CS1083 Week 9: More Recursion

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Outline

Recursive Binary Search

Parsing

Quicksort

MergeSort revisited

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Wrapper methods

```
public int positionOf(T target) {
    if (!isSorted) {
        Collections.sort(this);
        isSorted = true;
    }
    return bSearch(target, 0, this.size()-1);
}
```

BinarySearchList2

Iteration vs. recursion

```
while (lo<=hi){
    int mid=(lo+hi)/2;
    int diff =
        get(mid)
        .compareTo(it);
    if (diff==0)
        return mid;
    if (diff<0)
        lo=mid+1;
    else
        hi=mid-1;
}
return -1;
```

```
private int bSearch(T it,int lo,
                    int hi){
    if (lo>hi) return -1;
    int mid = (lo+hi)/2;
    int diff =
      get(mid).compareTo(it);
    if (diff==0)
        return mid;
    if (diff<0)
        return bSearch(it,
                       mid+1,hi);
    else
        return bSearch(it,
                       lo,mid-1);
}
```

Recursive Binary Search

- Add tracing

```
private int bSearch(T it,int left,int right){
    if (left>right)
        return -1;
    int mid = (left+right)/2;
    int diff =
      get(mid).compareTo(it);
    if (diff==0)
        return mid;
    if (diff<0)
        return bSearch(it,mid+1,right);
    else
        return bSearch(it,left,mid-1);
}
```

Recursive Binary Search

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Grammars

- Formal languages (like Java!) are often described by grammars.
- Grammars are naturally recursive.
- Here is the grammar for a simple arithmetic expression:

```
expression -> ( expression )
expression -> expression op expression
expression -> number
op -> + | - | * | /
```

Easy direction: generate some expressions

```
expression -> expression op expression
           -> expression +  expression
           -> 100+32
```

Digression: StreamTokenizer

```
StreamTokenizer st=new StreamTokenizer(new FileReader("table
int type=st.nextToken();
while (type!= StreamTokenizer.TT_EOF) {
    switch(type){
    case StreamTokenizer.TT_WORD:
        System.out.println("word: "+st.sval);
        break;
    case StreamTokenizer.TT_NUMBER:
        System.out.println("number: "+st.nval);
        break;
    }
    type=st.nextToken();
}
```

StreamTokenizerExample

Try to match the three possibilities in order

```
expression -> ( expression )
expression -> expression op expression
expression -> number
```

StringParser2

```
private double eval(int level) {
    double left;
    int token=tokens.nextToken();
    if (token=='('){
        left=eval(level+1);
        if (tokens.nextToken()!=')') die("expected ')'");
    } else if (token==StreamTokenizer.TT_NUMBER)
        left=tokens.nval;
    else
        die("Syntax error");
}
```

Try each operator

```
    double right=0.0;
    int op=tokens.nextToken();
    if (op=='+' || op=='-' || op=='*' || op=='/'){
      // rule 2
      right=eval(level);
      return arith(level,op,left,right);
    } else {
      // oops, we read too far.
      // Turns out to be rule 3
      tokens.pushBack();
      return left;
    }

}
```

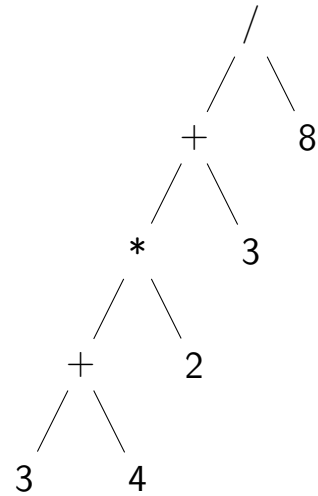
Example

```
StringParser2 Parser=
      new StringParser2(
        "(((3+4)*2)+3)/8)");

double d=Parser.eval(0);
```

```
enter eval
  enter eval
    enter eval
      enter eval
        enter eval
          enter eval
          return 4.0
        return 3.0+4.0=7.0
        enter eval
        return 2.0
      return 7.0*2.0=14.0
      enter eval
      return 3.0
```

Example

```
StringParser2 Parser=
      new StringParser2(
        "(((3+4)*2)+3)/8)");

double d=Parser.eval(0);
```



Recursive Binary Search

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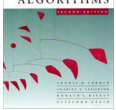
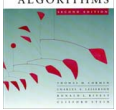
Quicksort

MergeSort revisited



Quicksort

- Proposed by C.A.R. Hoare in 1962.
- Divide-and-conquer algorithm.
- Sorts “in place” (like insertion sort, but not like merge sort).
- Very practical (with tuning).



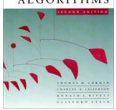
Divide and conquer

Quicksort an n-element array:

1. ***Divide:*** Partition the array into two subarrays around a ***pivot*** x such that elements in lower subarray $\leq x \leq$ elements in upper subarray.

2. ***Conquer:*** Recursively sort the two subarrays.
3. ***Combine:*** Trivial.

Key: *Linear-time partitioning subroutine.*

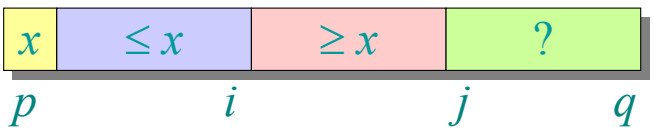


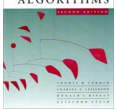
Partitioning subroutine

```
PARTITION(A, p, q)   ▷ A[p . . q]
    x ← A[p]         ▷ pivot = A[p]
    i ← p
    for j ← p + 1 to q
        do if A[j] ≤ x
                then  i ← i + 1
                      exchange A[i] ↔ A[j]
    exchange A[p] ↔ A[i]
    return i
```

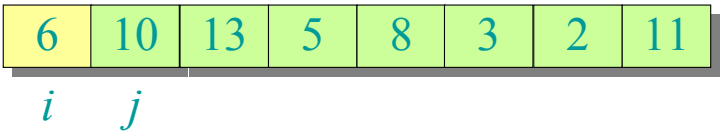
Running time $= O(n)$ for n elements.

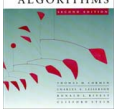
Invariant:





Example of partitioning





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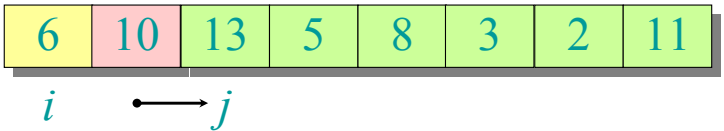

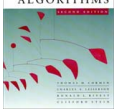

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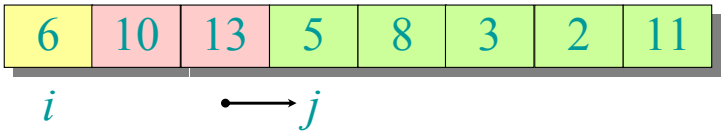

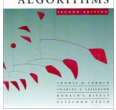

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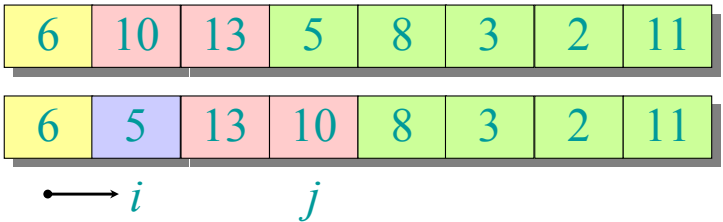

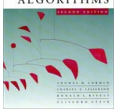

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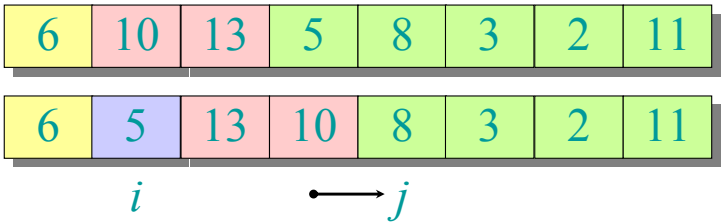

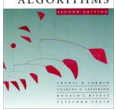

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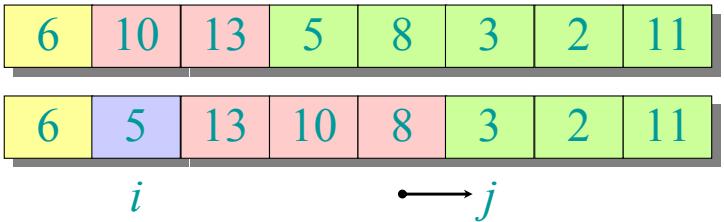

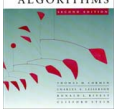

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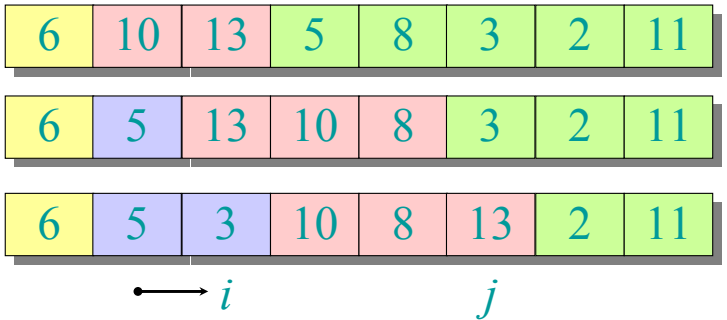

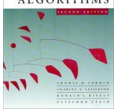

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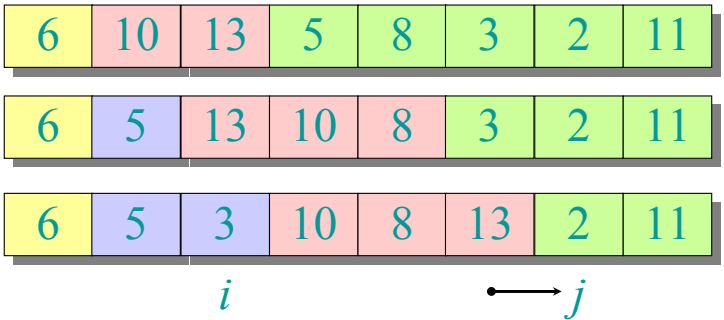

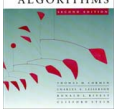

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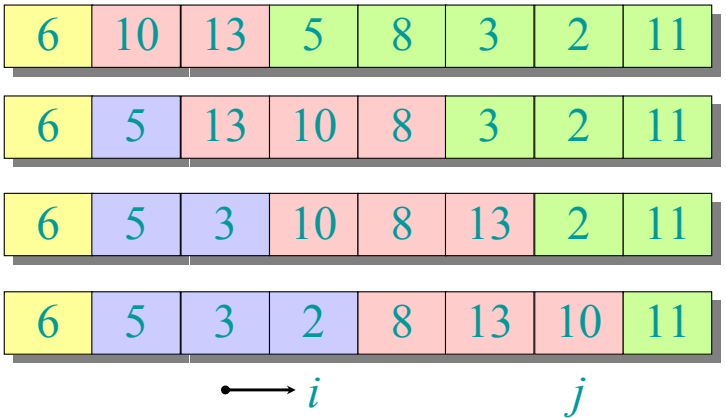

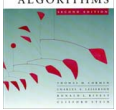

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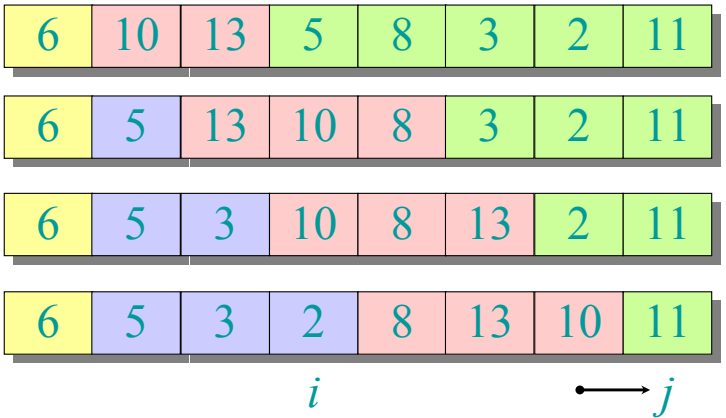

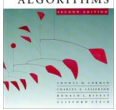

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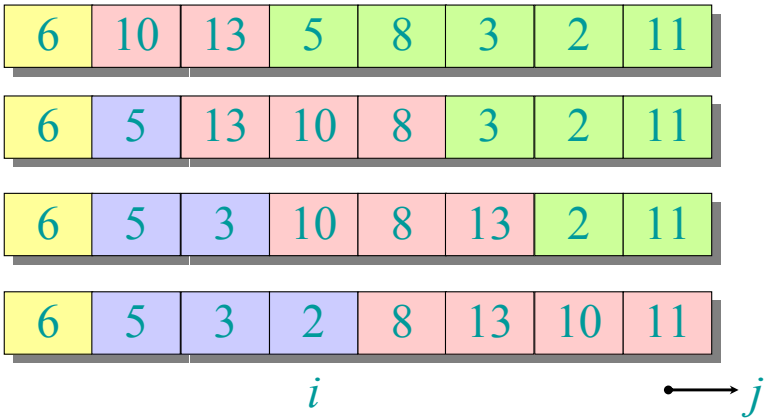
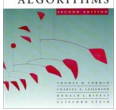
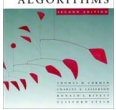




Example of partitioning

6	10	13	5	8	3	2	11
6	5	13	10	8	3	2	11
6	5	3	10	8	13	2	11
6	5	3	2	8	13	10	11
2	5	3	6	8	13	10	11

i



Pseudocode for quicksort

```
QUICKSORT(A, p, r)
    if p < r
        then q ← PARTITION(A, p, r)
             QUICKSORT(A, p, q–1)
             QUICKSORT(A, q+1, r)
```

Initial call: QUICKSORT$(A, 1, n)$

Quicksort in Java

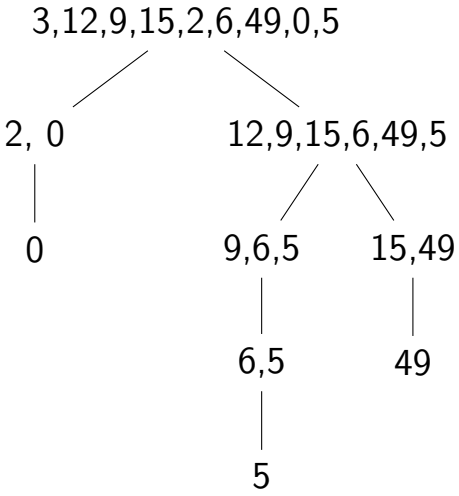
```
public static void quickSort(int[] a, int from, int to){

    if (from >= to) return;

    int mid=partition(a,from,to);

    quickSort(a, from, mid-1);
    quickSort(a, mid+1, to);
}
```

QuickSort

Partition in Java

```
private static int partition(int[] a,int from,int to){
    int pivot = a[from];
    int i = from;
    for (int j=from+1; j<= to; j++)
        if(a[j] <= pivot ) {
            i++;
            int temp=a[i];
            a[i]=a[j];
            a[j]=temp;
        }
    a[from]=a[i];
    a[i]=pivot;
    return i;
}
```

Quicksort example



- Here we choose pivot from first element; how could we do better?

Recursive Binary Search

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Quicksort

MergeSort revisited

Replacing Bottom levels of MergeSort

MyMergeSort2

```
public static void sort(int[] a){
    if (a.length <= 1) return;
    if (a.length <= 26){
        IntSelectionSort.sort(a);
    } else {









    }
}
```

Replacing Bottom levels of MergeSort

MyMergeSort2

```
    public static void sort(int[] a){
        if (a.length <= 1) return;
        if (a.length <= 26){
            IntSelectionSort.sort(a);
        } else {
            int mid=a.length/2;
            int[] left=Arrays.copyOfRange(a,0,mid);
            int[] right=Arrays.copyOfRange(a,mid,
                                           a.length);



        }
    }
```

Replacing Bottom levels of MergeSort

MyMergeSort2

```
public static void sort(int[] a){
    if (a.length <= 1) return;
    if (a.length <= 26){
        IntSelectionSort.sort(a);
    } else {
        int mid=a.length/2;
        int[] left=Arrays.copyOfRange(a,0,mid);
        int[] right=Arrays.copyOfRange(a,mid,
                                       a.length);
        sort(left); sort(right);
        merge(left,right,a);
    }
}
```

Comparing Sort algorithms

```
for (int len=increment; len <= steps*increment; len+=increme
    int[] a = ArrayUtil.randomIntArray(len, 1000000);
    switch(sort){
    case 0:
        clock.start();
        IntSelectionSort.sort(a);
        clock.stop();
        break;
    case 1:
        clock.start();
        MyMergeSort.sort(a);
        clock.stop();
        break;
    case 2:
        clock.start();
        MergeSort.sort(a);
```

SortTest

A clock utility class

StopWatch

```
public void start()
{
    if (isRunning) return;
    isRunning = true;
    startTime = System.nanoTime();
}

public void stop()
{
    if (!isRunning) return;
    isRunning = false;
    long endTime = System.nanoTime();
    elapsedTime = elapsedTime + endTime - startTime;
}
```