CS3383 Unit 0, Lecture 1: Deeper into Asymptotics

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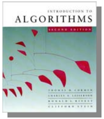
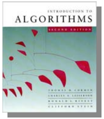
Asymptotics

- big-Theta = big-O *and* big-Omega
- Root vs. Log
- An example we didn't get to
- Introducing little-o and little-omega
- Coming back to our example



Θ-notation (tight bounds)

$$\Theta(g(n)) = O(g(n)) \cap \Omega(g(n))$$

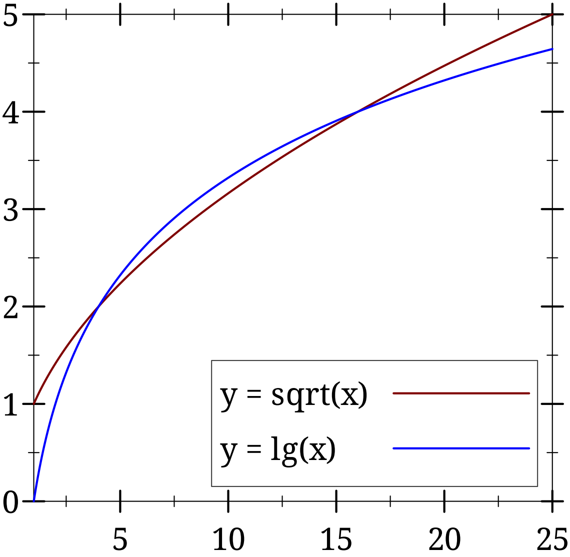


Θ-notation (tight bounds)

$$\Theta(g(n)) = O(g(n)) \cap \Omega(g(n))$$

EXAMPLE: $\frac{1}{2}n^2 - 2n = \Theta(n^2)$

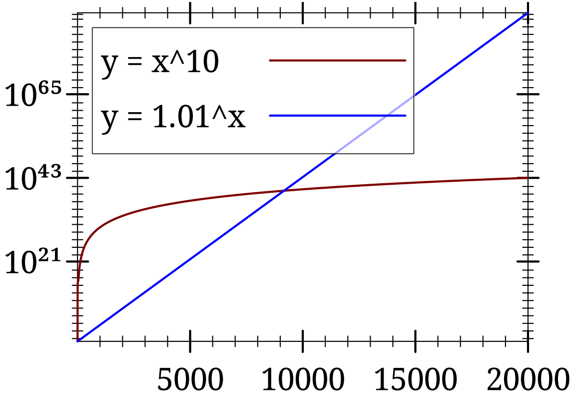
Root vs lg, revisited



- note there are 2 crossing points

Exponential versus Polynomial

$$(1.01)^n \in \Omega(n^{10})$$



CLRS3.13

$$\lim_{n\to\infty} \frac{n^b}{a^n} = 0 \quad \forall a > 1$$

- How to prove?
- How does it help?

Strengthening big-O

Definition ($f(n) \in O(g(n))$)

$$\exists c \exists n_0 \quad \forall n > n_0 \quad 0 \leq f(n) \leq cg(n)$$

Definition ($f(n) \in o(g(n))$)

$$\forall c > 0 \; \exists n_0 \quad \forall n > n_0 \quad 0 \leq f(n) < cg(n)$$

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example

$f \in o(g) \implies f \in O(g)$, but not vice-versa.

$$2n^2 \in o(n^3)$$
$$2n^2 \in O(n^2)$$
$$2n^2 \notin o(n^2)$$

Strengthening big-Omega

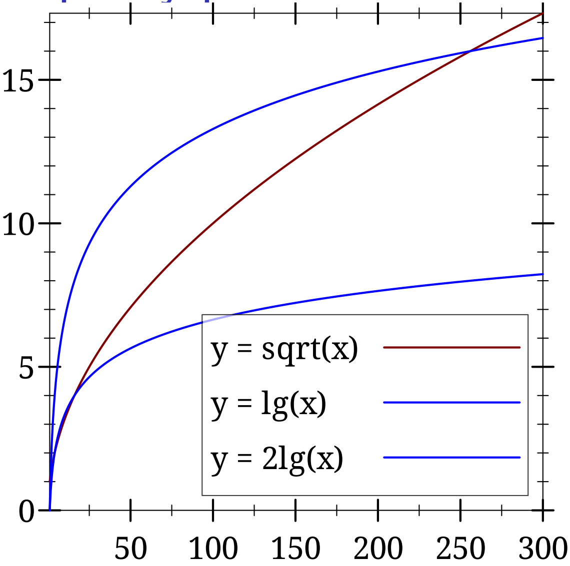
Definition ($f(n) \in \Omega(g(n))$)

$$\exists c \exists n_0 \quad \forall n > n_0 \quad 0 \leq cg(n) \leq f(n)$$

Definition ($f(n) \in \omega(g(n))$)

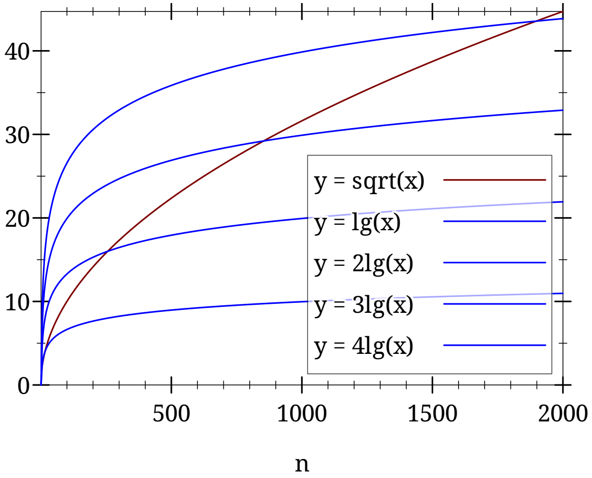
$$\forall c > 0 \; \exists n_0 \quad \forall n > n_0 \quad 0 \leq cg(n) < f(n)$$

Ooh a pretty pattern



- Want: $\sqrt{n} \in \omega(\lg(n))$
- For $c = 1$, $n_0 = 16$ (last class)
- For $c = 2$, $n_0 = 256$

The pattern is a lie



- For $c = 3$, $n_0 \approx 853.25$
- For $c = 4$, $n_0 \approx 1897.41$.
- The exact solution, is not *nice*

Sometimes calculus is the easy way...

Definition ($f(n) \in \omega(g(n))$)

$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$

A useful rule (CLRS3.13)

For any b, and any $a > 1$

$$\lim_{n\to\infty} \frac{a^n}{n^b} = \infty$$