable of reasoning about multiphase physical systems, such as reasoning about what will happen if two types of liquid with different densities are combined together, or to reason about what will happen if a gas and a liquid are combined together.

8.3.3 Large scale physical systems

Large scale physical systems exist in the real physical world. Almost all of the models discussed above are quite simple compared to the real physical world. It is therefore necessary to develop a theory capable of reasoning about large scale physical systems. For instance, a full understanding of an internal combustion engine requires an understanding about physical processes and geometry interactions in the engine.

8.3.4 Naive physics systems

Naive physics systems deal with our daily activities. For example, an activity of making a salad, which is a complicated process for a machine to understand. It deals with multiple pieces of flexible materials (lettuce) with rather complex tools (the salad tongs). It deals with liquid of varying viscocities (salad dressings) and granular substances like croutons and bacon bits. We may require a theory which is capable of reasoning about naive physics systems for building a robot performing these actions.

8.3.5 Integration with robotics

Reasoning about physical systems is, in principle, closely related to robotics. It enables a robot to reason about, for example, where fluid flows, or where something might go (process motion); hence it can tell a robot where something might go if an object is dropped.

8.3.6 Friendly and intelligent user interface

Reasoning about physical systems enables the construction of algorithm that generates causal descriptions explaining how physical systems work. Unfortunately, none of the reasoning systems reviewed have a friendly user interface, except for the commercial tool G2. Therefore, it is necessary to have an intelligent and friendly user interface for such systems.

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